Problem Solving With Rational Numbers in Decimal Form

2.2

***MathLinks 9*, pages 55–62**

Suggested Timing
80–100 minutes

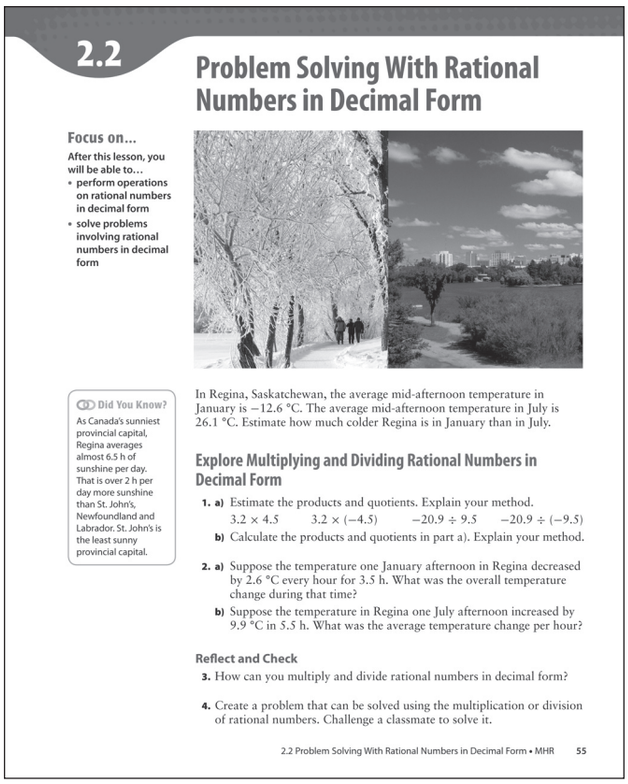
Materials
- ruler
- integer chips (optional)
- hundred grids (optional)
- base ten blocks (optional)

Blackline Masters
Master 2 Communication Peer Evaluation
Master 4 Number Lines
BLM 2–3 Chapter 2 Warm-Up
BLM 2–7 Section 2.2 Extra Practice
BLM 2–8 Section 2.2 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
N3 Demonstrate an understanding of rational numbers by:
- comparing and ordering rational numbers
- solving problems that involve arithmetic operations on rational numbers.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6, 8, 10, 12, Math Link
Typical	#1–4, 6, 8, 10, 12, 13, 16, 18, 22, Math Link
Extension/Enrichment	#1–3, 16, 20, 22, 24–29



2.2

Problem Solving With Rational Numbers in Decimal Form

Focus on...

After this lesson, you will be able to...
- perform operations on rational numbers in decimal form
- solve problems involving rational numbers in decimal form

Did You Know?
As Canada's sunniest provincial capital, Regina averages almost 6.5 h of sunshine per day. That is over 2 h per day more sunshine than St. John's, Newfoundland and Labrador. St. John's is the least sunny provincial capital.

In Regina, Saskatchewan, the average mid-afternoon temperature in January is −12.6 °C. The average mid-afternoon temperature in July is 26.1 °C. Estimate how much colder Regina is in January than in July.

Explore Multiplying and Dividing Rational Numbers in Decimal Form

1. a) Estimate the products and quotients. Explain your method.
3.2 × 4.5 3.2 × (−4.5) −20.9 ÷ 9.5 −20.9 ÷ (−9.5)
b) Calculate the products and quotients in part a). Explain your method.

2. a) Suppose the temperature one January afternoon in Regina decreased by 2.6 °C every hour for 3.5 h. What was the overall temperature change during that time?
b) Suppose the temperature in Regina one July afternoon increased by 9.9 °C in 5.5 h. What was the average temperature change per hour?

Reflect and Check

3. How can you multiply and divide rational numbers in decimal form?

4. Create a problem that can be solved using the multiplication or division of rational numbers. Challenge a classmate to solve it.

2.2 Problem Solving With Rational Numbers in Decimal Form • MHR 55

Planning Notes

Have students complete the warm-up questions on **BLM 2–3 Chapter 2 Warm-Up** to reinforce material learned in previous sections.

Refer students to the two photographs on page 55 that show winter and summer scenes in Regina. The paragraph below the photographs provides data on winter and summer temperatures in Regina. Students have previously performed operations with integers, but not with negative decimals, so they are likely to use integers to complete the estimates. A reasonable approach would be to use the integers −13 and +26 to arrive at an estimate of 39 °C. To assist students, you might ask:
- Which operation will you use? (The most likely answer is subtraction, but some students may indicate addition if they think in terms of subtracting by adding the opposite.)
- Is −12.6 closest to −11, −12, or −13?
- Would using values of +30 and −10 be a reasonable way to determine the estimate? Explain.

If students have difficulty with mental estimation, encourage them to use a vertical number line to model the estimation with integers.

You might point out that, while students are limited to estimation at this stage, they will return to the same context in Practise #10, where they will calculate an exact answer.

If you do not live in Regina, you may wish to ask students to research the average mid-afternoon temperatures in your community in January and July, and to estimate how much colder your community is in January than in July.

For general interest, refer students to the Did You Know? that compares daily average amounts of sunshine in Canada's sunniest and least sunny provincial capitals, Regina and St. John's, respectively. A possible activity for students who are interested in weather and geography would be to predict the order of all the provincial and territorial capitals from most sunny to least sunny. These students might then use their research skills to check their prediction.

Explore Multiplying and Dividing Rational Numbers in Decimal Form

In this exploration, students build on their knowledge of operations with integers and positive decimals to multiply and divide negative decimals for the first time.

Because of the estimation with integers in the opening paragraph of the section, students can be expected to estimate with integers in #1a) of the exploration. Completion of the estimates will reactivate their skills with the sign rules for multiplication and division of integers. Students can then apply the sign rules to the multiplication and division of positive and negative rational numbers in decimal form. Comparison of the exact results with the estimates can be used to support the reasonableness of the calculated values.

Method 1 Have students work on the exploration in pairs or small groups and discuss their answers. Circulate to monitor progress. When appropriate, ask coaching questions to provide assistance or to encourage deeper understanding.

For #1a), you might ask:

- Which integers will you multiply or divide to determine each estimate?
- How do you know the sign of each estimate before you multiply or divide the two integers?
- When estimating 3.2×4.5, is it better to multiply 3×4 or 3×5? Explain.

For #1b), you might ask:

- How do you know the sign of each product or quotient before you multiply or divide the two decimals?
- Without multiplying, can you tell the relationship between the products 3.2×4.5 and $3.2 \times (-4.5)$? Explain.
- Without dividing, can you tell the relationship between the quotients $-20.9 \div 9.5$ and $-20.9 \div (-9.5)$? Explain.
- Do your estimates indicate that your calculated products are reasonable? Explain.

If students try to use their calculators to multiply or divide negative decimals in #1b), they may have difficulty. You may wish to encourage the use of pencil and paper instead. However, some students may realize that they can calculate by multiplying or dividing positive decimals and then applying the sign rules. Calculator use for operations involving negative decimals is included in Examples 1 and 2.

In #2, students can apply their new skills with negative decimals to solve problems. Encourage students to estimate as well as calculate in #2 and to use their estimates to check the reasonableness of their answers. If students have difficulty in modelling the problems with negative decimals, ask coaching questions to assist them by referring to their skills with integers.

For #2a), students might estimate that the temperature decreased by 2 ºC every hour for 3 h. You might then ask:

- What integer would you use to represent a temperature decrease of 2 ºC every hour?
- What integer would you use to represent a time of 3 h?
- What operation would you use to determine the overall temperature change?
- What would be the overall temperature change?

For #2b), students might estimate that the temperature increased by 10 ºC in 6 h. You might then ask:

- What integer would you use to represent a temperature increase of 10 ºC?
- What integer would you use to represent a time of 6 h?
- What operation would you use to determine the average temperature change per hour?
- What would be the average temperature change per hour?

For #3, encourage students to express and compare their ideas openly. Their answers may vary, depending on how they went about multiplying and dividing in #1b). Some students may mention calculator use. Some students may mention the sign rules. You might reinforce the importance of estimation by asking students how estimation can be helpful when they multiply and divide rational numbers.

The easiest way for students to create a problem in #4 is to modify #2a) or b). However, even this approach has a pitfall. Students may choose rates of temperature change that are unrealistic. Encourage students to show some originality by changing the context. If they have difficulty, ask them if they can recall some contexts that they came across when they studied integer operations. Examples might include:

- ascending or descending aircraft
- ascending or descending submarines
- bank deposits or withdrawals
- earning, saving, borrowing, or spending money

Students may wish to create a problem that can be solved using integers and then modify the numbers to introduce the use of positive and/or negative decimals. Stress that students should check carefully that they can solve their own problem before they challenge a classmate to solve it. Ask students to suggest improvements to each other's problems. You might consider having students share particularly original problems with the whole class.

Method 2 Have students answer the questions individually. Circulate to monitor progress and offer guidance, and ask coaching questions of individuals or the whole class when appropriate. Then, have students compare and discuss their answers to the exploration questions in groups, or conduct a discussion with the whole class.

Meeting Student Needs

- Poll students for the coldest and hottest temperatures they remember in their community. Ask students to ask an adult or Elder in the community for their memories.
- Have students compare the average mid-afternoon temperature in January in Regina with that in their community, then do the same with the temperature in July. Have them complete the estimation using the temperatures in their community.
- As an alternative, you may wish to introduce this section using statistics from Northern communities where the changing seasons can mean extreme weather. You might bring estimation into the discussion by having students estimate how much colder these communities are in January than in July.
- Assist students in recalling their skills and understanding of the sign rules for negative numbers.
- It may be better for your class to work through the Explore as a whole-class activity.

ELL

- Teach the following words in context: *average* and *estimate*.
- Ensure that students understand that *product* refers to the answer of a multiplication question and *quotient* refers to the answer of a division question.

Answers

Explore Multiplying and Dividing Rational Numbers in Decimal Form

1. a) Example: Round 4.5 to 5, and 3.2 to 3, so estimated product is 15; round −4.5 to −5, and 3.2 to 3, so estimated product is −15; round −20.9 to −20, and 9.5 to 10, so estimated quotient is −2; round −20.9 to −20, and −9.5 to −10, so estimated quotient is 2.

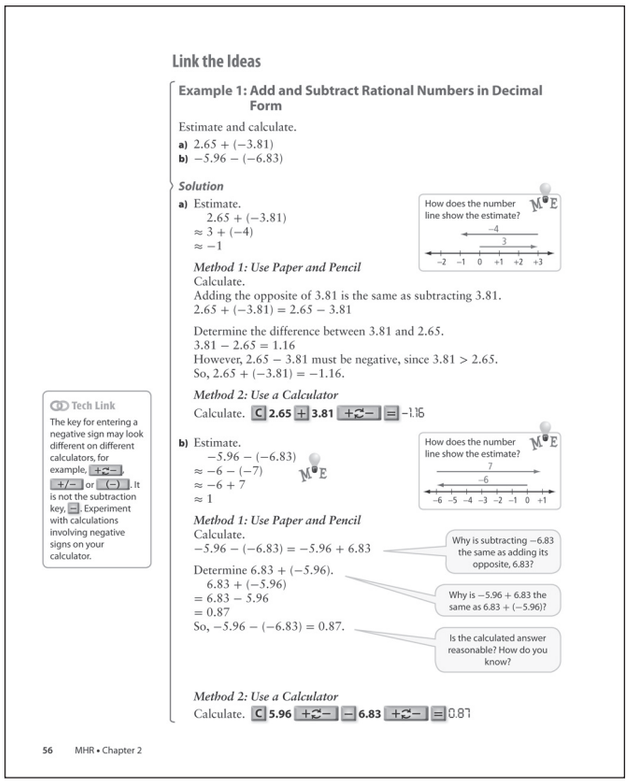
b) +14.4, −14.4, −2.2, +2.2. Example: The sign rules for multiplying and dividing integers

2. a) −9.1 °C **b)** +1.8 °C/h

3. Example: Multiply or divide the positive rational numbers and then use the sign rules to determine the sign of the product or quotient.

4. Example: Toby saves $4.60 from his allowance for 4.5 weeks. How much has he saved? Answer: $20.70

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore.	• It is important that students come away from the Explore with the understanding that the same sign rules apply when multiplying rational numbers in decimal form as when multiplying integers. • Some students will benefit from a discussion of how to use estimation to help them determine where to place the decimal in a product.



Link the Ideas

Example 1: Add and Subtract Rational Numbers in Decimal Form

Estimate and calculate.

a) $2.65 + (-3.81)$

b) $-5.96 - (-6.83)$

Solution

a) Estimate.

$2.65 + (-3.81)$

$\approx 3 + (-4)$

≈ -1

How does the number line show the estimate?

Method 1: Use Paper and Pencil

Calculate.

Adding the opposite of 3.81 is the same as subtracting 3.81.

$2.65 + (-3.81) = 2.65 - 3.81$

Determine the difference between 3.81 and 2.65.

$3.81 - 2.65 = 1.16$

However, $2.65 - 3.81$ must be negative, since $3.81 > 2.65$.

So, $2.65 + (-3.81) = -1.16$.

Method 2: Use a Calculator

Calculate. C 2.65 + 3.81 +/− = −1.16

Tech Link

The key for entering a negative sign may look different on different calculators, for example, +/−, +/= or (−). It is not the subtraction key, −. Experiment with calculations involving negative signs on your calculator.

b) Estimate.

$-5.96 - (-6.83)$

$\approx -6 - (-7)$

$\approx -6 + 7$

≈ 1

How does the number line show the estimate?

Method 1: Use Paper and Pencil

Calculate.

$-5.96 - (-6.83) = -5.96 + 6.83$

Why is subtracting −6.83 the same as adding its opposite, 6.83?

Determine $6.83 + (-5.96)$.

$6.83 + (-5.96)$

$= 6.83 - 5.96$

$= 0.87$

Why is −5.96 + 6.83 the same as 6.83 + (−5.96)?

So, $-5.96 - (-6.83) = 0.87$.

Is the calculated answer reasonable? How do you know?

Method 2: Use a Calculator

Calculate. C 5.96 +/− − 6.83 +/− = 0.87

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Link the Ideas

Example 1

This example presents strategies for adding and subtracting rational numbers in decimal form. Discuss the two parts of the example separately, but follow a similar approach in each part.

Before students begin part a), direct their attention to the estimated sum. The estimation is carried out using integer operations. For students who need a more visual reminder of integer addition, point out the number line in the thought bubble next to the estimate, and have students answer the question in the bubble.

Emphasize the importance of estimation when performing operations with rational numbers. Encourage students to check the reasonableness of their calculated answers by comparing them with their estimates.

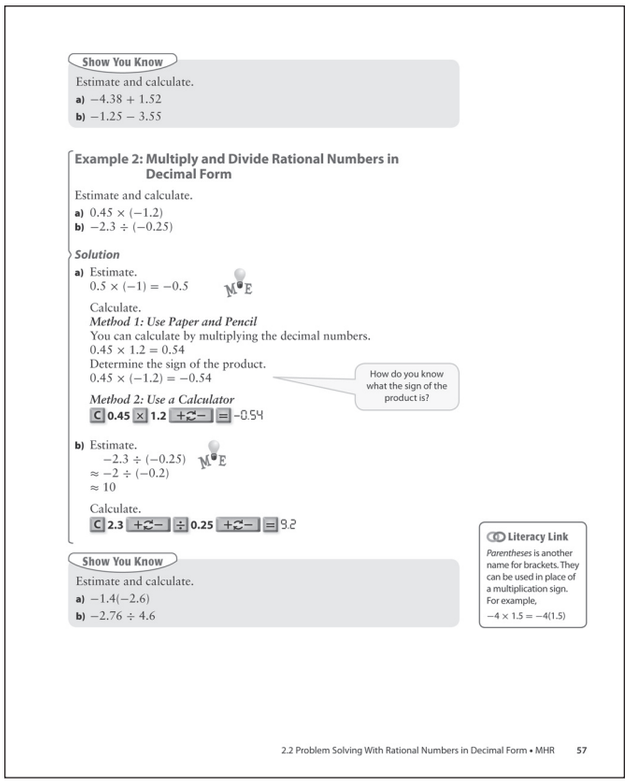
Next, discuss Method 1 of part a). You might ask:

- How do you know that adding the opposite of 3.81 is the same as subtracting 3.81? (If students are unsure, you might consider illustrating this concept by returning to the integers in the estimate. For example, you might use diagrams of integer chips on an overhead projector to show that $3 + (-4)$ and $3 - (+4)$ give the same result. Modelling the latter calculation involves the introduction of a zero pair before the removal of the appropriate chips.)
- How do the differences $3.81 - 2.65$ and $2.65 - 3.81$ compare? (They are opposites in the same way that $4 - 3$ and $3 - 4$ give results that are opposites. Therefore, the subtraction $2.65 - 3.81$ can be carried out by subtracting $3.81 - 2.65$ and then changing the sign of the difference.)
- Is the sign of the sum correct? How do you know? (Make sure that students realize the importance of checking the sign of the sum against the sign of the estimate, especially when they are adding a positive rational number and a negative rational number. You might consider presenting students with a sum such as $3.21 + (-2.25)$, so that they see a case in which a positive rational number and a negative rational number have a positive sum. Stress that, unlike with multiplication, students do not have a general rule for determining the sign of the sum. You might ask if there are specific cases in which the sign of the sum is obvious. Guide students to understand that two positive rational numbers always have a positive sum, and two negative rational numbers always have a negative sum.)

Finally, when you feel that students have grasped the concepts, discuss the calculator sequence in Method 2 of part a). Stress that inputting errors are common, so students should check their calculated value against an estimate.

Refer students to the Tech Link on the same page. Make sure that they are able to obtain the correct answer, using their own model of calculator. Have them record their own calculator sequence if it differs from the one shown.

Before students begin part b), direct their attention to the estimated difference, and stress the importance of estimation. The estimation is carried out using integer operations. For students who need a more visual reminder of integer subtraction, point out the number line in the thought bubble next to the estimate, and have students answer the question in the bubble. The number line shows the addition of -6 and 7. You might ask students how they know that adding 7 is the same as subtracting -7. If students are unsure, you might wish to use diagrams of integer chips on an overhead projector to show that $-6 + 7$ and $-6 - (-7)$ give the same result. Modelling the latter calculation involves the introduction of a zero pair before the removal of the appropriate chips.



Show You Know

Estimate and calculate.

a) $-4.38 + 1.52$

b) $-1.25 - 3.55$

Example 2: Multiply and Divide Rational Numbers in Decimal Form

Estimate and calculate.

a) $0.45 \times (-1.2)$

b) $-2.3 \div (-0.25)$

Solution

a) Estimate.

$0.5 \times (-1) = -0.5$

Calculate.

Method 1: Use Paper and Pencil

You can calculate by multiplying the decimal numbers.

$0.45 \times 1.2 = 0.54$

Determine the sign of the product.

$0.45 \times (-1.2) = -0.54$

How do you know what the sign of the product is?

Method 2: Use a Calculator

C 0.45 × 1.2 +/− = -0.54

b) Estimate.

$-2.3 \div (-0.25)$

$\approx -2 \div (-0.2)$

≈ 10

Calculate.

C 2.3 +/− ÷ 0.25 +/− = 9.2

Literacy Link

Parentheses is another name for brackets. They can be used in place of a multiplication sign. For example,

$-4 \times 1.5 = -4(1.5)$

Show You Know

Estimate and calculate.

a) $-1.4(-2.6)$

b) $-2.76 \div 4.6$

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Next, discuss Method 1 of part b). Refer students to the thought bubbles and make sure that they can explain why subtracting -6.83 is the same as adding 6.83, and why $-5.96 + 6.83$ is the same as $6.83 + (-5.96)$. The estimate in part b) should already have reminded students of subtracting by adding the opposite, and students should be aware from their work in previous grades that two numbers can be added in either order.

You might ask:

- Is the sign of the difference correct? How do you know?

Ensure that students realize the importance of checking the sign of the difference against the sign of the estimate when they subtract rational numbers. To reinforce the idea that they have no sign rule to follow, you might ask students to show calculations that meet the following conditions:

- the subtraction of two positive decimal numbers to give a positive result
- the subtraction of two positive decimal numbers to give a negative result
- the subtraction of two negative decimal numbers to give a negative result

Finally, when you feel that students have grasped the concepts, discuss the calculator sequence in Method 2. Again, refer students to the Tech Link.

When students complete the Show You Know, encourage them to check each other's answers, discuss their chosen methods, and make improvements to their solutions.

Example 2

This example presents strategies for multiplying and dividing rational numbers in decimal form.

Before students begin part a), direct their attention to the estimated product. Guide students in a discussion of the estimate by asking probing questions, such as:

- Is the sign of the product correct in the estimate? How do you know?
- Do you think that $0.4 \times (-1)$ would give as good an estimate? Why or why not?
- Are there other rational numbers you might use to estimate the product? Explain.

Next, discuss Method 1. You might ask:

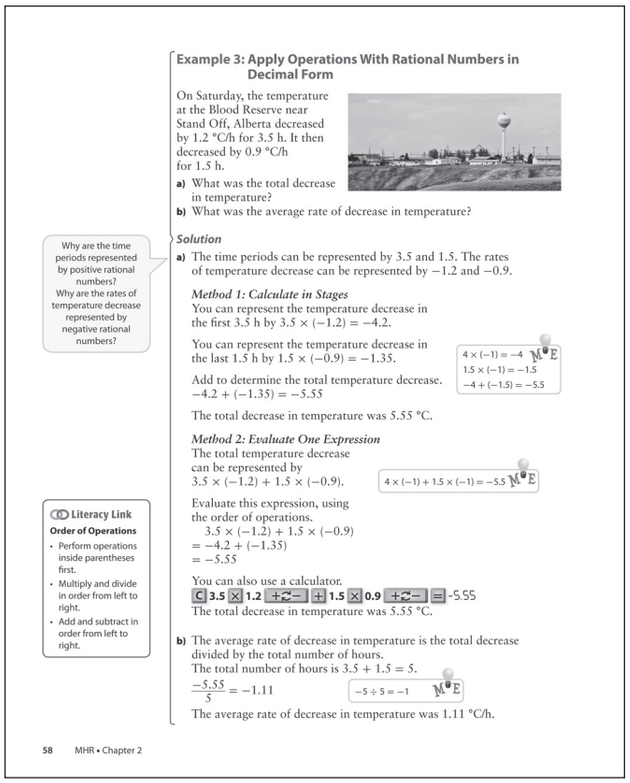
- How could you decide the sign of the product before you multiply?
- Does the calculated product seem reasonable? How do you know?

When students are ready to consult the calculator sequence in Method 2, remind them of the Tech Link beside Example 1. Ask students to make sure that they are able to calculate the correct answer using their own model of calculator. Have them record their own calculator sequence if it differs from the one shown in the example. You might ask if there is an alternative to including the negative sign in the calculator sequence. The multiplication could be carried out using positive decimals, with the sign decided using the sign rule for multiplication, as in Method 1.

In part b), again begin with the estimate. You might ask:

- Is the sign of the quotient correct in the estimate? How do you know?
- Do you think that using $-2.5 \div (-0.25)$ would be a good way to estimate? Why or why not?
- Are there other rational numbers you might use to estimate the quotient? Explain.

You might follow up on students' responses to these questions by mentioning the usefulness of compatible numbers in estimation. You might ask students why we prefer to estimate using $-2 \div (-0.2)$ or $-2.5 \div (-0.25)$, rather than, say, $-2.1 \div (-0.2)$.



Example 3: Apply Operations With Rational Numbers in Decimal Form

On Saturday, the temperature at the Blood Reserve near Stand Off, Alberta decreased by 1.2 °C/h for 3.5 h. It then decreased by 0.9 °C/h for 1.5 h.

a) What was the total decrease in temperature?

b) What was the average rate of decrease in temperature?

Solution

Why are the time periods represented by positive rational numbers? Why are the rates of temperature decrease represented by negative rational numbers?

a) The time periods can be represented by 3.5 and 1.5. The rates of temperature decrease can be represented by −1.2 and −0.9.

Method 1: Calculate in Stages

You can represent the temperature decrease in the first 3.5 h by $3.5 \times (-1.2) = -4.2$.

You can represent the temperature decrease in the last 1.5 h by $1.5 \times (-0.9) = -1.35$.

$4 \times (-1) = -4$
$1.5 \times (-1) = -1.5$
$-4 + (-1.5) = -5.5$

Add to determine the total temperature decrease.
$-4.2 + (-1.35) = -5.55$

The total decrease in temperature was 5.55 °C.

Method 2: Evaluate One Expression

The total temperature decrease can be represented by $3.5 \times (-1.2) + 1.5 \times (-0.9)$.

$4 \times (-1) + 1.5 \times (-1) = -5.5$

Evaluate this expression, using the order of operations.

$3.5 \times (-1.2) + 1.5 \times (-0.9)$
$= -4.2 + (-1.35)$
$= -5.55$

Literacy Link
Order of Operations
- Perform operations inside parentheses first.
- Multiply and divide in order from left to right.
- Add and subtract in order from left to right.

You can also use a calculator.

C 3.5 × 1.2 +/− + 1.5 × 0.9 +/− = -5.55

The total decrease in temperature was 5.55 °C.

b) The average rate of decrease in temperature is the total decrease divided by the total number of hours.
The total number of hours is $3.5 + 1.5 = 5$.

$\frac{-5.55}{5} = -1.11$

$-5 \div 5 = -1$

The average rate of decrease in temperature was 1.11 °C/h.

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As before, ask students to make sure that they are able to calculate the correct answer using their own model of calculator. You might ask:

- How could you decide the sign of the quotient before you divide?
- Does the calculated quotient seem reasonable? How do you know?

You may wish to challenge students to complete the division using paper and pencil instead of a calculator.

When students begin the Show You Know, you might ask them to state the sign of the answer to each part before they do any calculations. Alternatively, after the calculations are complete, you may wish to ask students if their answers agree with the sign rules. You might also check understanding by asking students why they were required to complete estimates in the Show You Know. Encourage students to check each other's answers, discuss their chosen methods, and make improvements to their solutions.

Literacy Link Point out the Literacy Link before students complete the Show You Know. To check understanding, ask students to show another way to write the expression in part a) of the Show You Know.

Example 3

This example presents an application of operations with rational numbers to consolidate and extend the work with temperatures that students did in the section opener and the Explore. It also introduces the order of operations for calculations with rational numbers. You may wish to mention that the photo shows the Blood Reserve near Standoff, Alberta.

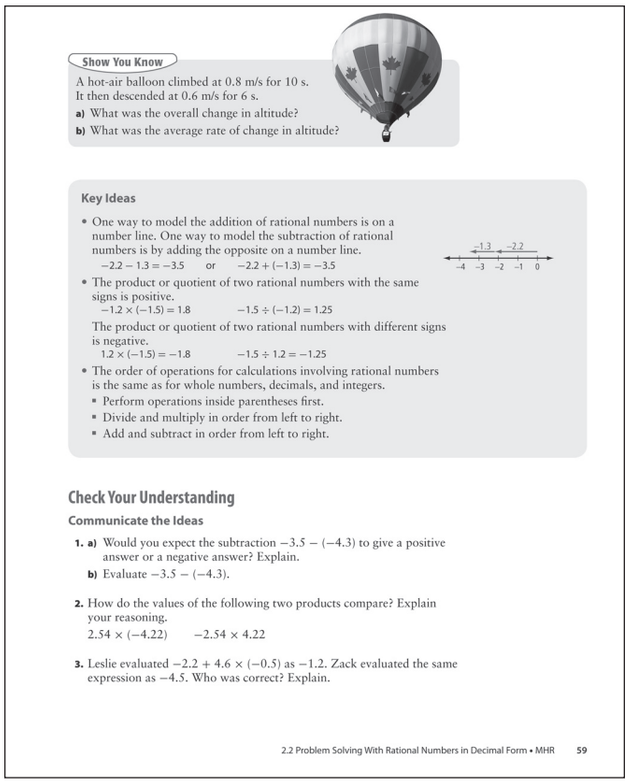
Point out the thought bubble at the beginning of part a). Ask students to explain why the positive and negative rational numbers are used to represent the quantities given in the problem.

As students examine Method 1 in part a), you might ask:

- Why is multiplication used to determine the temperature decrease in each time period?
- Why is addition used to determine the total temperature decrease?
- How could you represent the addition on a number line?
- Does the estimate suggest that the answer is reasonable? Explain.
- What other reasonable estimates can you suggest?

Point out the summary statement at the end of Method 1. Stress that a summary statement is an essential part of the solution to a word problem. Point out that the negative sign in −5.55 is consistent with a decrease in temperature.

Literacy Link Before students examine Method 2 in part a), point out the Literacy Link that deals with the order of operations. Stress that the order of operations with rational numbers is the same as with whole numbers, integers, and decimals.



Show You Know

A hot-air balloon climbed at 0.8 m/s for 10 s. It then descended at 0.6 m/s for 6 s.

a) What was the overall change in altitude?

b) What was the average rate of change in altitude?

Key Ideas

- One way to model the addition of rational numbers is on a number line. One way to model the subtraction of rational numbers is by adding the opposite on a number line.
  $-2.2 - 1.3 = -3.5$ or $-2.2 + (-1.3) = -3.5$
- The product or quotient of two rational numbers with the same signs is positive.
  $-1.2 \times (-1.5) = 1.8$ $-1.5 \div (-1.2) = 1.25$
  The product or quotient of two rational numbers with different signs is negative.
  $1.2 \times (-1.5) = -1.8$ $-1.5 \div 1.2 = -1.25$
- The order of operations for calculations involving rational numbers is the same as for whole numbers, decimals, and integers.
  - Perform operations inside parentheses first.
  - Divide and multiply in order from left to right.
  - Add and subtract in order from left to right.

Check Your Understanding

Communicate the Ideas

1. a) Would you expect the subtraction $-3.5 - (-4.3)$ to give a positive answer or a negative answer? Explain.
   b) Evaluate $-3.5 - (-4.3)$.
2. How do the values of the following two products compare? Explain your reasoning.
   $2.54 \times (-4.22)$ -2.54×4.22
3. Leslie evaluated $-2.2 + 4.6 \times (-0.5)$ as -1.2. Zack evaluated the same expression as -4.5. Who was correct? Explain.

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As students examine Method 2 in part a), you might ask:

- Why does the expression shown represent the total temperature decrease? (If students have difficulty, refer them to Method 1. Ask how the stages in Method 1 help them to understand the expression used in Method 2.)
- Which is easier to understand, Method 1 or Method 2? Explain.
- Which of the two methods is more efficient? Explain.
- Do you prefer Method 1 or Method 2? Explain.

After students have discussed the two methods, draw their attention to the calculator sequence at the end of Method 2. Emphasize that this sequence works on a calculator that follows the order of operations. Ask students to repeat the calculation on their own calculator. Ask if they need to modify the keying sequence to obtain the correct answer, e.g., through the use of parentheses. Provide assistance to individual students who have difficulty.

As students examine part b), you might ask:

- Why is division used to determine the average rate of decrease?
- Does the estimate suggest that the answer is reasonable? Explain.
- What other reasonable estimate can you suggest?

Again, stress the importance of the summary statement at the end of the solution to explain the meaning of the calculated value, -1.11.

When students are ready to begin the Show You Know, challenge them to write a single expression for part a) and evaluate it using the order of operations. Suggest that students who have difficulty should instead work in stages. After these students have completed their solutions, ask if they can now use their stages to construct a single expression that represents the problem. Ask students if they now prefer one method over the other. Make it clear that students are free to choose either method when solving a problem.

Key Ideas

This section summarizes operations with rational numbers in decimal form. To check students' understanding, you might ask how the number line models the given examples of addition and subtraction. Emphasize that while students are now working with a different category of numbers than in previous grades, the order of operations remains the same. Students could prepare their own list of Key Ideas, with their own examples, and include it in their Foldable.

Meeting Student Needs

- It may be better for your class to work through the examples as a whole-class activity. Assign part of the Show You Know activities as small-group or pair activities. Then, assign the rest of the Show You Know activities as individual student work.

Common Errors

- Some students may have difficulty in performing mental estimations of sums and differences with integers.

R_x Encourage students to model integer addition on a number line and to model integer subtraction by adding the opposite on a number line.

- Some students may make inputting or procedural errors when using a calculator to perform operations with rational numbers, but may accept the result in the calculator display without question.

R_x Stress the importance of using an estimate to check the reasonableness of a calculated answer, whether it is obtained using paper and pencil or a calculator. Also, encourage students to check that the sign of a product or quotient agrees with the sign rule for multiplication or division.

- Some students may try to apply the sign rules for multiplication and division when performing addition and subtraction.

R_x Demonstrate that this approach is not valid. For example, show on a number line that adding two negative decimals gives a negative sum, not a positive sum, and that the subtraction of negative decimals may give a positive or a negative result. If students lack confidence in working with negative decimals, have them check that the sign rules for multiplication and division do not apply to the addition and subtraction of integers.

Answers

Example 1: Show You Know

a) Example: Estimate: -2. Answer: -2.86

b) Example: Estimate: -5. Answer: -4.8

Example 2: Show You Know

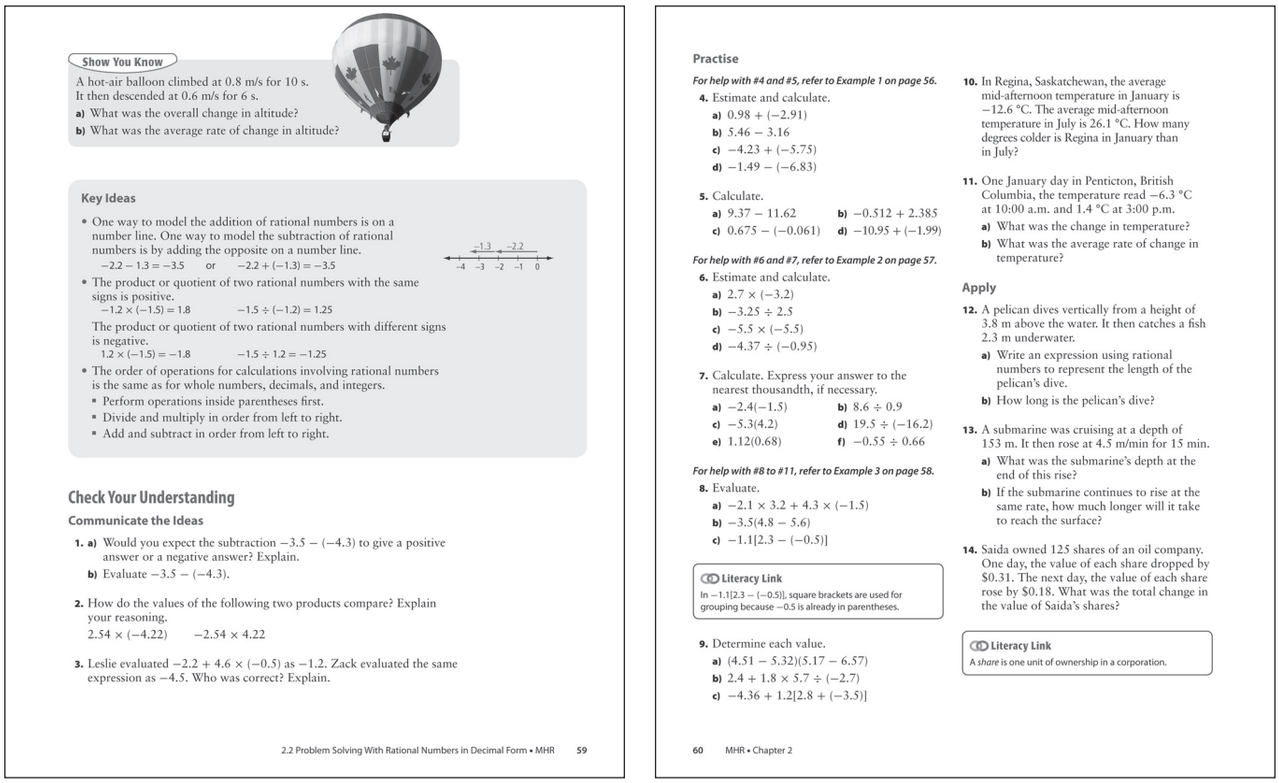
a) Example: Estimate: 3. Answer: 3.64

b) Example: Estimate: -0.6. Answer: -0.6

Example 3: Show You Know

a) $+4.4$ m **b)** $+0.275$ m/s

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may wish to provide students with **Master 4 Number Lines** to use as they work on Example 1. • Encourage students to estimate their answers before completing them. Some students may benefit from a number line to visualize the process. • Encourage students to use a solution method that does not involve a calculator before they solve using a calculator. It is important that they can recognize a reasonable estimate and answer to the question before calculating using a calculator.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from using hundred grids and base ten blocks to multiply the numbers.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • For students who are having difficulty, go over the order of operations with them and encourage them to complete only one step per line, showing all of their work. This will help them keep track of where they are in the process and help you identify where they are going off track.



Show You Know

A hot-air balloon climbed at 0.8 m/s for 10 s. It then descended at 0.6 m/s for 6 s.

a) What was the overall change in altitude?

b) What was the average rate of change in altitude?

Key Ideas

- One way to model the addition of rational numbers is on a number line. One way to model the subtraction of rational numbers is by adding the opposite on a number line.
  $-2.2 - 1.3 = -3.5$ or $-2.2 + (-1.3) = -3.5$
- The product or quotient of two rational numbers with the same signs is positive.
  $-1.2 \times (-1.5) = 1.8$ $-1.5 \div (-1.2) = 1.25$
  The product or quotient of two rational numbers with different signs is negative.
  $1.2 \times (-1.5) = -1.8$ $-1.5 \div 1.2 = -1.25$
- The order of operations for calculations involving rational numbers is the same as for whole numbers, decimals, and integers.
  - Perform operations inside parentheses first.
  - Divide and multiply in order from left to right.
  - Add and subtract in order from left to right.

Check Your Understanding

Communicate the Ideas

1. a) Would you expect the subtraction $-3.5 - (-4.3)$ to give a positive answer or a negative answer? Explain.

b) Evaluate $-3.5 - (-4.3)$.

2. How do the values of the following two products compare? Explain your reasoning.
$2.54 \times (-4.22)$ -2.54×4.22

3. Leslie evaluated $-2.2 + 4.6 \times (-0.5)$ as -1.2. Zack evaluated the same expression as -4.5. Who was correct? Explain.

Practise

For help with #4 and #5, refer to Example 1 on page 56.

4. Estimate and calculate.
a) $0.98 + (-2.91)$
b) $5.46 - 3.16$
c) $-4.23 + (-5.75)$
d) $-1.49 - (-6.83)$

5. Calculate.
a) $9.37 - 11.62$ **b)** $-0.512 + 2.385$
c) $0.675 - (-0.061)$ **d)** $-10.95 + (-1.99)$

For help with #6 and #7, refer to Example 2 on page 57.

6. Estimate and calculate.
a) $2.7 \times (-3.2)$
b) $-3.25 \div 2.5$
c) $-5.5 \times (-5.5)$
d) $-4.37 \div (-0.95)$

7. Calculate. Express your answer to the nearest thousandth, if necessary.
a) $-2.4(-1.5)$ **b)** $8.6 \div 0.9$
c) $-5.3(4.2)$ **d)** $19.5 \div (-16.2)$
e) $1.12(0.68)$ **f)** $-0.55 \div 0.66$

For help with #8 to #11, refer to Example 3 on page 58.

8. Evaluate.
a) $-2.1 \times 3.2 + 4.3 \times (-1.5)$
b) $-3.5(4.8 - 5.6)$
c) $-1.1[2.3 - (-0.5)]$

Literacy Link
In $-1.1[2.3 - (-0.5)]$, square brackets are used for grouping because -0.5 is already in parentheses.

9. Determine each value.
a) $(4.51 - 5.32)(5.17 - 6.57)$
b) $2.4 + 1.8 \times 5.7 \div (-2.7)$
c) $-4.36 + 1.2[2.8 + (-3.5)]$

10. In Regina, Saskatchewan, the average mid-afternoon temperature in January is -12.6 °C. The average mid-afternoon temperature in July is 26.1 °C. How many degrees colder is Regina in January than in July?

11. One January day in Penticton, British Columbia, the temperature read -6.3 °C at 10:00 a.m. and 1.4 °C at 3:00 p.m.
a) What was the change in temperature?
b) What was the average rate of change in temperature?

Apply

12. A pelican dives vertically from a height of 3.8 m above the water. It then catches a fish 2.3 m underwater.
a) Write an expression using rational numbers to represent the length of the pelican's dive.
b) How long is the pelican's dive?

13. A submarine was cruising at a depth of 153 m. It then rose at 4.5 m/min for 15 min.
a) What was the submarine's depth at the end of this rise?
b) If the submarine continues to rise at the same rate, how much longer will it take to reach the surface?

14. Saida owned 125 shares of an oil company. One day, the value of each share dropped by $0.31. The next day, the value of each share rose by $0.18. What was the total change in the value of Saida's shares?

Literacy Link
A *share* is one unit of ownership in a corporation.

Check Your Understanding

Communicate the Ideas

These questions allow students to explain aspects of operations on rational numbers in decimal form.

In #1, students may think in terms of relative sizes (i.e., subtracting a smaller number from a larger number) or they may think in terms of making an estimate (with or without using a number line). You might ask students to explain why it would or would not be a good idea to use $-4 - (-4)$ to determine an estimate.

In #2, students can reason from the sign rule for multiplication.

In #3, students observe the effect of not following the order of operations but instead operating from left to right, which is a common error.

Practise

You may wish to have students work in pairs or small groups when completing the Practise questions. Encourage students to compare their solution methods and discuss their advantages and disadvantages.

Literacy Link Before students work on #8 or 9, refer them to the Literacy Link on grouping with square brackets. You might ask students why the value -0.5 is in round brackets in #8c). Students should know from their earlier work with integers that a negative number is written in brackets if it follows an operation symbol. Students may have already encountered square brackets as grouping symbols when applying the order of operations. Remind students that the order of operations applies to calculations involving both square and round brackets.

Before students complete #10, you may wish to remind them that they met this context at the beginning of section 2.2. At that stage, they did not know how to perform operations on negative rational numbers, so they were limited to estimation with integers. Now, they can calculate an exact answer and compare it with their earlier estimate.

15. In dry air, the temperature decreases by about 0.65 °C for each 100-m increase in altitude.

a) The temperature in Red Deer, Alberta, is 10 °C on a dry day. What is the temperature outside an aircraft 2.8 km above the city?

b) The temperature outside an aircraft 1600 m above Red Deer is −8.5 °C. What is the temperature in the city?

16. Bella is more comfortable working with integers than with positive and negative decimal numbers. This is her way of understanding −4.3 + 2.5.

−4.3 is $\frac{-43}{10}$ or −43 tenths.

2.5 is $\frac{25}{10}$ or 25 tenths.

−43 tenths + 25 tenths is −18 tenths.

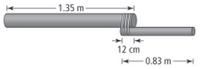
−18 tenths is $\frac{-18}{10}$ or −1.8.

So, −4.3 + 2.5 = −1.8.

a) Use Bella's method to determine 6.1 + (−3.9).

b) How could you modify Bella's method to determine 1.25 − 3.46?

17. Two wooden poles measured 1.35 m and 0.83 m in length. To make a new pole, they were attached by overlapping the ends and tying them together. The length of the overlap was 12 cm. What was the total length of the new pole in metres?



18. Determine the mean of each set of numbers. Express your answer to the nearest hundredth, if necessary.

a) 0, −4.5, −8.2, 0.4, −7.6, 3.5, −0.2

b) 6.3, −2.2, 14.9, −4.8, −5.3, 1.6

19. A company made a profit of $8.6 million in its first year. It lost $5.9 million in its second year and lost another $6.3 million in its third year.

a) What was the average profit or loss per year over the first three years?

b) The company broke even over the first four years. What was its profit or loss in the fourth year?

20. Research to find out the current price of gasoline in Calgary. It is 300 km from Calgary to Edmonton. How much less would it cost to drive this distance in a car with a fuel consumption of 5.9 L/100 km than in a car with a fuel consumption of 9.4 L/100 km? Give your answer in dollars, expressed to the nearest cent.

WWW Web Link

To find out prices of gas in Calgary, go to www.mathlinks9.ca and follow the links.

21. Andrew drove his car 234 km from Dawson to Mayo in Yukon Territory in 3 h. Brian drove his truck along the same route at an average speed of 5 km/h greater than Andrew's average speed. How much less time did Brian take, to the nearest minute?

22. An aircraft was flying at an altitude of 2950 m. It descended for 3 min at 2.5 m/s and then descended for 2.5 min at 2.8 m/s. What was the plane's altitude after the descent?

23. One week in October in Iqaluit, Nunavut, the daily high temperatures were −4.6 °C, −0.5 °C, 1.2 °C, 2.3 °C, −1.1 °C, 1.5 °C, and −3.0 °C. What was the mean daily high temperature that week?

Did You Know?

Iqaluit is the capital of Nunavut. This territory covers almost $\frac{1}{5}$ of the area of Canada but has less than $\frac{1}{1000}$ of the Canadian population. Over $\frac{3}{5}$ of the people who live in Nunavut are Inuit.

WWW Web Link

For more information about Nunavut, go to www.mathlinks9.ca and follow the links.

24. Copy and complete each statement.

a) ■ + 1.8 = −3.5

b) −13.3 − ■ = −8.9

c) ■ × (−4.5) = −9.45

d) −18.5 ÷ ■ = 7.4

25. Create a word problem that involves operations with rational numbers in decimal form. Make sure you can solve your problem. Then, have a classmate solve your problem.

Extend

26. Four points, A, B, C, and D, have the following coordinates:

A(0.75, 0.81)
B(0.75, −0.65)
C(−1.22, −0.65)
D(−1.22, 0.81)

What is the area of quadrilateral ABCD, to the nearest hundredth of a square unit?

27. The mean of six rational numbers is −4.3.

a) What is the sum of the rational numbers?

b) If five of the numbers each equal −4.5, what is the sixth number?

28. Evaluate each expression.

a) $3.6 + 2y$, $y = -0.5$

b) $(m - 1.8)(m + 1.8)$, $m = 1.7$

c) $\frac{4.5}{q} - \frac{q}{4.5}$, $q = -3.6$

29. Add one pair of parentheses or square brackets to the left side of each equation to make a true statement.

a) 3.5 × 4.1 − 3.5 − 2.8 = −0.7

b) 2.5 + (−4.1) + (−2.3) × (−1.1) = 4.29

c) −5.5 − (−6.5) ÷ 2.4 + (−1.1) = −0.5

Math Link

Play this game with a partner or in a small group. You will need two dice and one coin.

- For each turn, roll the two dice and toss the coin. Then, repeat.
- Create numbers of the form ■.■ from the result of rolling the two dice.
- Tossing heads means the rational numbers are positive. Tossing tails means the rational numbers are negative.
- Record the two pairs of numbers.
- Choose one number from each pair so that the sum of the chosen numbers is as close as possible to zero. Record the sum of the chosen numbers.
- In each round, the player with the sum one closest to zero wins two points. If there is a tie, each tied player wins one point.
- The winner is the first player with ten points. If two or more players reach ten points in the same round, keep playing until one player is in the lead by at least two points.

The four possible sums are
1.2 + (−5.6) 1.2 + (−6.5)
2.1 + (−5.6) 2.1 + (−6.5)
Estimation shows that 2.1 + (−5.6) is closest to zero, so calculate this sum.
2.1 + (−5.6) = −3.5

Apply

Most of the Apply problems require students to represent given quantities using rational numbers and then to choose appropriate operations. As students work on the problems, encourage them to compare solutions, discuss strategies, identify errors, and improve solutions.

Literacy Link Before students complete #14, you may wish to draw their attention to the related Literacy Link. You may wish to elaborate by indicating that large corporations may have many shareholders and that individual shareholders may own many shares. The price of one share varies greatly from one corporation to another and varies over time for one corporation. Share prices for different corporations typically range from a few cents to a few hundred dollars. Students can readily find examples on the Internet or in the business section of a newspaper. Students may find the related Web Link that follows in this TR useful.

WWW Web Link

For share prices and to learn more about share prices and investment decisions, go to www.mathlink9.ca and follow the links.

When students complete #16, you might ask:

- What do you think of Bella's method?
- What are the advantages and disadvantages of Bella's method in comparison with methods you already know?
- Does Bella's method make it any easier for you to understand the addition and subtraction of rational numbers? Explain.

In #17, some students may not pay attention to the units of measure and may try to evaluate an expression such as 1.35 + 0.83 − 12. If so, show the importance of checking that numerical answers make sense in the context of a problem. The length of the pole is clearly not a negative quantity. Some students may use units correctly but may subtract the length of the overlap twice. If so, use the diagram to explain why the result is not reasonable, or show the effect of an overlap on total length by demonstrating with two rulers.

For #20, point out the Web Link that follows it.

Before students complete #23, you might draw attention to the Did You Know? and Web Link that follow it. You might ask:

- If Canada's population was evenly distributed across its whole area, about how many people would live in Nunavut?
- About how many people actually live in Nunavut?
- About how many people who live in Nunavut are Inuit?

Apply #25 follows up on #4 in the Explore. Students have now come across many more suitable contexts that they may use. As before, encourage originality. Emphasize that students should be able to solve their own problem before challenging a classmate to solve it. Ask students to suggest improvements to each other's problems.

Extend

In #26, students observe a link between operations with rational numbers and coordinate geometry.

In #27, students extend their work from #18 by working backward from the mean.

In #28, students extend their work on the order of operations with rational numbers by first substituting given values into algebraic expressions.

In #29, students will again see how determining the correct value of an expression depends on the application of the order of operations.

Literacy Link Have students develop a Frayer model showing what they already know about decimal numbers at the beginning of section 2.2. Have them revisit their Frayer model at the end of the section.

Math Link

In this Math Link, students apply their skills in adding rational numbers as they play a game with coins and dice. Before they begin playing, point out the examples shown pictorially in the Math Link. Suggest that students have a few trial runs so that they are clear on the rational numbers that they generate. You might also ask students to read the thought bubble in the Math Link and explain how estimation establishes the sum closest to zero in this case.

Students will be asked to design their own game in the Wrap It Up! at the end of this chapter. Therefore, you might ask them to think about the design of the present game and about ways that the game could be modified. You might ask:

- Suppose you used playing cards to generate pairs of numbers, instead of using the two dice. Which cards would you use? How would you use them? Would the game be harder or easier than before? Explain.
- What could you use instead of coins to decide whether the rational numbers are positive or negative?
- If points were awarded for the sum that is furthest from zero, instead of closest to zero, would the game be any harder or easier? Explain.
- If the game involved products instead of sums, would it be any harder or easier? Explain.
- Would you change the way that points are awarded in the game? Explain.

Meeting Student Needs

- Provide **BLM 2–7 Section 2.2 Extra Practice** to students who would benefit from more practice.

ELL

- Have students draw diagrams for the word problems to help them understand what these problems are asking. Clarify key words for each word problem.
- For #13, provide a picture of a submarine so that students understand what the question is asking.

Gifted and Enrichment

- For #26, challenge students to make up related problems involving perimeter and/or area by using coordinates that meet certain criteria. (Example: A square with all its vertices in the third quadrant.) You might also challenge students to create problems similar to #29. Then, have them exchange their problems with a student of similar ability.

Common Errors

- Some students may have difficulty in recalling the correct order of operations.

R_x Encourage the use of a mnemonic, such as BEDMAS or Please Excuse My Dear Aunt Sally (parentheses, exponents, multiplication, division, addition, and subtraction).

- Some students may have difficulty in deciding which operation(s) to use when solving problems.

R_x Encourage students to try many problems and to compare solutions with their classmates' solutions. Ask students to consider which solutions are the most efficient. Also, encourage students to check that answers to problems make sense in relation to the given data.

- Some students may fail to provide a summary statement in their solution to an application problem.

R_x Emphasize that the solution to an application problem should end with a summary statement that clearly answers the original question. You could refer to Example 3 in this section and point out why it was necessary to explain the meanings of the calculated values −5.55 and −1.11.

Answers

Communicate the Ideas

1. a) Example: A positive answer is expected. Rewriting the subtraction as adding the opposite, it becomes $-3.4 + 4.3$. Since 4.3 is further from zero on the number line and positive, the answer will be positive.

b) 0.8

2. Example: The products of the two rational numbers are the same because the values are the same, and each product has a positive and a negative multiplier.

3. Zack is correct. Example: Follow the order of operations. Multiply 4.6 by −0.5, and then add −2.2.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • For #1, some students will benefit from the use of a number line. They will need to recall the rules for adding and subtracting integers. Some students may wish to change the question to adding the opposite. • For #2, it is not necessary for students to carry out the multiplication as it is for them to demonstrate an understanding of the rules for multiplying positive and negative rational numbers. They will need to recall the sign rules for multiplying integers. • Consider having students assess each other's answers to one or more of the questions, using **Master 2 Communication Peer Evaluation**.
Assessment *for* Learning	
Practise and Apply Have students do #4, 6, 8, 10, 12, and 13. Students who have no problems with these questions can go on to the remaining Apply questions.	• You may wish to provide number lines to students to use as they work on these questions. • For students having difficulty with #4 and 6, coach them on the rules for operations on integers. Have them estimate their answers first. If the challenge relates to the sign of the answer, have students create addition/subtraction and multiplication/division sign charts and place them into their Foldable for future reference. • For #8, it will be necessary for students to recall the order of operations. Providing students with the acronym BEDMAS or Please Excuse My Dear Aunt Sally may be helpful. • For students having difficulty with #10, 12, and 13, it may be beneficial for them to have a vertical number line. This will assist them in visualizing the rise and drop in temperature or altitude.
Math Link The Math Link on page 62 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 85.	• Students who need help getting started could use **BLM 2–8 Section 2.2 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Literacy Link Have students work in pairs to develop a Frayer model on decimal numbers.	• Use students' Frayer models to identify their misconceptions about decimals. • At the end of section 2.2, have students revisit their Frayer model and make additions and improvements.
Math Learning Log Have students respond to the following prompts: • Describe how to use a number line to add rational numbers. • Two ways I can complete $-4.7 - (-5.6)$ are …	• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. • Some students may need an addition expression given to them in order to show their understanding for the first question. Keeping the decimal values to tenths will make the task simpler to graph on a number line.