Surface Area

1.3

***MathLinks 9*, pages 26–35**

Suggested Timing
120–135 minutes

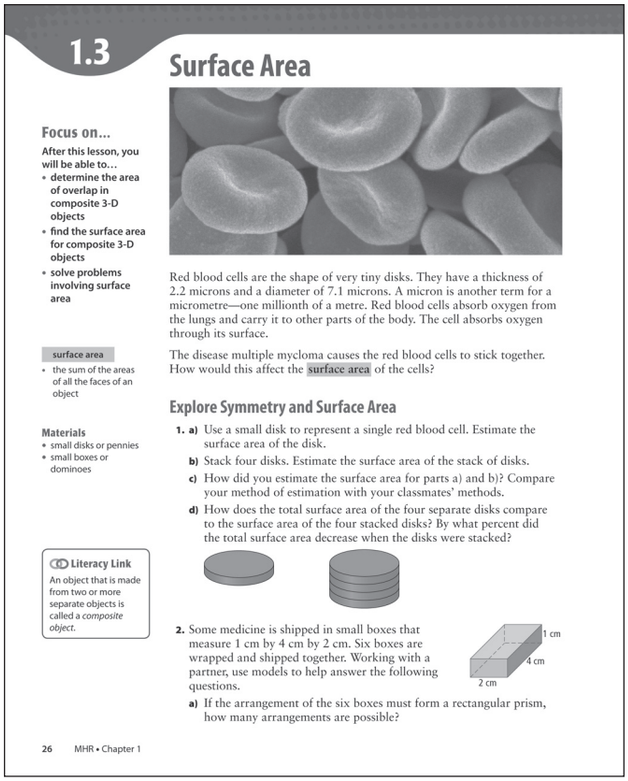
Materials
- small disks or pennies
- small boxes or dominoes
- centimetre cubes (optional)
- decks of playing cards, memo pads, business cards, and notepads (optional)

Blackline Masters
Master 2 Communication Peer Evaluation
Master 7 Isometric Dot Paper
Master 8 Centimetre Grid Paper
BLM 1–4 Chapter 1 Warm-Up
BLM 1–11 Section 1.3 Extra Practice
BLM 1–12 Section 1.3 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
SS2 Determine the surface area of composite 3-D objects to solve problems.
SS5 Demonstrate an understanding of line and rotation symmetry.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 4, 6, 8, 10, 12, 18, Math Link
Typical	#1–4, 6, 8, 10, 12, one of 13 or 14, one of 16–19, Math Link
Extension/Enrichment	#1–3, 14, 17, 20–23



1.3 Surface Area

Focus on…
After this lesson, you will be able to…
- determine the area of overlap in composite 3-D objects
- find the surface area for composite 3-D objects
- solve problems involving surface area

surface area
- the sum of the areas of all the faces of an object

Materials
- small disks or pennies
- small boxes or dominoes

Literacy Link
An object that is made from two or more separate objects is called a *composite object*.

Red blood cells are the shape of very tiny disks. They have a thickness of 2.2 microns and a diameter of 7.1 microns. A micron is another term for a micrometre—one millionth of a metre. Red blood cells absorb oxygen from the lungs and carry it to other parts of the body. The cell absorbs oxygen through its surface.

The disease multiple mycloma causes the red blood cells to stick together. How would this affect the surface area of the cells?

Explore Symmetry and Surface Area

1. a) Use a small disk to represent a single red blood cell. Estimate the surface area of the disk.
   b) Stack four disks. Estimate the surface area of the stack of disks.
   c) How did you estimate the surface area for parts a) and b)? Compare your method of estimation with your classmates' methods.
   d) How does the total surface area of the four separate disks compare to the surface area of the four stacked disks? By what percent did the total surface area decrease when the disks were stacked?

2. Some medicine is shipped in small boxes that measure 1 cm by 4 cm by 2 cm. Six boxes are wrapped and shipped together. Working with a partner, use models to help answer the following questions.
   a) If the arrangement of the six boxes must form a rectangular prism, how many arrangements are possible?

1 cm, 4 cm, 2 cm

26 MHR • Chapter 1

Planning Notes

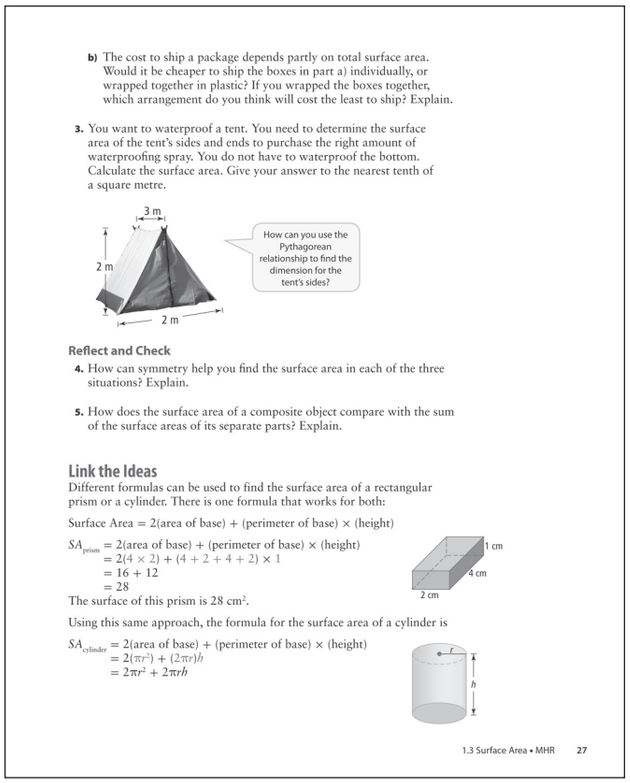
Have students complete the warm-up questions on **BLM 1–4 Chapter 1 Warm-Up** to reinforce material learned in previous grades.

Help students remember how to calculate surface area by discussing what they learned the previous year about the Pythagorean relationship.

Students may wish to use **Master 7 Isometric Dot Paper and Master 8 Centimetre Grid Paper** as they work through the section.

Explore Symmetry and Surface Area

In this Explore, students compare the surface areas of individual cylinders and prisms to stacks of solids. Students should consider how much the surface area changes when objects are stacked and how symmetry can help simplify surface-area calculations.



b) The cost to ship a package depends partly on total surface area. Would it be cheaper to ship the boxes in part a) individually, or wrapped together in plastic? If you wrapped the boxes together, which arrangement do you think will cost the least to ship? Explain.

3. You want to waterproof a tent. You need to determine the surface area of the tent's sides and ends to purchase the right amount of waterproofing spray. You do not have to waterproof the bottom. Calculate the surface area. Give your answer to the nearest tenth of a square metre.

Reflect and Check

4. How can symmetry help you find the surface area in each of the three situations? Explain.

5. How does the surface area of a composite object compare with the sum of the surface areas of its separate parts? Explain.

Link the Ideas

Different formulas can be used to find the surface area of a rectangular prism or a cylinder. There is one formula that works for both:

Surface Area = 2(area of base) + (perimeter of base) × (height)

SA_{prism} = 2(area of base) + (perimeter of base) × (height)
= 2(4 × 2) + (4 + 2 + 4 + 2) × 1
= 16 + 12
= 28

The surface of this prism is 28 cm².

Using this same approach, the formula for the surface area of a cylinder is

$SA_{cylinder}$ = 2(area of base) + (perimeter of base) × (height)
= $2(\pi r^2) + (2\pi r)h$
= $2\pi r^2 + 2\pi rh$

Method 1 Complete #1 as a class, seeking responses from students. Then, have students work in small groups to complete #2 to 5. Have each small group present their findings relating to #4 and 5.

Method 2 Have students work in pairs. Give each pair the materials needed to model the situations. Circulate as students work on the Explore. To check for understanding, you may wish to ask questions such as:

- Why do you think the sticking of blood cells together could be a problem?
- What is your estimate of the dimensions of your disk/penny?
- How do you find the area of a circle?
- In estimating the area of a circle, what is a good value to use for π?
- How does the total surface area change when you put two disks together? By how much does it change? Do you think this pattern will continue?

Literacy Link You may wish to direct students' attention to the Literacy Link on page 26, which describes a composite figure. For many students, this will be the first time they come across this term, though they may have worked with composite figures before.

Meeting Student Needs

- Students may benefit from working through the Explore and examples as a whole-class activity. As an alternative, consider having students discuss their work with a partner, then in small groups, and then as a whole class.
- In the Explore, each student should have models for small cylinders and rectangular prisms.

ELL

- Teach the word *area* in context by having students put their hands on the area of their desktop. Have them identify *perimeter* by directing them to use their finger to outline the outside of their desktop.
- Explain that *sum* means to add.
- Teach the word *stack* in context by modelling stacking pennies. Explain that to stack things means to put them one on top of the other.
- Rephrase *shipped*: mailed, sent to, goes to.
- Clarify what each question is asking. For example, students may be able to explain all the possible measurements of the boxes into a rectangle; however, they might not understand that this is what is being asked in #2a).
- For #3, explain what a tent is used for and what waterproofing a tent means.

Gifted and Enrichment

- Encourage students to create a method of finding the surface area of irregular shapes. Challenge students to calculate the amount of gold needed to plate small items, such as earrings or rings, and larger items, such as vases.

Answers

Explore Symmetry and Surface Area

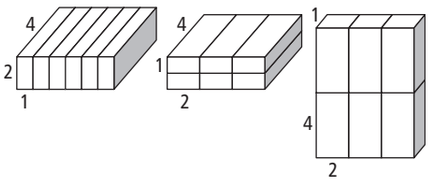
1. a) Example: 144 square microns

b) Example: 288 square microns

c) Example: round π to 3, round radius to 4, round thickness to 2

d) The surface area of the stacked disks is less than the surface area for four separate disks. The difference is approximately 50%.

2. a) There are 12 possible arrangements. Examples:



b) Combining the boxes together in a rectangular prism reduces the surface area, making it cheaper to ship. The smallest surface area possible is 80 cm^2.

3. 17.2 m^2.

4. Example: Knowing that parts of an object are symmetrical means you only have to calculate the surface area once, then double it for the symmetrical area. For example, because the tent is symmetrical, the formula for surface area of a triangle needs to be used only once, then doubled for the front and back surface area.

5. Example: A composite object has a smaller surface area than the sum of the surface areas of each separate part. This happens because when two objects are grouped together (stacked, or joined in some way) the area where the objects touch is no longer included in surface area.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Encourage students to generalize their findings. Check responses for conceptual understandings.	• Help students explore the concept that symmetry implies "sameness" and that opposite sides of prisms and cylinders have the same size and dimensions. Holding a class discussion for #5 should benefit students by allowing them to compare results and to compare their thinking.

b) The cost to ship a package depends partly on total surface area. Would it be cheaper to ship the boxes in part a) individually, or wrapped together in plastic? If you wrapped the boxes together, which arrangement do you think will cost the least to ship? Explain.

3. You want to waterproof a tent. You need to determine the surface area of the tent's sides and ends to purchase the right amount of waterproofing spray. You do not have to waterproof the bottom. Calculate the surface area. Give your answer to the nearest tenth of a square metre.

How can you use the Pythagorean relationship to find the dimension for the tent's sides?

Reflect and Check

4. How can symmetry help you find the surface area in each of the three situations? Explain.

5. How does the surface area of a composite object compare with the sum of the surface areas of its separate parts? Explain.

Link the Ideas

Different formulas can be used to find the surface area of a rectangular prism or a cylinder. There is one formula that works for both:

Surface Area = 2(area of base) + (perimeter of base) × (height)

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= 2(4 × 2) + (4 + 2 + 4 + 2) × 1
= 16 + 12
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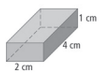
The surface of this prism is 28 cm².



Using this same approach, the formula for the surface area of a cylinder is

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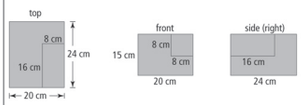
Example 1: Calculating Surface Area of a Solid

Consider the solid shown, in which all angles are right angles.



a) What are the dimensions of the cutout piece?
b) What is the total surface area of the solid?

Solution

a) The cutout notch is a right rectangular prism. The dimensions of the notch are 8 cm by 8 cm by 16 cm.

b) *Method 1: Find the Surface Area of Each Face*
You need to find the area of nine faces, including the faces of the notch. Number the faces to help keep track of the faces you have completed. Let the left face be #7, the back #8, and the bottom #9.

Why do you subtract 8 × 8 in the calculation for face 1?

Face	Calculation	Surface Area (cm²)
1	15 × 20 − (8 × 8)	236
2	20 × 24 − (8 × 16)	352
3	8 × 16	128
4	8 × 8	64
5	8 × 16	128
6	15 × 24 − (8 × 16)	232
7 (left side)	15 × 24	360
8 (back)	15 × 20	300
9 (bottom)	20 × 24	480
	Total Surface Area:	2280

The total surface area of the solid is 2280 cm².

Link the Ideas

Example 1

This example illustrates a connection to the views of rectangular solids studied in grade 8 by providing the dimensions on the views rather than on the representation of the solid. Reinforce that the surface area can be found several ways and that symmetry can help shorten the calculations, as shown in Method 2. You may wish to ask questions such as:

- What is the width/height/length of the notch? Which view gives each dimension?
- How does the surface area of the notch compare with that of the same portion of the solid without the notch?
- What different ways can you use to find the total surface area of the block?
- How can you use symmetry to shorten your calculation of the total surface area?

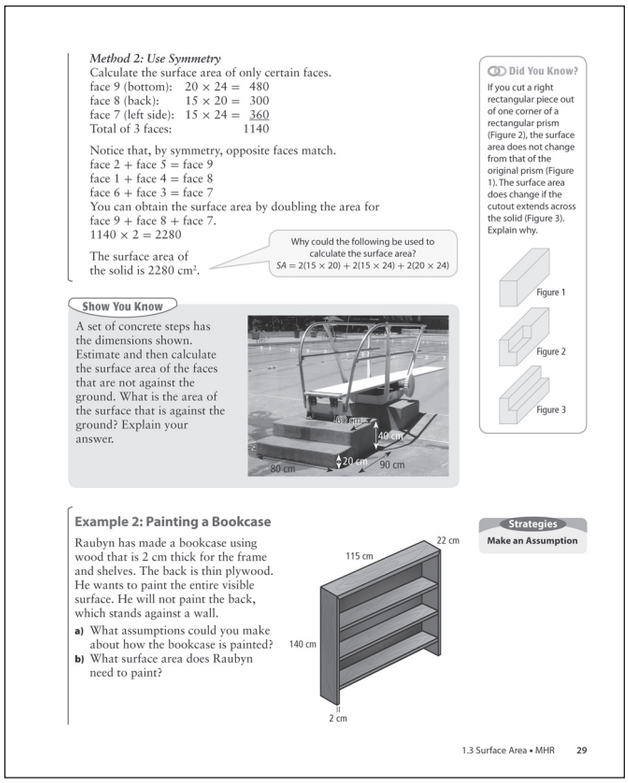
Draw students' attention to the information in the Did You Know? Ensure that all students understand why the surface areas of Figures 1 and 2 are the same while that of Figure 3 is somewhat less.

Example 2

This example uses a bookcase to have students calculate the surface area to be painted. Students should be encouraged not to paint hidden edges and to use symmetry to aid in their calculations. If possible, have one or two students build a small model of the bookcase. This would help all students to clearly see what is not painted if the bookcase is assembled prior to painting.

You may wish to discuss part a) with the class prior to students finding the surface area to be painted. Encourage students to check the thought bubbles as they work through the problem. The thought bubbles are intended to help students avoid common pitfalls and can be used as coaching questions.

Have students complete the Show You Know. After sufficient time for most students to try the questions, ask students to explain different ways of finding the surface area of the two triangular ends of the building.



Method 2: Use Symmetry
Calculate the surface area of only certain faces.
face 9 (bottom): $20 \times 24 = 480$
face 8 (back): $15 \times 20 = 300$
face 7 (left side): $15 \times 24 = 360$
Total of 3 faces: 1140

Notice that, by symmetry, opposite faces match.
face 2 + face 5 = face 9
face 1 + face 4 = face 8
face 6 + face 3 = face 7
You can obtain the surface area by doubling the area for face 9 + face 8 + face 7.
$1140 \times 2 = 2280$

The surface area of the solid is 2280 cm^2.

Why could the following be used to calculate the surface area?
$SA = 2(15 \times 20) + 2(15 \times 24) + 2(20 \times 24)$

Did You Know?
If you cut a right rectangular piece out of one corner of a rectangular prism (Figure 2), the surface area does not change from that of the original prism (Figure 1). The surface area does change if the cutout extends across the solid (Figure 3). Explain why.

Figure 1

Figure 2

Figure 3

Show You Know
A set of concrete steps has the dimensions shown. Estimate and then calculate the surface area of the faces that are not against the ground. What is the area of the surface that is against the ground? Explain your answer.

Example 2: Painting a Bookcase

Strategies
Make an Assumption

Raubyn has made a bookcase using wood that is 2 cm thick for the frame and shelves. The back is thin plywood. He wants to paint the entire visible surface. He will not paint the back, which stands against a wall.

a) What assumptions could you make about how the bookcase is painted?
b) What surface area does Raubyn need to paint?

Solution

a) Assumptions could include:
He paints the undersides of the three shelves.
The shelves are set inside the ends of the bookcase.
He paints the visible or inside back surface.
He does not paint the area of the base on which the bookcase stands.
Raubyn paints the bookcase after it is assembled.

Why would this assumption make a difference?

b) Group similar surfaces together.
Group 1: underside of top, and top and bottom of each of the three shelves.
Surface area $= 7 \times 111 \times 22$
$= 17\,094$

Why is this measurement 111 cm, rather than 115 cm?

Group 2: outside of top and sides.
Surface area $= 22 \times 115 + 2(22 \times 140)$
$= 8690$

Group 3: back of bookcase that shows inside and front edges of the three shelves.
Surface area $= 111 \times 138$
$= 15\,318$

Group 4: front edges of top and sides.
Surface area $= 2(2 \times 138) + 2 \times 115$
$= 782$

This measurement is 138, rather than 140, because the top piece is 2 cm thick. Notice that no surface area was subtracted to account for the back edges of the shelves, and none was added to account for the front edges. Using symmetry, explain why this works.

Total surface area:
$17\,084 + 8690 + 15\,318 + 782 = 41\,884$
The surface area Raubyn needs to paint is 41 884 cm^2.

WWW Web Link
For information on how to calculate the surface area of different shapes, go to www.mathlinks9.ca and follow the links.

Show You Know
Consider the building shown.
a) Estimate the outside surface area of the building.
b) Calculate the outside surface area. Determine your answer two different ways.
c) Which method do you prefer? Why?

Key Ideas

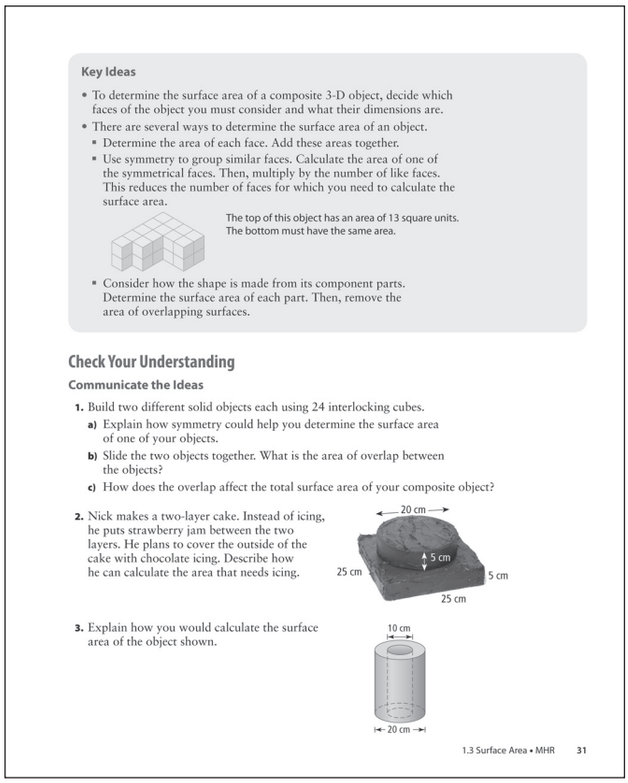
Have students make their own summary for section 1.3 prior to looking at the Key Ideas in the student resource. Then, have students compare their summary with other students and with the one in the student resource.

Meeting Student Needs

- Have some students continue to work with manipulatives, such as models of 3-D shapes, cans, or boxes, for as long as possible.
- Suggest that students create a chart when working on diagrams with multiple faces, such as the chart with Example 1. They can use headings such as Side Number, Shape, Equation, Substitution, and Surface Area. Students can add up their answers to determine the total surface area.
- Some students may find it easier to work from a net of the shape. This may assist your visual and kinesthetic learners.
- Encourage students to draw webs if they are having difficulty.
- It may be better for your class to be taught this part of section 1.3 over two class periods. Since many students have limited experience building or painting, it is helpful to have models for students to look at and to handle. You may wish to have one or two students make models for both Example 1 and Example 2.
- Students may benefit from working through the examples as a whole-class activity.
- Students may benefit from working through the Show You Know questions with a partner or in small groups.

ELL

- Teach the term *face* to English language learners. This meaning may be different from the meaning of face that they are familiar with.



Key Ideas

- To determine the surface area of a composite 3-D object, decide which faces of the object you must consider and what their dimensions are.
- There are several ways to determine the surface area of an object.
  - Determine the area of each face. Add these areas together.
  - Use symmetry to group similar faces. Calculate the area of one of the symmetrical faces. Then, multiply by the number of like faces. This reduces the number of faces for which you need to calculate the surface area.

The top of this object has an area of 13 square units. The bottom must have the same area.

  - Consider how the shape is made from its component parts. Determine the surface area of each part. Then, remove the area of overlapping surfaces.

Check Your Understanding

Communicate the Ideas

1. Build two different solid objects each using 24 interlocking cubes.
- **a)** Explain how symmetry could help you determine the surface area of one of your objects.
- **b)** Slide the two objects together. What is the area of overlap between the objects?
- **c)** How does the overlap affect the total surface area of your composite object?

2. Nick makes a two-layer cake. Instead of icing, he puts strawberry jam between the two layers. He plans to cover the outside of the cake with chocolate icing. Describe how he can calculate the area that needs icing.

3. Explain how you would calculate the surface area of the object shown.

Common Errors

- Some students may have difficulty determining which sides of a solid object to include in their calculations.

R_x Begin with small rectangular solids that can be picked up. For these objects, finding the total surface area would include all sides, unless otherwise stated in the questions. Next, ask students to find the exposed surface area for some immovable objects. In this case, one or more sides would be excluded from the calculations.

- Some students may believe that objects with notches always have smaller surface areas than those that do not.

R_x Use models of blocks like those shown in the Did You Know? on page 29. It is helpful if the cutouts are also available in order to show removed surfaces in relation to the original solids.

Answers

Example 1: Show You Know

Estimate: 1.6 m^2; Calculate: 1.64 m^2. The area of the surface against the ground is 7200 cm^2. The surface area can be calculated by considering a solid box, and then subtracting the surface area of missing pieces from each side.

Example 2: Show You Know

a) Estimate: 275 m^2

b) Method 1: Calculate individual surface areas for each outer face of the object and add for the total surface area. These include the front and back walls of the base of the building (bottom squares), side walls of the base of the building (bottom rectangles), front and back of the roof (upper triangles) and slopes of the roof (upper rectangles). The total surface area is 284 m^2.
Method 2: Calculate the surface area for the bottom (a rectangular prism) and the top (a triangular prism), and subtract the area where the two shapes overlap. Using this method, the surface area is also 284 m^2.

c) Example: Method 2 because it involves fewer calculations.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students draw the individual sides of the notched-out piece and label its dimensions. Calculate the area of these pieces and then of the remaining solid. Encourage students to make use of a table similar to the one shown in Method 1. • Have students explain which view relates to each side of the object and which dimension each one provides. • Have students indicate which method in Example 1 they prefer and why. • Encourage discussion about how symmetry can help in the calculation of surface area.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students of the need to identify their assumptions when calculating the surface area of complex objects. • Encourage students to always check whether sides are, or should be, excluded from the calculations involving surface area. Have them indicate any sides not included in their calculations, such as the bottom of large objects or sides that are tight together in composite objects. Encourage students to make use of a table similar to the one shown in Method 1 of Example 1.

Key Ideas

- To determine the surface area of a composite 3-D object, decide which faces of the object you must consider and what their dimensions are.
- There are several ways to determine the surface area of an object.
  - Determine the area of each face. Add these areas together.
  - Use symmetry to group similar faces. Calculate the area of one of the symmetrical faces. Then, multiply by the number of like faces. This reduces the number of faces for which you need to calculate the surface area.

The top of this object has an area of 13 square units. The bottom must have the same area.

  - Consider how the shape is made from its component parts. Determine the surface area of each part. Then, remove the area of overlapping surfaces.

Check Your Understanding

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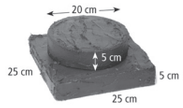


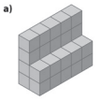
3. Explain how you would calculate the surface area of the object shown.

Practise

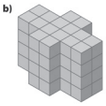
For help with #4 to #7, refer to Example 1 on pages 28–29.

4. Each object has been constructed from centimetre cubes. Estimate and then calculate the surface area.

a)

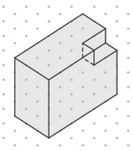


b)



5. The following objects have been drawn on isometric dot paper where the distance between dots is 2 cm. Determine the surface area of each object.

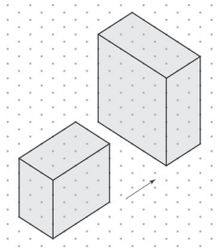
a)



b)

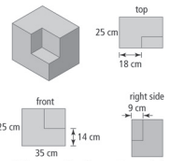
Note: The hole extends all the way through the block.

6. a) If you build the rectangular solids and slide them together as shown, what is the area of the overlap? Assume the dots are 1 cm apart.



b) What is the surface area when the solids are together?

7. Examine the solid and its views. All angles are right angles.



- **a)** What are the dimensions of the cutout piece?
- **b)** Explain how cutting out the corner piece will affect the surface area of the original rectangular solid.

Check Your Understanding

Communicate the Ideas

Have students work individually or in pairs to answer #1 and 2. Have a brief class discussion following the individual work time to draw out conceptual issues relating to common sides for composite objects. Then, you may wish to have students complete #3. Students may work in pairs to complete #3 if you do not use the question for assessment.

Practise

Students benefit from making choices, so you may wish to have students choose any two questions from #4 to 7. Of these, #7 is the most like Example 1. The others use different ways to show the dimensions on various solids. Students should then complete #8 or 9.

Apply

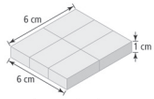
Suggested questions include #10, and 12 to15. You could assign these questions or give students some choice from among questions #10 to 12. Note that #10 involves finding the surface area for the different arrangements of ten small cubes. In #12, students find the surface area for a garage with a slanted roof. Note that #13 and 14 involve surface area for cylinders in different contexts, the first being drink mugs and, the second, chimneys. In #15, students find the surface area of a rectangular prism with a round hole in it.

Extend

Surface area is demonstrated through contexts in #20 to 23, from boiling grains of rice, to elephant ears, to birdbaths, and to swimming pools. Most questions require some thought or research outside the mathematics involved.

For help with #8 and #9, refer to Example 2 on pages 29–30.

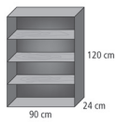
8. Six small boxes, all the same size, have been arranged as shown.



a) What are the dimensions of a single box?
b) What is the surface area for the arrangement of the six boxes?
c) What is the ratio of the answer in part b) to the total surface area of the six separate boxes?

Web Link
To see how surface area changes when a composite object is broken apart, go to www.mathlinks9.ca and follow the links.

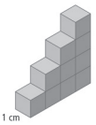
9. Examine the bookshelf. It is constructed of thin hardwood. The top, bottom, and all three shelves are the same size. There is an equal distance between the top, the shelves, and the base.



a) What is the surface area of one shelf? Include both sides, but ignore the edges.
b) What is the total surface area of the bookcase?
c) What is the fewest number of surfaces for which you need to find the surface area in order to answer part b)?

Apply

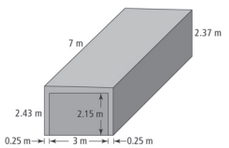
10. Use centimetre cubes to build the object shown.



a) What is the object's surface area?
b) Take the same ten cubes and build a rectangular prism. Estimate and then calculate whether the surface area remains the same. Explain with examples.

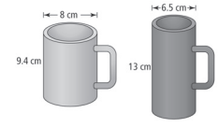
11. List places or situations in which surface area is important. Compare your list with those of your classmates.

12. Consider this drawing of a garage. The left side of the garage is attached to the house.



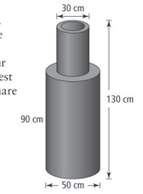
a) What is the difference in height between the left-hand and right-hand sides of the garage? Explain why you would want a slight slant to a roof?
b) Given that the house is attached to the left side of the garage, what is the surface area of the garage to the nearest hundredth of a square metre? What assumption(s) did you make in answering this question?

13. A mug for hot beverages is to be designed to keep its contents warm as long as possible. The rate at which the beverage cools depends on the surface area of the container. The larger the surface area of the mug, the quicker the liquid inside it will cool.

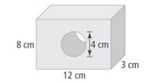


a) What is the surface area of each mug? Assume that neither has a lid.
b) Which is the better mug for keeping drinks warm? Justify your answer.

14. A chimney has the dimensions shown. What is the outside surface area of the chimney? Give your answer to the nearest hundredth of a square metre.

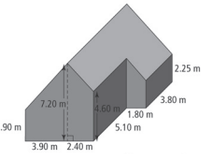


15. Twila made the object shown.



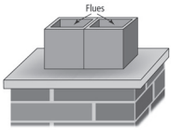
a) How can you use symmetry to help find the surface area of this object?
b) What is the surface area?

16. You are planning to put new shingles on the roof of the home shown.



a) How many times would you need to use the Pythagorean relationship in order to find the area of the roof of the building shown in the diagram?
b) What is the area of the roof that you cannot see in this figure, assuming that it is a rectangular roof?
c) One bundle of shingles covers approximately 2.88 m^2 and costs $26.95. What does it cost for shingles to cover the roof?

17. The hollow passages through which smoke and fumes escape in a chimney are called flues. Each flue shown is 2 cm thick, 20 cm high, and has a square opening that is 20 cm by 20 cm.



a) What are the outside dimensions of the two flues?
b) If the height of each flue is 30 cm, what is the outside surface area of the two flues? Hint: Do not forget the flat edges on top.

Literacy Link At the end of section 1.3, have students complete the bottom circle of the thematic map. Brainstorm and discuss as a class the information needed to complete the rectangle boxes and accompanying definitions.

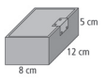
Math Link

This Math Link requires students to select objects such as playing cards, business cards, memo pads, and so on, and to find its dimensions and surface area. This Math Link is relatively short but forms the basis for further work in the Math Link: Wrap It Up! at the end of the chapter.

Meeting Student Needs

- Provide **BLM 1–11 Section 1.3 Extra Practice** to students who would benefit from more practice.
- Have a model showing the layer cake in #2. This helps students who are unfamiliar with layer cakes and also clearly shows the overlapping surfaces between layers.
- Consider having students work in pairs. They could work on one question together and then work individually on the next one. Ensure that each student works on a number of questions individually. Have them exchange their answers with a partner.
- Small models of the objects involved in the questions are helpful to give students a true visual image and to show overlapping sides when they are required to find the surface area of layers.

18. A small metal box is shown. What is the inside surface area of the box? What assumptions did you make in finding your answer?



19. A party planner buys two plain cakes for a meal she is planning. One cake is square and the other is round. Both cakes are 6 cm thick. The square cake measures 25 cm along each edge. The round cake has a diameter of 25 cm.

a) Sketch and label a diagram of each cake.

b) Show how to make four cuts to create eight equal pieces for each cake.

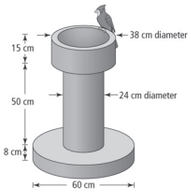
c) Estimate and then calculate how much the surface area increases after each cake is cut and the pieces are slightly separated.

Extend

20. Explain how surface area of individual grains of rice may affect the boiling of a cup of uncooked rice. Assume you have two kinds of rice. One has small grains and the other has larger ones. Consider each grain of rice to be cylindrical.

21. An elephant's ears are one of nature's best examples of the importance of surface area in heating and cooling. Research this phenomenon or another one that interests you. Write a brief report outlining the importance of surface area in heating and cooling. (Two other possible topics are why radiators have complex internal shapes and how a cactus minimizes surface area.)

22. The plan for a concrete birdbath is shown below. The bowl is a cylinder with a depth of 10 cm. If the bowl has a diameter of 30 cm, what is the exposed surface area of the birdbath, including the pillar and pedestal?



23. A swimming pool measures 25 m long and 10 m wide. It has a shallow end that is 1 m deep and gradually slopes down to a depth of 3 m at the deep end. The inside walls of the pool need repainting. Calculate the total area of the surfaces to be painted, to the nearest square metre.

Math Link

Your design company wants to create a new product that will have a design printed on it. Your project team has suggested playing cards, business cards, memo pads, and sticky notes. Choose one of these items.

a) What are the dimensions of your pack of cards or pad of paper?

b) What is the surface area of your pack of cards or pad of paper?

ELL

- For #1, ensure that students understand what is meant by *overlap*. Model a simple example. Make two solid shapes from four cubes, and then model the overlap and how it changes the total surface area of the new shape.

Common Errors

- Some students may struggle to identify overlapping sides in different situations.

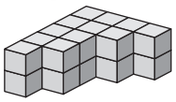
R_x Encourage students to identify the overlapping sides for a composite object before beginning to calculate the surface area. Then, have them sketch and label any overlapping surfaces. Check for assumptions and correctness in the overlaps.

Answers

Communicate the Ideas

1. a) Symmetry can be used to find the surface area of one face or side, and then can be doubled to account for the opposite, symmetrical side.

b) Example:



c) Example: the area of overlap reduces the total surface area by twice that amount.

2. Nick can first calculate the surface areas of each separate layer using the appropriate formulas and add them together. Then, he must subtract 2 times the area of overlap from the total. He should also subtract the bottom, as that usually is not iced.

3. Find the surface area of the total, large cylinder. Next, find the surface area of the smaller, cutout cylinder. Then, add these two surface areas together. From the total, subtract 2 times the area of one of the circles that form the end of the small cylinder.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1–3. Check responses for understanding.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Check each student's response to #1 to ensure understanding of the area of overlap when the objects are still together. • For #2 and 3, have students share as a class their strategies for the question and write them on the board. There are several approaches that can be used; therefore, students unsure of a starting point can choose whichever approach is logical for them. • For #2, one strategy is to find the visible surface area for the square layer and then add only the area of the side of the cylindrical layer. This way, they need not find the circular area.
Assessment *for* Learning	
Practise and Apply Have students do #4, 6, 8, 10, 12, and 18. Students who have no difficulty could go on to the remaining Apply questions.	• Provide additional coaching with Example 1 to students who need help with #4 and 6. Remind students to count the squares on specific sides of the object and then to use symmetry for opposite sides. The distances should be equal. Alternatively, they could copy the design and draw perpendicular lines of equal length from the object to the reflection line on opposite sides. Use parts or all of #5 and 7 to check for understanding. • You may wish to make cubes of some kind available to students to help them complete #4. • Provide additional coaching with Example 2 for those students struggling with #8 and then assign #9 to check for understanding. • Students having difficulty with #12 may benefit from working in a small group. Ask questions such as: – What are the missing dimensions for the front, back, right side, and top of the garage? How do you know? – Which side of the garage is higher? Why? How much higher? – How could you find the actual width of the roof? • For #18, students can use their knowledge of surface area and constructing shapes to answer the assumptions part of the question. It may also benefit the entire class to see some sample boxes that are exact sizes of the box in the question and some that are slightly smaller. The boxes will not fit into the other. One needs to be slightly larger for the other to slide inside.
Math Link The Math Link on page 35 is intended to help students work toward the chapter problem wrap-up titled Math Link: Wrap It Up! on page 39.	• Students who need help getting started could use **BLM 1–12 Section 1.3 Math Link**, which provides scaffolding. • Provide decks of playing cards, memo pads, business cards, and notepads so students can take necessary measurements for their surface-area calculations. • Remind students that you are measuring the contents of the box and not the box's dimensions. This will be calculated in the Math Link: Wrap It Up! • Allow students to use a variety of items that they could be making a box design for. If students are struggling with ideas and do not want to use the card idea, walk around the classroom with them and identify three additional possible items they could build a box for.
Assessment *as* Learning	
Literacy Link (page 3) Help students to recall the new terms introduced in this section by adding them to their map.	• Have students complete the bottom circle by adding terms from this section. Have students write their own definitions and create an example for each term.
Math Learning Log Have students complete the following prompts: • What are two methods for finding the surface area of a solid? • Compare finding the surface area of a rectangular prism and finding the surface area of a cylinder.	• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. • Depending upon students' learning styles, have them provide oral, written, or pictorial responses. • Encourage all learners to use models or draw diagrams to support their solution strategies. These could be included in their Foldable.