Comparing and Ordering Rational Numbers

2.1

***MathLinks 9*, pages 46–54**

Suggested Timing

80–100 minutes

Materials

- ruler

Blackline Masters

Master 2 Communication Peer Evaluation
Master 4 Number Lines
BLM 2–3 Chapter 2 Warm-Up
BLM 2–5 Section 2.1 Extra Practice
BLM 2–6 Section 2.1 Math Link

Mathematical Processes

- [x] Communication (C)
- [x] Connections (CN)
- [ ] Mental Math and Estimation (ME)
- [ ] Problem Solving (PS)
- [x] Reasoning (R)
- [ ] Technology (T)
- [x] Visualization (V)

Specific Outcomes

N3 Demonstrate an understanding of rational numbers by:
- comparing and ordering rational numbers
- solving problems that involve arithmetic operations on rational numbers.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6, 8, 10, 12, 14a), b), 16a), b), 18, Math Link
Typical	#1–4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 23, Math Link
Extension/Enrichment	#1–3, 20, 22, 25–30

Planning Notes

Have students complete the warm-up questions on **BLM 2–3 Chapter 2 Warm-Up** to reinforce material learned in previous sections.

Draw students' attention to the information in the Did You Know? on page 46. Also, point out the photographs on pages 46 and 47 and ask which one shows an urban area and which one shows a rural area.

2.1

Comparing and Ordering Rational Numbers

Focus on...

After this lesson, you will be able to...
- compare and order rational numbers
- identify a rational number between two given rational numbers

Cape Dorset, Nunavut

The percent of Canadians who live in rural areas has been decreasing since 1867. At that time, about 80% of Canadians lived in rural areas. Today, about 80% of Canadians live in urban areas, mostly in cities. The table shows changes in the percent of Canadians living in urban and rural areas over four decades.

Did You Know?

An urban area has a population of 1000 or more. In urban areas, 400 or more people live in each square kilometre. Areas that are not urban are called rural. What type of area do you live in?

Decade	Change in the Percent of Canadians in Urban Areas (%)	Change in the Percent of Canadians in Rural Areas (%)
1966–1976	+1.9	−1.9
1976–1986	+1.0	−1.0
1986–1996	+1.4	−1.4
1996–2006	+2.3	−2.3

How can you tell that some changes in the table are increases and others are decreases?

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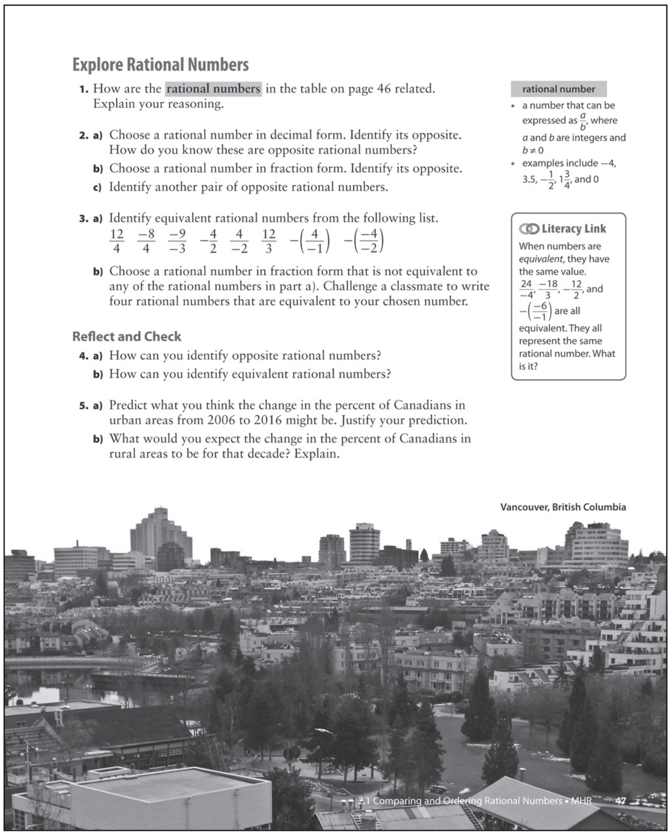
Have students use the information in the Did You Know? to decide if they live in an urban area or a rural area. If they are unsure, you may wish to provide them with the population and area of your community, so that they can determine if it meets the given criteria for an urban area. Then, point out the information in the first paragraph of the section. You may wish to discuss the significance of the year 1867 and what students know about Confederation.

Have students examine the table and discuss the question that follows it. If students have difficulty with the question, ask how the opening paragraph indicates which column in the table shows decreases and which column shows increases. You might also ask students how they have used signs to model increases and decreases with integers. After students have decided which numbers in the table represent decreases, point out that these numbers are negative decimals, which students are using for the first time. You might ask students if they have ever seen negative decimals before. They may have noticed that some temperature values are quoted in this way.


Explore Rational Numbers

1. How are the rational numbers in the table on page 46 related. Explain your reasoning.

2. a) Choose a rational number in decimal form. Identify its opposite. How do you know these are opposite rational numbers?
   b) Choose a rational number in fraction form. Identify its opposite.
   c) Identify another pair of opposite rational numbers.

3. a) Identify equivalent rational numbers from the following list.
   $\frac{12}{4}$ $\frac{-8}{4}$ $\frac{-9}{-3}$ $-\frac{4}{2}$ $\frac{4}{-2}$ $\frac{12}{3}$ $-\left(\frac{4}{-1}\right)$ $-\left(\frac{-4}{-2}\right)$
   b) Choose a rational number in fraction form that is not equivalent to any of the rational numbers in part a). Challenge a classmate to write four rational numbers that are equivalent to your chosen number.

rational number
- a number that can be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$
- examples include −4, 3.5, $-\frac{1}{2}$, $1\frac{3}{4}$, and 0

Literacy Link
When numbers are *equivalent*, they have the same value. $\frac{24}{-4}$, $\frac{-18}{3}$, $-\frac{12}{2}$, and $-\left(\frac{-6}{-1}\right)$ are all equivalent. They all represent the same rational number. What is it?

Reflect and Check

4. a) How can you identify opposite rational numbers?
   b) How can you identify equivalent rational numbers?

5. a) Predict what you think the change in the percent of Canadians in urban areas from 2006 to 2016 might be. Justify your prediction.
   b) What would you expect the change in the percent of Canadians in rural areas to be for that decade? Explain.

Vancouver, British Columbia

1.1 Comparing and Ordering Rational Numbers • MHR 47

You may wish to point out Apply #21 in this section as an example. Some students may be aware that negative decimals are used to represent decreases in share prices or stock market indices.

Explore Rational Numbers

In this exploration, students build on their knowledge of integers, positive decimals, and positive fractions to consider negative decimals and negative fractions for the first time.

Before students begin to answer the questions, draw their attention to the definition of a rational number. To encourage understanding of the definition, you might have a class discussion around the following:

- What type of number have you previously written in the form $\frac{a}{b}$? Give three examples.
- In your examples, are a and b both integers?
- Could you give examples in which b is 0? If not, explain why not.
- Do your examples qualify as rational numbers? Explain.
- Can the decimal 0.5 be expressed in the form $\frac{a}{b}$, where a and b are both integers?
- Is the decimal 0.5 a rational number?

You may also wish to reactivate students' understanding of opposites by asking:

- What are three examples of pairs of opposite integers?
- In a pair of opposite integers, how are the integers the same and how are they different?
- If you plotted a pair of opposite integers on a number line, how would their locations compare? (the same distance in opposite directions from 0)
- What is the sum of a pair of opposite integers? (Their sum is 0.)

Method 1 Have students work on the exploration in pairs or small groups and discuss their answers. Circulate to monitor progress. When appropriate, ask coaching questions to provide assistance or to encourage deeper understanding.

For #1, you might ask:

- Are all the percents in the table rational numbers?
- How are the percents in each row of the table the same? How are they different?

For #2, you might ask:

- Is any decimal number a rational number? Explain.
- How can you write the opposite of a rational number?
- Is any fraction a rational number? Explain.
- Can you use any integer value for the numerator? denominator? Explain.
- How could you use a number line to check that two rational numbers are opposites?

In #2a), students may choose a decimal number and its opposite from the table. If so, encourage them to suggest examples of their own. In #2b), students are likely to choose positive integers for the numerator and denominator, e.g., in $\frac{3}{4}$. Therefore, they can write its opposite, $-\frac{3}{4}$, by adding a negative sign. If students include negative integers in their chosen fractions, e.g., in $\frac{-2}{3}$, they may write $-\left(\frac{-2}{3}\right)$ as its opposite, rather than writing $\frac{2}{3}$. Both of these answers would be correct, but students may have difficulty in understanding the multiple negative signs until they have completed #3. In #2b), you may wish to guide students toward choosing positive integers for the numerator and denominator.

Literacy Link Before students begin work on #3, draw their attention to the Literacy Link on page 47 that deals with equivalent rational numbers. Check that students can answer the question in the Literacy Link. If they have difficulty, you might provide assistance by drawing on the first three of the following coaching questions supplied for #3.

Also, point out that a negative fraction can be rewritten with the negative sign assigned to either the numerator or the denominator. You might check student understanding by asking them to write each of the following in two other ways by moving the negative sign: $-\frac{1}{2}$, $-\frac{4}{5}$, $\frac{5}{-6}$. It may help students for you to point out that $-\left(\frac{-6}{-1}\right)$ is the same as $-1 \times \left(\frac{-6}{-1}\right)$.

For #3, you might ask:

- How are the values of equivalent fractions related?
- What is the value of $-(-1)$?
- For each rational number given in #3, what is its value expressed as an integer?
- How can you establish equivalence using the integer values you determined?
- How can you use multiplication to write equivalent fractions? Give an example.
- How can you use division to write equivalent fractions? Give an example.
- How can you make sure that your chosen rational number in part b) is not equivalent to any of the rational numbers given in part a)?

Encourage open discussion when students are completing #4 and 5, and encourage individuals to frame their responses in their own way. For example, in #4a), individuals may refer to differences in signs, to locations of points on a number line, or both.

Different students may reason in very different ways in #5a). For example, their approaches may include a ballpark estimate (e.g., about 2%) or an exact average of the four positive percents in the table. Encourage students to debate the merits of different approaches.

A student's answer to #5b) should be the opposite of that student's answer to #5a), but the depth of the explanation may vary widely. Some students may predict from the table that the two figures must be opposites, whereas others may go further by attempting to explain why the table includes opposites. The reason is that every person in Canada is a resident of either an urban area or a rural area. If students have difficulty with this concept, you might refer to the opening paragraph of the section and establish the approximate percents of Canadians living in urban and rural areas in 1867 (about 20% and 80%, respectively) and today (about 80% and 20%, respectively), and then compare the changes in the percent values for urban and rural areas.

Method 2 Have students answer the questions in the exploration individually. Circulate to monitor progress and offer guidance, and ask coaching questions of individuals or the whole class when appropriate. Then, have students compare and discuss their answers to the exploration questions in groups, or conduct a discussion with the whole class.

Meeting Student Needs

- Consider tying students' personal experiences to the section opener by asking:
  - Do you know someone who has moved to or from a rural area?
  - How did this person feel about the move?
  - Would you consider moving to a rural or urban area? Why or why not?
- Another real-life connection might be to research statistics regarding First Nations individuals who live on and off reserves. See the Web Link that follows in this TR. Discuss with students whether they know anyone who has left their First Nation community to live in a town or city.
- Consider preteaching the terms *rational number* and *opposites*. Have students practise writing rational numbers and opposites.
- It may help some students to better understand the term *rational numbers* if they come up with non-examples along with examples.
- Some students may need assistance in recalling what an integer is. Have students distinguish numbers that are integers from those that are not.
- Have students practise changing fractions to decimals, and decimals to fractions, and finding equivalent fractions.
- It may be better for your class to work through the Explore as a whole-class activity.

ELL

- Teach the following terms in context: *rural areas*, *decreasing*, *urban areas*, *decades*, *increases*, *rational numbers*, *opposite*, and *equivalent*.

Gifted and Enrichment

- Ask students to explain the following: Why is a seemingly large negative number smaller than a small positive one?

WWW Web Link

To research statistics about First Nations people living on and off reserves, go to www.mathlinks9.ca and follow the links.

Answers

Explore Rational Numbers

1. Each number in the first column has the same value as the number in the second column but with the opposite sign. Example: The sum of each set of two numbers is 0, therefore, they are opposites.

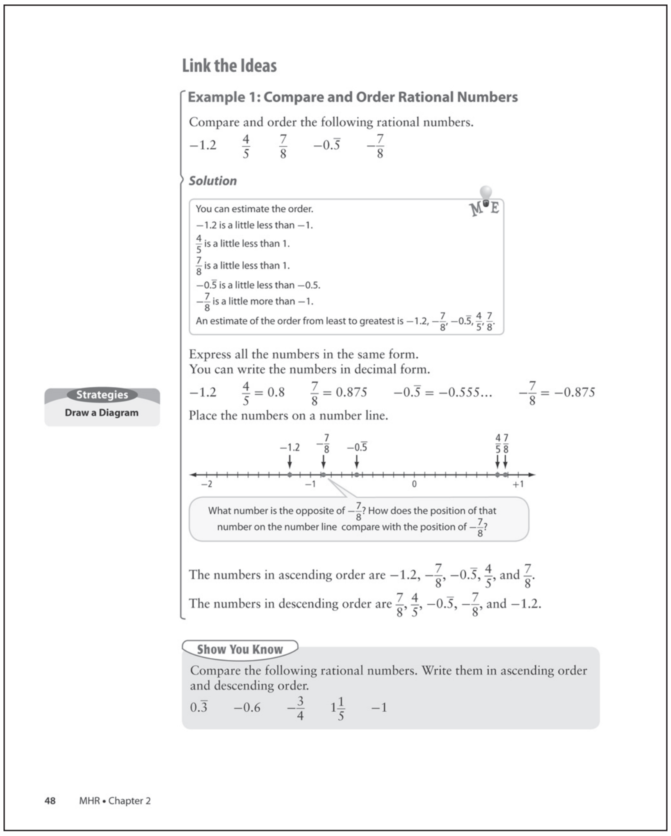
2. a) Example: 4.5 has an opposite −4.5.
b) Example: $\frac{7}{5}$ has an opposite $-\frac{7}{5}$.
c) Example: $\frac{-2}{9}$ has an opposite $\frac{-2}{-9}$.

3. a) $\frac{-8}{4}, -\frac{4}{2}, \frac{4}{-2}, -\left(\frac{-4}{-2}\right)$ are equal; $\frac{12}{4}, \frac{-9}{-3}$ are equal; $\frac{12}{3}, -\left(\frac{4}{-1}\right)$ are equal.
b) Example: $\frac{-8}{-4} = \frac{6}{3} = -\left(\frac{2}{-1}\right) = \frac{2}{1} = \frac{-4}{-2}$.

4. a) Example: Opposite rational numbers have a sum of zero.
b) Example: Equivalent rational numbers have the same value.

5. a) Example: The change will be +2.8 because the trend shows an increase of movement to the urban areas.
b) Example: The change will be −2.8, which is the opposite rational number of the predicted change in the urban population.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore.	• For students experiencing difficulty, encourage the use of the number line. Ensure that students are able to verbalize that opposites are the same distance from zero. If students have not drawn this conclusion, go through several examples, identifying the distances before moving on. • If students are having difficulty finding equivalent fractions, start with simpler fractions, such as $\frac{1}{2}, \frac{1}{3}, \frac{3}{4}$ and review how to find equivalents. Have students complete several examples before moving on. • You may wish to provide students with **Master 4 Number Lines** to use as they work on the Explore.



Link the Ideas

Example 1: Compare and Order Rational Numbers

Compare and order the following rational numbers.

$-1.2 \quad \frac{4}{5} \quad \frac{7}{8} \quad -0.\overline{5} \quad -\frac{7}{8}$

Solution

You can estimate the order.
-1.2 is a little less than -1.
$\frac{4}{5}$ is a little less than 1.
$\frac{7}{8}$ is a little less than 1.
$-0.\overline{5}$ is a little less than -0.5.
$-\frac{7}{8}$ is a little more than -1.
An estimate of the order from least to greatest is $-1.2, -\frac{7}{8}, -0.\overline{5}, \frac{4}{5}, \frac{7}{8}$.

Strategies
Draw a Diagram

Express all the numbers in the same form.
You can write the numbers in decimal form.

$-1.2 \quad \frac{4}{5} = 0.8 \quad \frac{7}{8} = 0.875 \quad -0.\overline{5} = -0.555... \quad -\frac{7}{8} = -0.875$

Place the numbers on a number line.

The numbers in ascending order are $-1.2, -\frac{7}{8}, -0.\overline{5}, \frac{4}{5}$, and $\frac{7}{8}$.
The numbers in descending order are $\frac{7}{8}, \frac{4}{5}, -0.\overline{5}, -\frac{7}{8}$, and -1.2.

Show You Know

Compare the following rational numbers. Write them in ascending order and descending order.

$0.\overline{3} \quad -0.6 \quad -\frac{3}{4} \quad 1\frac{1}{5} \quad -1$

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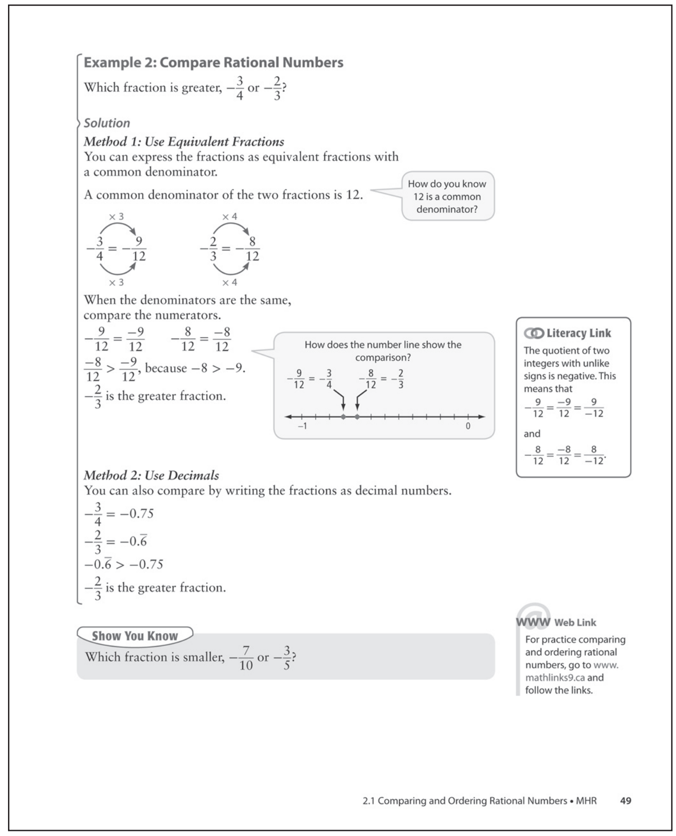

Example 2: Compare Rational Numbers

Which fraction is greater, $-\frac{3}{4}$ or $-\frac{2}{3}$?

Solution

Method 1: Use Equivalent Fractions
You can express the fractions as equivalent fractions with a common denominator.

A common denominator of the two fractions is 12.

How do you know 12 is a common denominator?

$-\frac{3}{4} = -\frac{9}{12}$ (× 3) $\qquad -\frac{2}{3} = -\frac{8}{12}$ (× 4)

When the denominators are the same, compare the numerators.

$-\frac{9}{12} = \frac{-9}{12} \qquad -\frac{8}{12} = \frac{-8}{12}$

$\frac{-8}{12} > \frac{-9}{12}$, because $-8 > -9$.

$-\frac{2}{3}$ is the greater fraction.

Literacy Link
The quotient of two integers with unlike signs is negative. This means that
$-\frac{9}{12} = \frac{-9}{12} = \frac{9}{-12}$
and
$-\frac{8}{12} = \frac{-8}{12} = \frac{8}{-12}$.

Method 2: Use Decimals
You can also compare by writing the fractions as decimal numbers.

$-\frac{3}{4} = -0.75$

$-\frac{2}{3} = -0.\overline{6}$

$-0.\overline{6} > -0.75$

$-\frac{2}{3}$ is the greater fraction.

Show You Know

Which fraction is smaller, $-\frac{7}{10}$ or $-\frac{3}{5}$?

Web Link
For practice comparing and ordering rational numbers, go to www.mathlinks9.ca and follow the links.

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Link the Ideas

Example 1

This example illustrates the use of a number line for comparing and ordering rational numbers. As students examine the example, ask probing questions such as:

- Why are the five numbers expressed in the same form before they are plotted on the number line?
- How might the thought bubble below the number line help you plot negative decimals?
- If you have a list of numbers in ascending order, what is the easiest way to write the numbers in descending order?

If you wish, you might ask students what the > and < symbols mean, and you might show that the final two lines of the solution could be written as follows.

The numbers in ascending order are
$-1.2 < -\frac{7}{8} < -0.\overline{5} < \frac{4}{5} < \frac{7}{8}$.

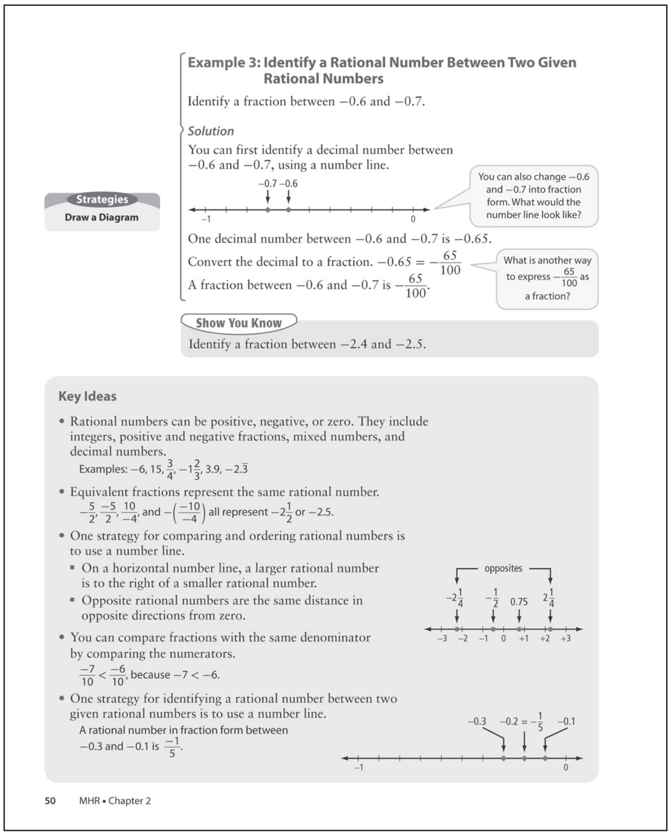
The numbers in descending order are
$\frac{7}{8} > \frac{4}{5} > -0.\overline{5} > -\frac{7}{8} > -1.2$.

Point out that, in this case, estimation can be used to establish the order, as shown at the beginning of the solution. You might support students' understanding by asking some related questions:

- Is -1.8 greater or less than -2? How do you know?
- Is -0.6 greater or less than -0.5? How do you know?
- How useful would estimation be in ordering $\frac{5}{6}, \frac{7}{8}$, and $\frac{6}{7}$?

If you wish to extend students' thinking, you might ask why the numbers are expressed in decimal form, rather than fraction form, before they are plotted in Example 1. Working in decimal form permits a comparison on the basis of place values. A difficulty with fraction form is the need to use common denominators to make the comparison, as will be described in Example 2. Plotting $\frac{3}{4}, \frac{4}{5}$, and $\frac{5}{6}$, for example, on the same number line would involve dividing it into sixtieths, so that the three points could be plotted at $\frac{45}{60}, \frac{48}{60}$, and $\frac{50}{60}$. A further difficulty with working in fraction form is that, while many students will recognize the fractional values of certain repeating decimals (e.g., $0.\overline{3}$ as $\frac{1}{3}$), they do not know a general method for converting a repeating decimal to a fraction.

When students complete the Show You Know, you might consider asking them how they could use a vertical number line to make the comparison.



Example 3: Identify a Rational Number Between Two Given Rational Numbers

Identify a fraction between −0.6 and −0.7.

Solution

You can first identify a decimal number between −0.6 and −0.7, using a number line.

Strategies: Draw a Diagram

You can also change −0.6 and −0.7 into fraction form. What would the number line look like?

One decimal number between −0.6 and −0.7 is −0.65.

Convert the decimal to a fraction. $-0.65 = -\frac{65}{100}$

A fraction between −0.6 and −0.7 is $-\frac{65}{100}$.

What is another way to express $-\frac{65}{100}$ as a fraction?

Show You Know

Identify a fraction between −2.4 and −2.5.

Key Ideas

- Rational numbers can be positive, negative, or zero. They include integers, positive and negative fractions, mixed numbers, and decimal numbers.
  Examples: $-6, 15, \frac{3}{4}, -1\frac{2}{3}, 3.9, -2.\overline{3}$
- Equivalent fractions represent the same rational number.
  $-\frac{5}{2}, \frac{-5}{2}, \frac{10}{-4},$ and $-\left(\frac{-10}{-4}\right)$ all represent $-2\frac{1}{2}$ or −2.5.
- One strategy for comparing and ordering rational numbers is to use a number line.
  - On a horizontal number line, a larger rational number is to the right of a smaller rational number.
  - Opposite rational numbers are the same distance in opposite directions from zero.

- You can compare fractions with the same denominator by comparing the numerators.
  $\frac{-7}{10} < \frac{-6}{10}$, because $-7 < -6$.
- One strategy for identifying a rational number between two given rational numbers is to use a number line.
  A rational number in fraction form between −0.3 and −0.1 is $\frac{-1}{5}$.

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Example 2

Method 1 of this example illustrates the use of common denominators in the comparison of two rational numbers in fraction form. The example shows that the comparison can then be made by comparing the numerators.

You might reinforce that the numerical comparison of the fractions is more straightforward when the denominators are positive. In this case, a comparison of the numerators directly compares the fractions. Therefore, the example shows the comparison of $\frac{-8}{12}$ and $\frac{-9}{12}$, not of $\frac{8}{-12}$ and $\frac{9}{-12}$. The difficulty with using a negative common denominator is the need to recognize that the smaller numerator is in the larger fraction.

Point out the thought bubble that includes the number line, and have students answer the question in the bubble. Ask students if they prefer to make the comparison numerically or by using a number line, and to explain their preference.

Literacy Link Point out the Literacy Link on page 49 that explains that the quotient of two integers with unlike signs is negative. Have students generate their own example to show their understanding.

Method 2 involves the conversion of fractions to decimals, and the comparison of the decimals. Remind students that Example 1 also involved the conversion of two fractions to decimals as part of a comparison.

As students examine Example 2, ask probing questions such as:

- Could you use mental math to help with the solution? If so, how?
- Is there a different common denominator that you could use to compare the fractions? If so, give an example.
- Is there any advantage to using 12 as the common denominator? Explain.
- How do you know that $-8 > -9$?
- How do you know that $-0.\overline{6} > -0.75$?
- Would you prefer to plot the fractional values or the decimal values on a number line? Why?
- Which of the methods shown do you prefer? Why?

Encourage students to use a method of their choice to complete the Show You Know. Have students explain their choice of method to their group or the class. There may be some variations not shown in the example, such as writing the fractions as decimals and then plotting the decimals on a number line.

Example 3

This example shows students how to identify a rational number that is between two given rational numbers. In this case, the two given rational numbers are in decimal form. You might ask students:

- Could you solve the problem without using a number line?
- Can you identify two other fractions between −0.6 and −0.7?
- How would you complete the example if the two rational numbers were given in fraction form, as $-\frac{6}{10}$ and $-\frac{7}{10}$? (One approach to this question would be to convert the given fractions to decimals and proceed as already shown in the example.

Another approach would be to write equivalent fractions, such as $-\frac{60}{100}$ and $-\frac{70}{100}$, and to determine a fraction that is between them, such as $-\frac{61}{100}$ or $-\frac{62}{100}$, without using a number line. A third possibility would be to plot $-\frac{6}{10}$ and $-\frac{7}{10}$ on a number line, as prompted by the first thought bubble in the example. One way to determine a fraction between them would be to create smaller divisions on the line. For example, if the divisions were changed to twentieths, $-\frac{6}{10}$ would be located at $-\frac{12}{20}$, and $-\frac{7}{10}$ would be located at $-\frac{14}{20}$. The number line would show that $-\frac{13}{20}$ is between the two points.)

You might challenge students to complete the Show You Know in two ways, by working in decimal form and by working in fraction form. Ask:

- Which method did you find easier to use?
- Did you use a number line? Why or why not?

Key Ideas

This section summarizes the meaning of rational numbers and how they can be expressed, compared, and ordered. Students could prepare their own list of Key Ideas and include it in their Foldable, especially if they have used methods not described in the section.

Meeting Student Needs

- It may be better for your class to work through the examples as a whole-class activity. Assign the Show You Know activities as small-group or pair activities. Then, assign a similar Show You Know activity as individual student work.
- For Example 2, it may help your students to begin with an example of positive fractions (e.g., $\frac{3}{4}$ and $\frac{2}{3}$), before moving on to negative fractions. This will serve two purposes: it will be a connection to prior learning, and it will reinforce the difference between negative and positive rational numbers (i.e., while $\frac{3}{4}$ is greater than $\frac{2}{3}$, $-\frac{3}{4}$ is less than $-\frac{2}{3}$).

ELL

- Read through and discuss the Key Ideas with students to make sure that they understand these concepts.

Common Errors

- Some students may have difficulty in understanding that integers are rational numbers.

R_x Ask students what the denominator would be if they were asked to write any natural number, such as 2 or 3, as a fraction in lowest terms. When students are clear that 2 can be expressed as $\frac{2}{1}$ and 3 can be expressed as $\frac{3}{1}$, refer students to the definition of rational numbers and ask if 2 and 3 qualify as rational numbers. Then, extend the discussion to negative integers, such as -2 and -3, which can be expressed as $\frac{-2}{1}$ and $\frac{-3}{1}$.

- Some students may have difficulty in writing equivalent fractions that include negative signs.

R_x First, check that students can write equivalent positive fractions by multiplying or dividing the numerator and denominator of a given fraction by the same number. Then, have students apply the same techniques to express the negative fraction $-\frac{2}{4}$ in equivalent forms, such as $-\frac{1}{2}$ and $-\frac{4}{8}$. If students are unsure of the equivalence of these three fractions, ask them to plot all three on a number line.

- Some students may have difficulty in deciding whether fractions that include more than one negative sign are positive or negative.

R_x Remind students of the sign rules for division of integers. Then, ask students how they could include negative signs in a fraction such as $\frac{3}{4}$ so that the fraction stays positive. The sign rules will indicate that $\frac{-3}{4}$ and $\frac{3}{-4}$ are both negative, whereas $\frac{-3}{-4}$ is positive.

- Some students may ignore negative signs when plotting negative rational numbers on a number line.

R_x Stress that rational numbers include integers, positive and negative decimals, and positive and negative fractions and mixed numbers. Ask students where points that represent positive and negative numbers are located in relation to 0 on a number line. You might also ask how the positions of points that represent two opposite rational numbers, such as 0.5 and -0.5, are related.

WWW Web Link

For an activity that involves determining a fraction with a value between two fractions on a number line, go to www.mathlinks9.ca and follow the links.

Answers

Example 1: Show You Know

The numbers in ascending order are $-1 < -\frac{3}{4} < -0.6 < 0.\overline{3} < 1\frac{1}{5}$.

The numbers in descending order are $1\frac{1}{5} > 0.\overline{3} > -0.6 > -\frac{3}{4} > -1$.

Example 2: Show You Know

$-\frac{7}{10}$

Example 3: Show You Know

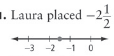
Example: $-\frac{49}{20}$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may wish to provide students with a number line that has regular increments between integers already marked. Encourage students to mark points such as $+/-\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$. Have them write their decimal equivalents below the fraction. This will provide a visual reference point for students to use for plotting and organizing the given values. A number line labelled with these values placed into the Foldable would serve as a good future reference for students. • Assist students in remembering the terms *descending* and *ascending*. Point out that *descending* begins with *d* as in *down* (or *decreasing*); the numbers go down from greatest to least. *Ascending* is the opposite; the numbers go up from least to greatest. • You may wish to provide students with number lines to use as they work on this example.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Once students have found equivalent fractions, encourage them to divide the units between integers on their number lines, in equal intervals to match their denominator. Alternatively, they could use the numerators and visualize them on a number line in order to place equivalent fractions in ascending or descending order. • A number line like the one described above will provide a visual reference point for students to use for plotting, organizing, and comparing decimal values.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students may require assistance in determining how -0.7 could be compared to $-\frac{65}{100}$. Help students to see that -0.7 could be written as $-\frac{7}{10}$ or $-\frac{70}{100}$. Provide students with number lines that have tenths marks or equal intervals between integer values to help them find values between two given rational numbers. • You may wish to provide students with number lines to use as they work on this example.

Check Your Understanding

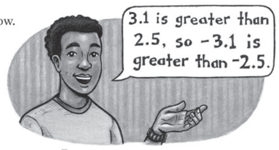
Communicate the Ideas

1. Laura placed $-2\frac{1}{2}$ incorrectly on a number line, as shown.



How could you use the idea of opposites to show Laura how to plot $-2\frac{1}{2}$ correctly?

2. Is Dominic correct? Show how you know.



3. Tomas and Roxanne were comparing −0.9 and $-\frac{7}{8}$. Tomas wrote −0.9 as a fraction, and then he compared the two fractions. Roxanne wrote $-\frac{7}{8}$ as a decimal, and then she compared the two decimals.

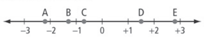
a) Which method do you prefer? Explain.

b) Which is greater, −0.9 or $-\frac{7}{8}$? Explain how you know.

Practise

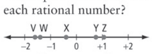
For help with #4 to #9, refer to Example 1 on page 48.

4. Match each rational number to a point on the number line.



a) $\frac{3}{2}$ **b)** −0.7 **c)** $-2\frac{1}{5}$ **d)** $\frac{14}{5}$ **e)** $-1\frac{1}{3}$

5. Which point on the number line matches each rational number?



a) $-1\frac{2}{5}$ **b)** $\frac{3}{4}$ **c)** $1\frac{1}{20}$ **d)** $-1\frac{3}{5}$ **e)** $-0.\overline{4}$

6. Place each number and its opposite on a number line.

a) $\frac{8}{9}$ **b)** −1.2 **c)** $2\frac{1}{10}$ **d)** $-\frac{11}{3}$

7. What is the opposite of each rational number?

a) $-4.\overline{1}$ **b)** $\frac{4}{5}$ **c)** $-5\frac{3}{4}$ **d)** $\frac{9}{8}$

8. Compare $1\frac{5}{6}$, $-1\frac{2}{3}$, −0.1, 1.9, and $-\frac{1}{5}$. Write the numbers in ascending order.

9. Compare $-\frac{3}{8}$, $1.\overline{8}$, $\frac{9}{5}$, $-\frac{1}{2}$, and −1. Write the numbers in descending order.

For help with #10 to #13, refer to Example 2 on page 49.

10. Express each fraction as an equivalent fraction.

a) $-\frac{2}{5}$ **b)** $\frac{10}{6}$ **c)** $-\frac{9}{12}$ **d)** $\frac{-4}{3}$

11. Write each rational number as an equivalent fraction.

a) $\frac{-1}{3}$ **b)** $\frac{-4}{-5}$ **c)** $-\left(\frac{-5}{-4}\right)$ **d)** $\frac{7}{-2}$

12. Which value in each pair is greater?

a) $\frac{1}{3}, -\frac{2}{3}$ **b)** $-\frac{9}{10}, \frac{7}{10}$ **c)** $-\frac{1}{2}, -\frac{3}{5}$ **d)** $-2\frac{1}{8}, -2\frac{1}{4}$

13. Which value in each pair is smaller?

a) $\frac{4}{7}, \frac{2}{3}$ **b)** $-\frac{4}{3}, -\frac{5}{3}$ **c)** $-\frac{7}{10}, -\frac{3}{5}$ **d)** $-1\frac{3}{4}, -1\frac{4}{5}$

For help with #14 to #17, refer to Example 3 on page 50.

14. Identify a decimal number between each of the following pairs of rational numbers.

a) $\frac{3}{5}, \frac{4}{5}$ **b)** $-\frac{1}{2}, -\frac{5}{8}$ **c)** $-\frac{5}{6}, 1$ **d)** $-\frac{17}{20}, -\frac{4}{5}$

15. What is a decimal number between each of the following pairs of rational numbers?

a) $1\frac{1}{2}, 1\frac{7}{10}$ **b)** $-2\frac{2}{3}, -2\frac{1}{3}$ **c)** $1\frac{3}{5}, -1\frac{7}{10}$ **d)** $-3\frac{1}{100}, -3\frac{1}{50}$

16. Identify a fraction between each of the following pairs of rational numbers.

a) 0.2, 0.3 **b)** 0, −0.1 **c)** −0.74, −0.76 **d)** −0.52, −0.53

17. Identify a mixed number between each of the following pairs of rational numbers.

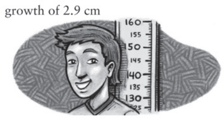
a) 1.7, 1.9 **b)** −0.5, 1.5 **c)** −3.3, −3.4 **d)** −2.01, −2.03

Apply

18. Use a rational number to represent each quantity. Explain your reasoning.

a) a temperature increase of 8.2 °C

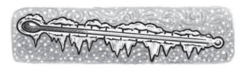
b) growth of 2.9 cm



c) 3.5 m below sea level

d) earnings of $32.50

e) 14.2 °C below freezing



Check Your Understanding

Communicate the Ideas

These questions allow students to explain ways of representing and comparing rational numbers.

In #1, students explain how to plot a rational number correctly on a number line. Encourage students to redraw the number line correctly to help with their explanation.

In #2, students examine the importance of the sign in making comparisons between rational numbers. Students should use a strategy (e.g., drawing a number line) to check Dominic's statement.

In #3, students consider different methods for comparing two rational numbers, and state and explain their preference. You might ask students to consider whether or not their answer to #3a) would be the same if they were using a number line.

Practise

You may wish to have students work in pairs or small groups when completing the Practise questions. Encourage comparisons of solution methods.

When students have completed #8 and 9, you might have them discuss whether they prefer to work in fraction form or decimal form and explain why. Plotting the numbers expressed in fraction form can involve some sizeable common denominators (e.g., at least 30 in #8). For #10 and 11, you might have students compare their answers. Each part of these questions has an infinite number of correct answers. For #12 to 17, you might have students consider whether they prefer to work in fraction form or in decimal form when making the comparisons, and have them explain why. Their responses may vary from one question to another and may depend on whether they choose to use a number line or not.

Apply

These questions provide a range of contexts for students to solve problems involving rational numbers.

Before students consider #18, you may wish to remind them of similar questions that they completed with integers in grades 7 and 8. You might begin by asking students to use an integer to represent each of the following quantities:

- a temperature decrease of 5 ºC
- 4 m above sea level
- earning $20
- spending $15

19. The table includes the melting points and boiling points of six elements known as the noble gases.

Noble Gas	Melting Point (°C)	Boiling Point (°C)
Argon	−189.2	−185.7
Helium	−272.2	−268.6
Neon	−248.67	−245.92
Krypton	−156.6	−152.3
Radon	−71.0	−61.8
Xenon	−111.9	−107.1

a) Which noble gases have a melting point that is less than the melting point of argon?
b) Which noble gases have a boiling point that is greater than the boiling point of krypton?
c) Arrange the melting points in ascending order.
d) Arrange the boiling points in descending order.

Science Link

For many years, the noble gases were known as the *inert gases*. Most chemists thought that these gases would not react with other chemicals. In 1962, Neil Bartlett, a chemist at the University of British Columbia, proved them wrong.

WWW Web Link

To learn more about Neil Bartlett and to research Canadian scientific discoveries, go to www.mathlinks9.ca and follow the links.

20. a) Kwasi said that he ignored the fractions when he decided that $-2\frac{1}{5}$ is smaller than $-1\frac{9}{10}$. Explain his thinking.
b) Naomi said that she ignored the integer −1 when she decided that $-1\frac{1}{4}$ is greater than $-1\frac{2}{7}$. Explain her thinking.

21. The table shows the average early-morning temperature for seven communities in May.

Community	Average Early-Morning Temperature (°C)
Churchill, Manitoba	−5.1
Regina, Saskatchewan	3.9
Edmonton, Alberta	5.4
Penticton, British Columbia	6.1
Yellowknife, Northwest Territories	−0.1
Whitehorse, Yukon Territory	0.6
Resolute, Nunavut	−14.1

a) Write the temperatures in descending order.
b) Which community has an average temperature between the values for Whitehorse and Churchill?

22. Replace each ■ with >, <, or = to make each statement true.
a) $\frac{-9}{6}$ ■ $\frac{3}{-2}$
b) $-\frac{3}{5}$ ■ $-0.\overline{6}$
c) $-1\frac{3}{10}$ ■ $-\left(\frac{-13}{-10}\right)$
d) -3.25 ■ $-3\frac{1}{5}$
e) $-\frac{8}{12}$ ■ $-\frac{11}{15}$
f) $-2\frac{5}{6}$ ■ $-2\frac{7}{8}$

23. Is zero a rational number? Explain.

24. Give an example of a fraction in lowest terms that satisfies the following conditions.
a) greater than 0, with the denominator greater than the numerator
b) between 0 and −1, with the denominator less than the numerator
c) less than −2, with the numerator less than the denominator
d) between −1.2 and −1.3, with the numerator greater than the denominator

2.1 Comparing and Ordering Rational Numbers • MHR 53

25. Which integers are between $\frac{11}{5}$ and $\frac{15}{-4}$?

26. Which number in each pair is greater? Explain each answer.
a) 0.4 and 0.44
b) $0.\overline{3}$ and 0.33
c) −0.7 and −0.77
d) −0.66 and $-0.\overline{6}$

27. Identify the fractions that are between 0 and −2 and that have 3 as the denominator.

Extend

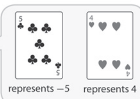
28. How many rational numbers are between $\frac{2}{3}$ and $0.\overline{6}$? Explain.

29. Replace each ■ with an integer to make each statement true. In each case, is more than one answer possible? Explain.
a) ■.5 < −1.9
b) $\frac{■}{-4} = -2\frac{1}{4}$
c) $\frac{-3}{■} = -\frac{-15}{5}$
d) −1.5■2 > −1.512
e) $-\frac{3}{4} < -0.7$■
f) $-5\frac{1}{2} > \frac{11}{■}$
g) $-2\frac{3}{5} = \frac{■}{10}$
h) $\frac{8}{■} < -\frac{2}{3}$

30. Determine the value of x.
a) $\frac{4}{-5} = \frac{x}{-10}$
b) $\frac{x}{3} = \frac{6}{-9}$
c) $\frac{5}{x} = -\frac{20}{12}$
d) $\frac{-6}{-5} = \frac{30}{x}$

Math Link

Play the following game with a partner or in a small group. You will need one deck of playing cards.
- Remove the jokers, aces, and face cards from the deck.
- Red cards represent positive integers. Black cards represent negative integers.
- In each round, the dealer shuffles the cards and deals two cards to each player.
- Use your two cards to make a fraction that is as close as possible to zero.
- In each round, the player with the fraction closest to zero wins two points. If there is a tie, each tied player wins a point.
- The winner is the first player with ten points. If two or more players reach ten points in the same round, keep playing until one player is in the lead by at least two points.



With a five of clubs and a four of hearts, you can make $\frac{4}{-5}$ or $\frac{-5}{4}$. Choose $\frac{4}{-5}$ because it is closer to zero.

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You might also take the opportunity to ask students how they know that each of their answers to #18 is a rational number, i.e., by showing that each answer can be written as the quotient of two integers, where the divisor is not 0. For example, the answer −3.5 for #18c) can be written as $\frac{-35}{10}$, $\frac{35}{-10}$, $\frac{-7}{2}$, $\frac{7}{-2}$, $\frac{-70}{20}$, and $\frac{70}{-20}$, and as an infinite number of other quotients of two integers.

For #19, discuss the information about noble gases in the Science Link as a point of interest.

If students have difficulty with #20, you might suggest the use of a number line to make the situation clearer. In part a), the two numbers are on either side of −2, so the comparison can be made without considering the fractions. In part b), both numbers are between −1 and −2, so only the fractions need be compared.

To answer #24, students need to know that a negative sign in a fraction may be assigned to the numerator or denominator. For example, in part b), some students may write $-\frac{1}{2}$ as a fraction in lowest terms between 0 and −1. However, this answer does not clearly satisfy the remaining condition "with the denominator less than the numerator." Assigning the negative sign to the denominator to give $\frac{1}{-2}$ satisfies this condition.

Extend

In #28, the idea is reinforced that the same rational number can be expressed in different ways.

In the parts of #29 that include inequalities, students can experiment with different values of the unknown numbers. You might extend these parts by asking how many answers are possible. There may be an infinite number, as in part a), or a finite number, as in part d).

Students can complete each part of #30 by first determining the relationship between the numerators or the denominators. For example, in part d), the values of the numerators can be used to show that both the numerator and denominator of $\frac{-6}{-5}$ can be multiplied by −5 to obtain the equivalent fraction. To use this approach in part c), students will need to assign the negative sign in $-\frac{20}{12}$ to either the numerator or the denominator. You might have students explore both possibilities to show that the value of x is not affected by this choice.

Literacy Link Have students develop a Frayer model showing what they already know about integers at the beginning of section 2.1. Have them revisit their Frayer model at the end of the section.

Math Link

In this Math Link, students apply their skills in comparing rational numbers in fraction form as they play a card game. Students will be asked to design their own game in the Wrap It Up! at the end of this chapter. Therefore, you might ask them to think about the design of the present game and about ways in which the game might be modified. You might ask:

- Why are the jokers, aces, and face cards removed from the deck?
- Is there an alternative to removing these cards? If so, what might it be?
- Suppose the dealer deals three cards to each player. How many different fractions could you make with your cards?
- Suppose you rolled a red die to create positive integers and a black die to create negative integers, instead of using red and black cards. Would the game be harder or easier than before? Explain.
- Suggest other ways of creating positive and negative integers to be used in the game.
- Would you change the way that points are awarded in the game? Explain.

If students suggest only minor variations in answer to the final question above, you might ask:

- Does the number of points awarded to the winner of a round need to be a fixed number? Explain.
- Could you use a die to decide the number of points awarded to the winner of a round? If so, how?
- Are there other ways to decide the number of points awarded to the winner of a round? If so, what are they?
- If you changed the way that points are awarded in a round, would you need to change the way in which the winner of the game is decided? Explain.

Meeting Student Needs

- Provide **BLM 2–5 Section 2.1 Extra Practice** to students who would benefit from more practice.

ELL

- Teach the following terms in context: *plot*, *incorrectly*, *melting point*, *boiling point*, *denominator*, *numerator*, *greater than*, and *less than*.
- Have students read through the Check Your Understanding questions and write down any terms they do not understand. Define the terms using either examples or pictures.

Gifted and Enrichment

- For #19, challenge students to find out how Neil Bartlett, a chemist at the University of British Columbia, proved that noble gases react with other chemicals. They may find the related Web Link on student resource page 53 helpful.

Common Errors

- Some students may have difficulty in understanding how to plot negative rational numbers on a number line.

R_x Stress the concept of opposites, i.e., the idea that, on a horizontal number line, the points that represent a pair of opposites are equal distances to the right of 0 and to the left of 0. You might check students' understanding by having students plot a pair of opposites, such as +3.5 and −3.5, on a vertical number line.

- Some students may ignore the bar notation in repeating decimals and, as a result, make errors in questions such as #22b), 26b), and 26d).

R_x Remind students that bar notation is used to represent repeating decimals. Stress that the same notation is used for positive and negative repeating decimals.

- Some students may have difficulty in identifying zero as a rational number.

R_x Ask students to state the value of fractions such as $\frac{0}{1}$, $\frac{0}{2}$, and $\frac{0}{3}$. Ask students to provide two other ways of expressing zero as a fraction. Then, ask if zero is a rational number.

Answers

Communicate the Ideas

1. Example: If Laura can locate $+2\frac{1}{2}$ correctly, she could count the distance from 0 to $+2\frac{1}{2}$, and then count the same distance from 0 in the opposite direction.

2. No. Example: -3.1 is to the left of -2.5 on the number line. Therefore, -2.5 is greater than -3.1.

3. a) Example: Changing the fraction to a decimal and comparing the two decimal values is preferred because the answer is more apparent.

b) The fraction $-\frac{7}{8}$ is greater. Example: $-\frac{7}{8} = -0.875$, which is to the right of -0.9 on the number line.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students use **Master 2 Communication Peer Evaluation** to assess another student's answer to one of #1 to 3. • Students having difficulty with #1 and 2 may wish to refer to the Reflect and Check they completed for the Explore. In the discussion that occurred with #4 and 5, the equivalent distance from zero of opposites on a number line should have been pointed out. If students did not record this in their Foldables, encourage them to do so, modelling an example of their own for future reference. • It is important for students to have a good understanding of both methods presented in #3, although they may have a preference. Coach students who are able to complete only one approach in trying additional questions using the other method and using their preferred method to check their own work.
Assessment *for* Learning	
Practise and Apply Have students do #4, 6, 8, 10, 12, 14a), b), 16a), b), and 18. Students who have no problems with these questions can go on to the remaining Apply questions.	• You may wish to provide students with number lines to use as they work on these questions. • For #4, 6, and 8, it may be useful for some students to have number lines that are labelled with common fractions converted to decimal values. Ensure that the number line has sufficient space between increments so students can easily place multiple values if needed. • The class as a whole may benefit from having a wall number line showing -4 to 4, with increments of tenths in between, as well as the fractions $+/-\ \frac{1}{8}, \frac{1}{5}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\frac{1}{8}, 1\frac{1}{5}, 1\frac{1}{4}, 1\frac{1}{2}$, and $1\frac{3}{4}$ and their equivalent decimals. This would provide a quick visual reference for students in placing values, ordering them, and determining which is $>$, $<$, or $=$ to another value. • For #10, review the methods that were presented in the examples to determine the equivalent fraction. Determine which method students prefer and coach them accordingly. • Some students may benefit from a quick discussion of place values before starting #18. Remind them to write the fraction as they would read it (e.g., 8.2 is "eight and two tenths").
Math Link The Math Link on page 54 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 85.	• Students who need help getting started could use **BLM 2–6 Section 2.1 Math Link**, which provides scaffolding. • Provide number lines or have students reference the one in their Foldable or in the classroom to assist them in determining which rational numbers in fraction form are closer to zero.
Assessment *as* Learning	
Literacy Link At the beginning of section 2.1, have students work in pairs to develop a Frayer model on integers.	• Use students' Frayer models to identify their misconceptions about integers. • At the end of section 2.2, have students revisit their Frayer model and make additions and improvements.
Math Learning Log Have students respond to the following prompts: • The method I prefer for comparing rational numbers in fraction form is … • To identify a rational number between $\frac{3}{4}$ and $\frac{3}{5}$, I would …	• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. • Encourage students to use diagrams or models in their explanations.