Line Symmetry

MathLinks 9, pages 6-15

Suggested Timing

80–100 minutes

Materials

- scissors
- isometric dot paper
- tracing paper
- grid paper
- ruler
- coloured pencils

Blackline Masters

Master 2 Communication Peer Evaluation Master 7 Isometric Dot Paper Master 8 Centimetre Grid Paper BLM 1–4 Chapter 1 Warm-Up BLM 1–6 Section 1.1 Example 1 BLM 1–7 Section 1.1 Extra Practice BLM 1–8 Section 1.1 Math Link

Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Math and Estimation (ME)
- Problem Solving (PS)
- 🖌 Reasoning (R)
- Technology (T)
- Visualization (V)

Specific Outcomes

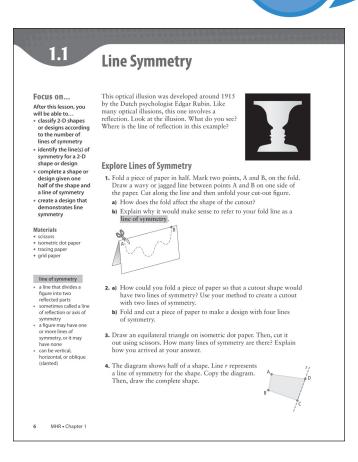
SS5 Demonstrate an understanding of line and rotation symmetry.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2, 5–7, 9, 12, 15, Math Link
Typical	#1, 2, 5, 7, 9, 10, 12–15, 16, 18, Math Link
Extension/Enrichment	#1, 2, 20–23, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 1–4 Chapter 1 Warm-Up to** reinforce material learned in previous grades.

Remind students that many illusions contain reflections. Reflections can be helpful in quickly identifying shapes. If the brain believes a reflection is present, we may see one half and erroneously assume the



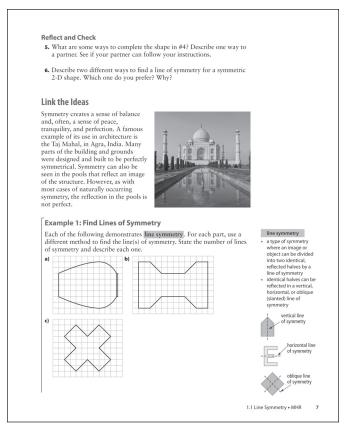
other half is the same. Or, if the other half is identical we may miss an embedded shape in our haste to complete the symmetry. In this sense, reflections and symmetry may be both helpful and harmful to our visual perception of a shape, design, or object.

Students may wish to use **Master 7 Isometric Dot Paper** and **Master 8 Centimetre Grid Paper** as they work through the section.

Explore Lines of Symmetry

In this exploration, students begin to examine symmetry using paper folding. This will establish a mental image that students can use throughout their study of line or reflection symmetry.

Method 1 Divide the class into pairs. As they complete #1 and 2, have them explore paper folding as a way to create shapes with different numbers of lines of symmetry. After some time, have a class discussion to ensure that all students found an answer to #2b). Then, have students complete the remaining questions #3 to 6.



Method 2 Divide the class into two groups. Have each group work through the exploration together. Have each group prepare a summary on flipchart paper of the Reflect and Check. Hang each group's summary on the wall. Conclude with a class discussion identifying the strengths and weaknesses of each summary.

As students work on the Explore, you may wish to ask questions such as:

- Do you think there is a pattern in folding a piece of paper to create lines of symmetry?
- How does where you end affect the lines of symmetry?
- What could you measure to draw an equilateral triangle?
- How could you construct an equilateral triangle on plain paper using a compass and straight edge?
- What is the measure of the angles in an equilateral triangle? How can this help you draw an equilateral triangle?
- What is the lowest number of points you must reflect in order to complete the shape in #4?

Discuss the alternative approach of drawing an equilateral triangle on isometric dot paper. Isometric dot paper considers either vertical or horizontal and diagonal distances to be equal depending upon the orientation of the paper. If you join two horizontal dots to the nearest dot above the ones you joined, you create a small equilateral triangle. To get a larger triangle, extend the horizontal distance to two or more units.

Meeting Student Needs

- Encourage students to see lines of symmetry by continuing to draw shapes on paper and then folding the paper to create lines of symmetry.
- Encourage students to use manipulatives such as rulers, MiraTM, and mirrors.
- Students may benefit from using flat mirrors, to see what a reflection will look like before completing a drawing using symmetry.
- Students may benefit from working through the Explore as a whole-class activity.

ELL

- Teach the following terms in context: *classify*, *optical illusion*, *reflection*, *cut-out*, *lines of symmetry*, *copy*, *vertical*, *horizontal*, and *oblique*.
- As you read through the instructions, model each step. This will help students to understand what to do and to learn the words in context.

Gifted and Enrichment

• Challenge students by asking them to create a unique optical illusion that uses symmetry to create its effect.

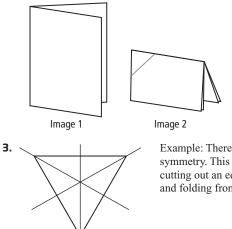
Web Link

For more examples of symmetry in art, go to www.mathlinks9.ca and follow the links.

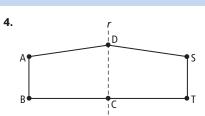
Answers

Explore Lines of Symmetry

- 1. a) The shape is symmetrical about the fold.b) Example: The fold is a line of symmetry because one side is a mirror-image (or flip) of the opposite side.
- **2.** a) Fold the paper in half, then turn the folded paper and fold again.b) Example:

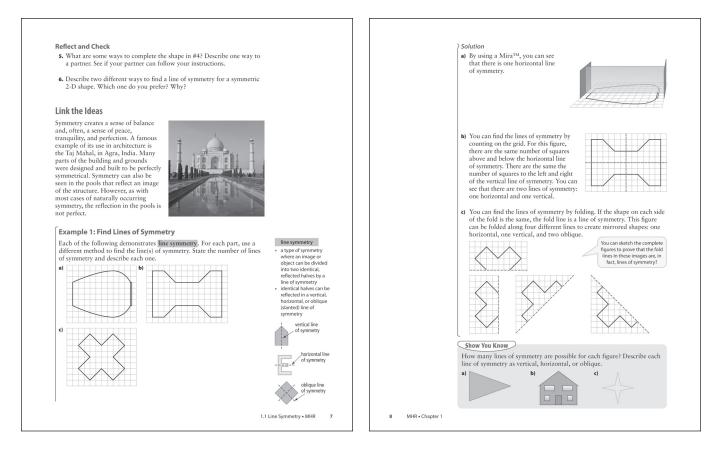


Example: There are 3 lines of symmetry. This can be shown by cutting out an equilateral triangle and folding from the vertices.



- **5.** Example: Each point can be reflected over the line of reflection by counting the distance to the line, or the shape can be cut out, flipped and traced.
- **6.** Example: Lines of symmetry may be found by counting distances of points on a grid, folding a cut-out shape, or using a mirror.

Assessment	Supporting Learning
Assessment as Learning	
Reflect and Check Listen as students discuss questions #5 and 6. Encourage students to use the correct terminology as they discuss appropriate methods. Suggest alternate wordings.	 Encourage students to both speak and write the words needed to describe the different ways to complete a shape or to find a line of symmetry. As much as possible, explanations should include sketches and diagrams. Students should be comfortable with the mathematical vocabulary associated with symmetry. Encourage students to enter the Key Words and additional vocabulary in their Foldable or notebook with terms and examples expressed in pictures, words, and symbols.



Link the Ideas

Draw students' attention to the definition of *line symmetry* on page 7. Ensure that they understand the difference between *line symmetry* and *line of symmetry* (defined on page 6).

Discuss with students the line symmetry found in the Taj Mahal. Students may wish to learn more about the Taj Mahal. Refer them to the related Web Link that follows.

Example 1

Encourage students to try Example 1 before looking at the solution. Success here may encourage some students who have traditionally struggled in mathematics to reason and think their way through the material. This is a great way to begin the year as it does not require much computational skill. Each part of Example 1 has an increasing number of lines of symmetry, so you may wish to assign part c) to students who have the greatest understanding of the concepts. You might find it helpful to concentrate on one diagram at a time. For example, for part a), you might ask questions such as:

• On the vase, what do you notice about the two sides of the vase?

- If the top of the vase must reflect on itself, what happens to the endpoints?
- What point on the top of the vase does not change position after reflection?
- What point on the bottom of the vase must reflect onto itself?
- Where must points that reflect onto themselves lie?

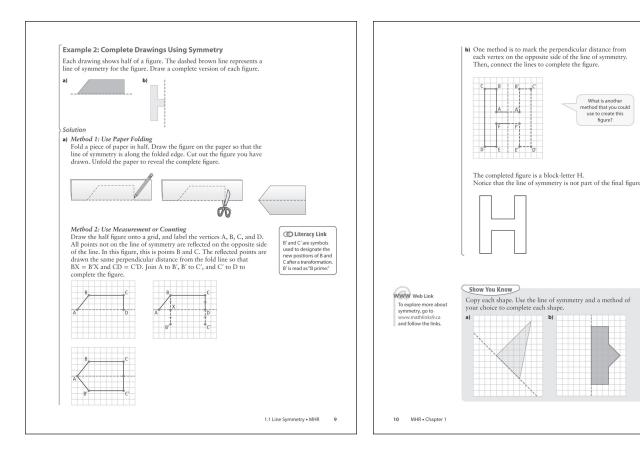
If you have the students work through the example on their own, you may wish to provide them with **BLM 1–6 Section 1.1 Example 1**.

Have students individually do the questions in Show You Know for formative assessment to determine if students clearly understand the concepts related to line symmetry.

Example 2

Before students consider either of the methods shown in Example 2, have them tell what they know about the halves of a reflected shape. You may wish to ask questions such as:

- What do you notice about the two halves of an object with line symmetry?
- What name is given to the line that divides a symmetric shape in half?



- How can you show if a shape is symmetric?
- What can you say about a point on a figure and its image on the other half if the figure has line symmetry?

Have students assess why the methods in Example 2 work. Students should also discuss which method works best under certain circumstances and which method they prefer for simple figures. Discuss with students why using a MiraTM is another option.

You may wish to use the following paper-folding directions as an alternative method for solving Example 2, or have students create their own paper-folding methods.

- Copy figure a) onto a piece of paper, so that the long edge is in the middle of the paper. Cut around the other three sides of the figure.
- **2.** Fold the flap over the line of symmetry and trace around it.
- **3.** Unfold the paper to reveal the complete figure.

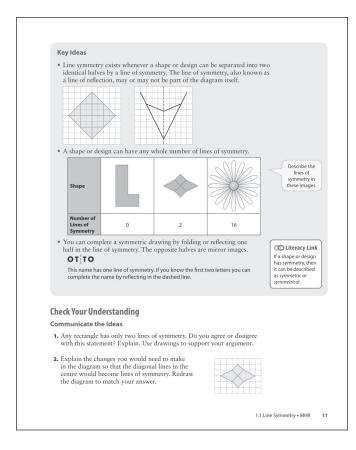
Have students individually select and complete one of the figures in Show You Know. Encourage students to state why they selected the figure they did. Some students will choose part b) since it has a vertical line of symmetry. Others will select part a) because it is a simpler figure, requiring the placement of only one image point. Try to discourage students from selecting part a) simply because it is the first question. It is better for their learning, and their understanding of their own learning, to make their own informed choices.

Key Ideas

The Key Ideas provide a detailed summary of what is most important in section 1.1. Have students prepare their own summary of the Key Ideas and record them in their Foldable. Encourage students to use different examples from those in the student resource. Students should be discouraged from copying the Key Ideas from either the student resource or another student. They should limit their summary to what is important to them, but it should also contain information of which they are less certain.



To learn more about symmetry and the Taj Mahal, go to www. mathlinks9.ca and follow the links.



Meeting Student Needs

- Have students identify familiar shapes and objects that exhibit line symmetry. For example, students might identify a beaded object such as a traditional Inuit parka. Have them demonstrate the symmetry in the design.
- Encourage students to make choices. They could be given a choice of which part of an example or Show You Know that they wish to work on. Have students exchange their work with a partner's to check each other's interpretations and/or reproductions.
- Students may benefit from working through the examples as a whole-class activity.
- Students may benefit from working through the Show You Know questions with a partner or in small groups.

ELL

- Teach the following words in context: *perpendicular* and *reflect*.
- Explain what it means to "Use the line of symmetry and a method of your choice to complete each shape." Remind students of the different ways to find the lines of symmetry (using a MiraTM, folding, etc.).

Common Errors

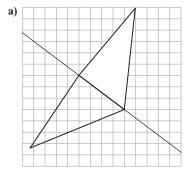
- Some students may always use folding to determine line symmetry or to determine the missing half of a symmetric figure.
- R_x Introduce more complex shapes and 3-D objects in an attempt to discourage single-method dependence on the part of students. Have students identify the strengths and weaknesses of the various methods used.
- Some students may generate incorrect image points when the line of symmetry is oblique.
- R_x Remind students that a point and its image are on opposite sides of the line of symmetry and that each one is the same perpendicular distance from the line of symmetry.

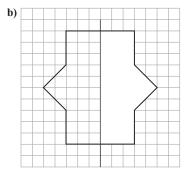
Answers

Example 1: Show You Know

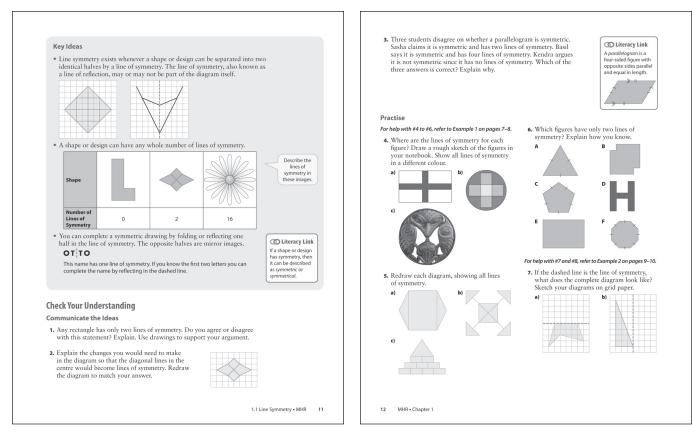
- a) 1; horizontal
- **b)** 1; vertical
- c) 4; 1 horizontal, 1 vertical, and 2 obliques

Example 2: Show You Know





Assessment	Supporting Learning
Assessment for Learning	
Example 1 Have students do the Show You Know related to Example 1.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Have students take a greater interest in the mathematics by encouraging them to use their own drawings and examples of symmetric shapes/objects. Introduce figures that are interesting and unusual, and that exhibit line symmetry. If students are not having difficulty, you may wish to use a complex drawing in which only a part of the shape has line symmetry. For example, a clock face may have symmetry but the numbers on it do not. Make sufficient resources available and encourage students to cut out shapes and physically fold papers to see the line symmetry. This will assist the visual and kinesthetic learners.
Example 2 Have students do the Show You Know related to Example 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Have students create questions for each other. Ensure that the lines of symmetry students work with vary from vertical to horizontal to oblique and that the placement of the completed half varies. For example, if the line symmetry is horizontal, the completed half of the object should sometimes appear above the line of symmetry and sometimes below it. This will ensure a deeper understanding of symmetry. For students having difficulty with the shapes presented, challenge them to draw another letter of the alphabet that has either vertical or horizontal line symmetry.



Check Your Understanding

Communicate the Ideas

Have students work individually or in small groups to answer the questions. In #1, students are asked to confirm that a rectangle has only two lines of symmetry. For #2, students are asked to modify a figure so that diagonal lines in the centre become lines of symmetry. In #3, students must decide which of three students has given the correct number of lines of symmetry for a particular parallelogram. This question uses the understandings of line symmetry from the first two questions.

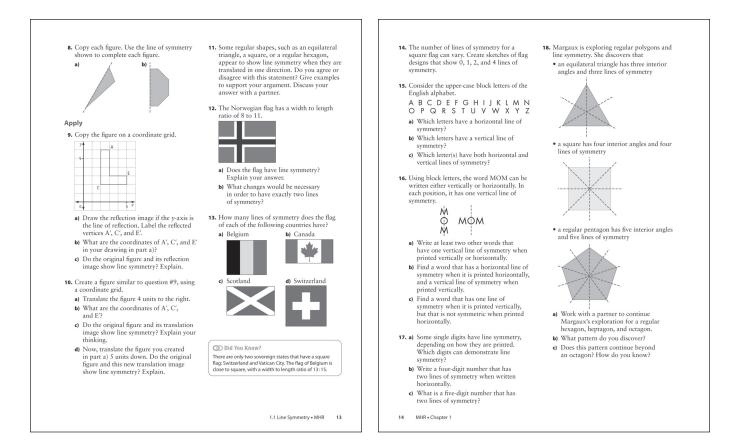
Literacy Link Direct students' attention to the Literacy Link box on page 12, which will assist students as they complete #3. The diagram will help them to visualize what is being asked. It may benefit some students to create a drawing of a parallelogram as they work on the question.

Practise

Students may be given a choice within each set of questions (#4 or 5, #7 or 8) since each pair requires similar understanding and skills. However, students should read both questions in the pair carefully to familiarize themselves with alternative ways of wording the questions. Students could work individually or in pairs on the Practise questions.

Apply

Students could work individually or in pairs on the Apply questions. In #9 to 11, students begin to relate symmetry to motion geometry. In #12 to 17 and 19, the mathematics of symmetry is related to applications that students should be familiar with. In #18, students see the importance of patterning in finding lines of symmetry for regular polygons. This question reinforces the importance of patterns in mathematics, in general. In #19, students are encouraged to determine whether an object or figure has true or perfect symmetry, or partial symmetry (some aspect is symmetric).



Extend

The Extend questions introduce additional mathematics and thoughts related to line symmetry. Questions 21 to 23 extend symmetry and motion geometry by having students identify single transformations that could be used to replace multiple reflections. In #24, students work through a question to do with the symmetry of 3-D objects.

Literacy Link It may be helpful for students to refer to the Literacy Link box on page 15 before they begin working on #24. Make sure students understand what it means for a 3-D figure to be cut by a plane.

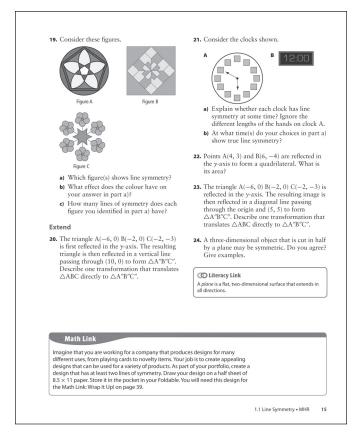
At the end of section 1.1, have students complete the right circle of the thematic map. Brainstorm and discuss as a class the information needed to complete the rectangle boxes and accompanying definitions.

Math Link

This Math Link gives students an opportunity to apply their understanding of line symmetry by creating a design to be used as part of the Math Link: Wrap It Up! The design should be applicable to a variety of products and have at least two lines of symmetry. Students may make their designs as complex as they wish.

Web Link

To find more examples of symmetry in mandalas, Inuit and Aboriginal art, and First Nations flags, go to www.mathlinks9.ca and follow the links.



Meeting Student Needs

- It may be more effective for some students to complete the Practise for Example 1 immediately after working on that Example. Follow the same procedure for Example 2. Then, have students complete the Apply activities.
- · Continue to encourage all students to use folding as a technique for confirming the location of lines of symmetry.
- Provide BLM 1–7 Section 1.1 Extra Practice to students who would benefit from more practice.

ELL

- Allow English language learners to provide their answers using their own language. Then, have them share their ideas in English.
- It may be better not to assign Communicate the Ideas questions to very new English language learners.
- For #4, ensure by modelling an example on the board that students know what it means to draw a rough sketch.
- For #16, new English language learners might have difficulty thinking of English words that fit this description. Consider providing six words and have students identify the words that the description would match.

• For #18, ensure that English language learners understand what interior angles means. Teach this word in context by pointing to the interior angle in the shape. To find out if new English language learners know the English name of each shape, point to the shape and ask them what it is called. If students do not know the name, take this opportunity to point to the shape and say the name. Then, have the student repeat the name. Make sure that students also understand how many sides a hexagon, heptagon, and octagon have.

Common Errors

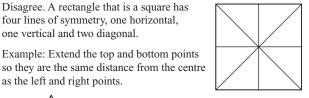
- Some students may be confused by colour when checking for line symmetry.
- $\mathbf{R}_{\mathbf{x}}$ Have students begin with only black-and-white diagrams, and include colour at a later time. Encourage students to approach questions in three parts:
 - 1. Check only shape or design.
 - 2. Check colour and shading.
 - 3. Examine for symmetry by considering both 1 and 2.
- Some students may believe that diagonals in rectangles and parallelograms are lines of symmetry for those shapes.
- $\mathbf{R}_{\mathbf{x}}$ Have students draw, cut out, and fold rectangles and parallelograms so they can see that diagonals are not lines of symmetry. Then, ask students to reflect points manually over diagonals using perpendicular distances to mathematically confirm what paper folding told them.

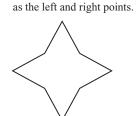
Answers

Communicate the Ideas

1. Disagree. A rectangle that is a square has four lines of symmetry, one horizontal, one vertical and two diagonal.

2. Example: Extend the top and bottom points





3. Kendra is correct. If the parallelogram is cut out, it does not fold to overlap itself. Therefore, there are no lines of symmetry.

Assessment	Supporting Learning
Assessment as Learning	
Communicate the Ideas Have all students complete #1 and 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. For both questions, encourage students to copy or draw shapes and to confirm lines of symmetry by folding. Alternatively, provide students with two congruent rectangles in two different colours, have them cut the rectangles in half diagonally and then place one diagonal cut of one colour over the other colour. The visual contrast in colour will identify that diagonals are not lines of symmetry in a rectangle. Remind students that, mathematically, all points reflected over a line of symmetry are equal perpendicular distances on opposite sides of the line of symmetry or line of reflection. Check student responses for understanding. Make sure they can explain why diagonals are not lines of symmetry in a rectangle. You may wish to have students use Master 2 Communication Peer Evaluation to assess each other's responses to one or more of the questions.
Assessment for Learning	
Practise and Apply Have students do #5–7, 9, 12, and 15. Students who have no problems with these questions can go on to the remaining Apply questions.	 Provide additional coaching with Examples 1 and 2 if students need help with #5 and 7. Coach struggling students through one part of #5 and then have them do selected questions from #4 to check for understanding. Follow the same procedure for #7 and 8. For #4, students will need access to coloured pencils. For #9, provide students with a grid on which they can draw the image. Encourage students to fold the grid to visually see the reflected image. An alternative approach is to provide small squares of tracing paper that students can copy the image onto, fold, and produce the reflected image. Students could use the same alternative strategies from #9 for #10; however, they should have a larger grid (10 by 10) on which to work. Again, encourage the use of tracing paper or a cutout of the geometric shape that could be used as a tracing master for transforming the shape. Students may benefit from copying the flag for #12 onto tracing paper and folding the paper to determine symmetry lines.
Math Link The Math Link on page 15 is intended to help students work toward the chapter problem wrap-up titled Math Link: Wrap It Up! on page 39.	 The design from this Math Link is to be included as part of the Math Link: Wrap It Up! on page 39. Students who need help getting started could use BLM 1–8 Section 1.1 Math Link, which provides scaffolding.
Assessment as Learning	
Literacy Link (page 3) Help students to recall the new terms introduced in this section by adding them to their map.	• Have students complete the right circle by adding the following terms: <i>line symmetry</i> and <i>line of symmetry</i> . Have students write their own definitions and create an example for each term.
 Math Learning Log Have students respond to the following prompts: What are the similarities and differences between the terminology associated with line symmetry and that of reflections? Draw separate figures showing zero, one, two, three, and four lines of symmetry. 	 Encourage students to use the What I Need To Work On section of their Foldable to note what they continue to have difficulties with. Encourage students to include pictures or diagrams when answering these questions. Have students look back through the work of section 1.1 and their Foldable for ideas to help answer the questions.