Problem Solving With Rational Numbers in Fraction Form

2.3

***MathLinks 9*, pages 63–71**

Suggested Timing
80–100 minutes

Materials
- ruler

Blackline Masters
Master 2 Communication Peer Evaluation
Master 4 Number Lines
BLM 2–3 Chapter 2 Warm-Up
BLM 2–9 Section 2.3 Extra Practice
BLM 2–10 Section 2.3 Math Link

Mathematical Processes
- [x] Communication (C)
- [x] Connections (CN)
- [x] Mental Math and Estimation (ME)
- [x] Problem Solving (PS)
- [x] Reasoning (R)
- [ ] Technology (T)
- [x] Visualization (V)

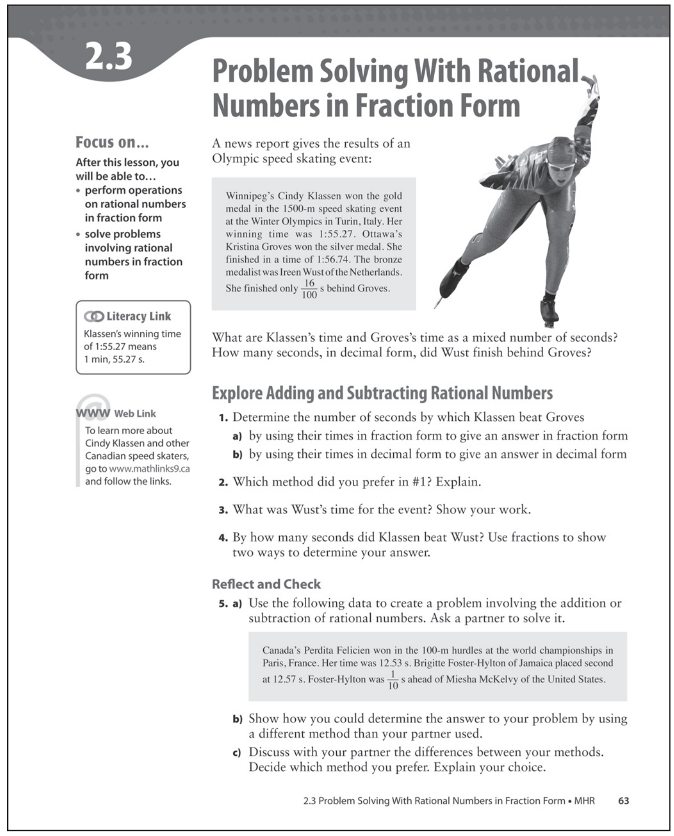
Specific Outcomes
N3 Demonstrate an understanding of rational numbers by:
- comparing and ordering rational numbers
- solving problems that involve arithmetic operations on rational numbers.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 7, 9, 12, 13, Math Link
Typical	#1–5, 7, 9, 12, 13 or 14, 15, one of 16–18, 20, Math Link, History Link
Extension/Enrichment	#1–4, 17, 19 or 20, 21–27, History Link

Planning Notes

Have students complete the warm-up questions on **BLM 2–3 Chapter 2 Warm-Up** to reinforce material learned in previous sections.

Refer students to the photograph on page 63 that shows Cindy Klassen winning the gold medal in the 1500-m speed skating event at the Winter Olympics in Turin, Italy. The news report with the photograph includes numbers expressed in time notation and in fraction form.



2.3

Problem Solving With Rational Numbers in Fraction Form

Focus on...
After this lesson, you will be able to...
- perform operations on rational numbers in fraction form
- solve problems involving rational numbers in fraction form

Literacy Link
Klassen's winning time of 1:55.27 means 1 min, 55.27 s.

Web Link
To learn more about Cindy Klassen and other Canadian speed skaters, go to www.mathlinks9.ca and follow the links.

A news report gives the results of an Olympic speed skating event:

> Winnipeg's Cindy Klassen won the gold medal in the 1500-m speed skating event at the Winter Olympics in Turin, Italy. Her winning time was 1:55.27. Ottawa's Kristina Groves won the silver medal. She finished in a time of 1:56.74. The bronze medalist was Ireen Wust of the Netherlands. She finished only $\frac{16}{100}$ s behind Groves.

What are Klassen's time and Groves's time as a mixed number of seconds? How many seconds, in decimal form, did Wust finish behind Groves?

Explore Adding and Subtracting Rational Numbers

1. Determine the number of seconds by which Klassen beat Groves
   a) by using their times in fraction form to give an answer in fraction form
   b) by using their times in decimal form to give an answer in decimal form
2. Which method did you prefer in #1? Explain.
3. What was Wust's time for the event? Show your work.
4. By how many seconds did Klassen beat Wust? Use fractions to show two ways to determine your answer.

Reflect and Check

5. a) Use the following data to create a problem involving the addition or subtraction of rational numbers. Ask a partner to solve it.

> Canada's Perdita Felicien won in the 100-m hurdles at the world championships in Paris, France. Her time was 12.53 s. Brigitte Foster-Hylton of Jamaica placed second at 12.57 s. Foster-Hylton was $\frac{1}{10}$ s ahead of Miesha McKelvy of the United States.

   b) Show how you could determine the answer to your problem by using a different method than your partner used.
   c) Discuss with your partner the differences between your methods. Decide which method you prefer. Explain your choice.

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Literacy Link Refer students to the Literacy Link on page 63 to make sure that they know how to interpret the times expressed in time notation. To check for understanding, you might ask students to write a time of 3 min, 14.95 s in time notation.

Before students answer the first question in the opening paragraph, you might ask them to explain what mixed numbers are and to give two examples. Some students may give their answers to the first question incorrectly in the form 1:55 $\frac{27}{100}$, i.e., using minutes and seconds. If so, ask students to express the total number of seconds in 1:55.27 as a decimal before converting this total to a mixed number of seconds. The second question involves a conversion from a fraction to the equivalent decimal. Since the fraction is expressed with a denominator of 100, this conversion should be straightforward. After students complete this question, you might check understanding and activate some previous skills by asking:

- What is $\frac{16}{100}$ expressed in lowest terms?
- How did you determine your answer?

After students have determined the fraction in lowest terms correctly as $\frac{4}{25}$, ask:

- What is $\frac{4}{25}$ expressed in decimal form? (Some students may unnecessarily carry out the division at this point.)
- Why did you not need to do any calculations to answer the previous question? (Students already know that $\frac{4}{25}$ and $\frac{16}{100}$ are equivalent, and that the latter equals 0.16 in decimal form.)

Explore Adding and Subtracting Rational Numbers

In this exploration, students solve problems using both decimals and fractions. In #1 and 3, students make use of their answers from the questions in the opening paragraph.

To avoid any confusion surrounding signs in #1 and 4, you might first ask students a question with simpler numbers. For example, Ben and Freya raced through their garden from the back fence to the front door. Ben took 6 s. Freya took 5 s. By how many seconds did Freya beat Ben? It is important for students to realize that the winner takes less time and that to get the positive answer of 1 s, they should subtract the smaller time from the greater time.

Method 1 Have students work on the exploration in pairs or small groups and discuss their answers. When appropriate, ask coaching questions of individuals, groups, or the whole class.

For #1, ask:

- How can you estimate the answer?
- Does your estimate show that the answer is reasonable?
- Which operation can you use to solve the problem?
- How can you carry out the operation when the times are in fraction form?
- How can you carry out the operation when the times are in decimal form?
- Are the numerical answers from the two methods equal? If not, what do you know about the answers? (At least one of them is wrong.)

For #2, ask:

- Is one method easier to understand than the other? Explain.
- Is one method faster than the other? Explain.
- Is it easier to make mistakes with one method than the other? Explain.

For #3, ask:

- How can you estimate the answer?
- Is an estimate useful for this problem?
- Which operation can you use to solve the problem?
- Could you get the answer by carrying out the operation on numbers in decimal form? Explain.
- Could you get the answer by carrying out the operation on numbers in fraction form? Explain.
- Which form would you rather work with here? Explain.

For #4, one method is to subtract Klassen's time from Wust's time. Another is to add the number of seconds by which Klassen beat Groves and the number of seconds by which Groves beat Wust. You might ask:

- Do both of your methods involve the same operation? Explain.
- Which of your two methods do you prefer? Why?
- Are the answers from the two methods equal? If not, what do you know about the answers?
- What are two ways to estimate the answer?
- Would you prefer to solve the problem with decimals instead of fractions? If so, which solution method would you choose? Why?

In #5a), students can create a problem by modelling it on #1, 3, or 4. If they choose to model the problem on #1, you might suggest that they make their problem more open by not specifying the form that must be used in the solution. Leaving the problem more open will permit different methods in #5b). Emphasize that students must check that they can solve their own problem before asking a partner to solve it. If the wording of a problem proves to be unclear, have students clarify the problem on the basis of their partner's feedback.

In #5c), encourage open discussion between partners. Consider extending the discussion to larger groups or the whole class if there are interesting differences of opinion. Stress that students are free to choose their own preferred strategy when solving a problem, but encourage them to identify strategies that work most efficiently and most reliably for them.

Method 2 Have students answer #1 to 4 individually. Circulate to monitor progress and offer guidance, and ask coaching questions of individuals or the whole class when appropriate. Then, have students compare and discuss their answers to these questions with a partner before they complete #5.

You might challenge students who are particularly interested in sports to research the results of other events that are decided on the basis of times, and to write problems based on the data they uncover. For results of past Olympics events, refer to the Web Link that follows in this TR. Encourage students to share their problems with their partners or groups, or with the whole class.

Meeting Student Needs

- It may be better for your class to work through the Explore as a whole-class activity.
- If possible, find some video footage of speed skating. Discuss how the event works and the distances and times involved.
- As an alternative to the section opener, have students find data about sports events in their school or area. Another suggestion is to have students research data on Aboriginal athletes and/or Aboriginal sports events, such as the North American Indigenous Games (NAIG). See the following Web Link in this TR for some relevant sites.

ELL

- Teach the following terms in context: *speed skating*, *seconds*, *fraction form*, and *decimal form*.

WWW Web Link

For data on Summer and Winter Olympics events, go to www.mathlinks9.ca and follow the links.

To research Aboriginal athletes and Aboriginal sports events, go to www.mathlinks9.ca and follow the links.

Answers

Explore Adding and Subtracting Rational Numbers

1. a) $\frac{147}{100}$ s **b)** 1.47 s

2. Example: Decimal form is preferred because the times were given in decimal form.

3. 1:56.90. To obtain the answer, add 0.16 to 1:56.74.

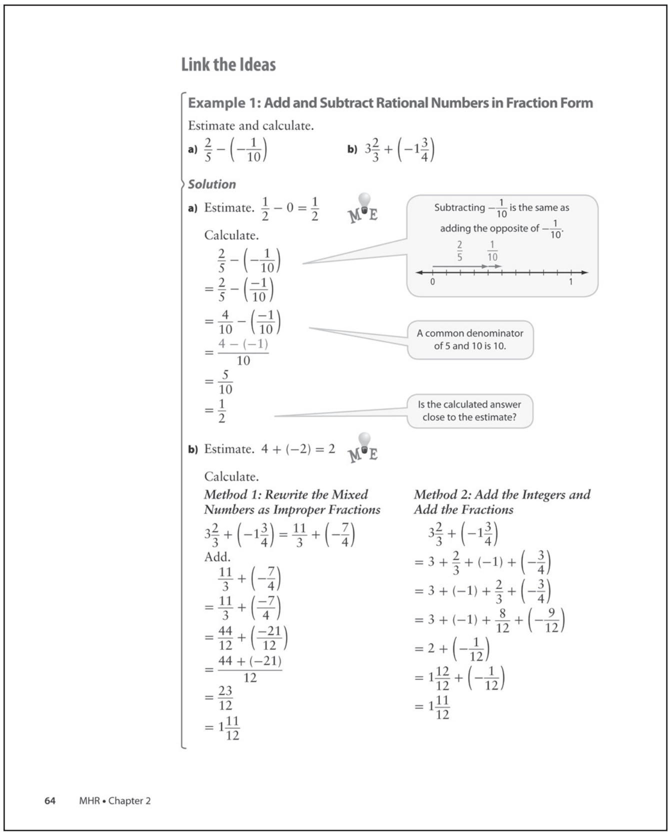
4. 0:01.63 s. Example: The answer could be found by subtracting 1:55.27 from 1:56.90, or by performing the following fraction subtraction: $56\frac{9}{10} - 55\frac{27}{100}$.

5. a) Example: What was Miesha McKelvy's time? Answer: 12.67 s.

b) Example: $12.57 + 0.1 = 12.67$, or using fractions: $12\frac{57}{100} + \frac{1}{10} = 12\frac{67}{100}$.

c) Example: Although both methods involved addition, one method used fractions and the other method used decimals. Decimals might be preferred because the calculation is more apparent.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore.	• For students who have difficulty generating their own word problem, suggest they pattern their problem after the one in the Explore. • After partner discussion in #5c), it would be beneficial to take the discussion up with the class. For each case, it may be beneficial to make a list of pros and cons for decimal form vs. fraction form. The summary could be placed in their Foldable.



Link the Ideas

Example 1: Add and Subtract Rational Numbers in Fraction Form

Estimate and calculate.

a) $\frac{2}{5} - \left(-\frac{1}{10}\right)$ **b)** $3\frac{2}{3} + \left(-1\frac{3}{4}\right)$

Solution

a) Estimate. $\frac{1}{2} - 0 = \frac{1}{2}$

Calculate.

$$\begin{aligned} &\frac{2}{5} - \left(-\frac{1}{10}\right) \\ &= \frac{2}{5} - \left(\frac{-1}{10}\right) \\ &= \frac{4}{10} - \left(\frac{-1}{10}\right) \\ &= \frac{4 - (-1)}{10} \\ &= \frac{5}{10} \\ &= \frac{1}{2} \end{aligned}$$

Subtracting $-\frac{1}{10}$ is the same as adding the opposite of $-\frac{1}{10}$.

A common denominator of 5 and 10 is 10.

Is the calculated answer close to the estimate?

b) Estimate. $4 + (-2) = 2$

Calculate.

Method 1: Rewrite the Mixed Numbers as Improper Fractions

$3\frac{2}{3} + \left(-1\frac{3}{4}\right) = \frac{11}{3} + \left(-\frac{7}{4}\right)$

Add.

$$\begin{aligned} &\frac{11}{3} + \left(-\frac{7}{4}\right) \\ &= \frac{11}{3} + \left(\frac{-7}{4}\right) \\ &= \frac{44}{12} + \left(\frac{-21}{12}\right) \\ &= \frac{44 + (-21)}{12} \\ &= \frac{23}{12} \\ &= 1\frac{11}{12} \end{aligned}$$

Method 2: Add the Integers and Add the Fractions

$$\begin{aligned} &3\frac{2}{3} + \left(-1\frac{3}{4}\right) \\ &= 3 + \frac{2}{3} + (-1) + \left(-\frac{3}{4}\right) \\ &= 3 + (-1) + \frac{2}{3} + \left(-\frac{3}{4}\right) \\ &= 3 + (-1) + \frac{8}{12} + \left(-\frac{9}{12}\right) \\ &= 2 + \left(-\frac{1}{12}\right) \\ &= 1\frac{12}{12} + \left(-\frac{1}{12}\right) \\ &= 1\frac{11}{12} \end{aligned}$$

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Link the Ideas

Example 1

This example extends students' previous work on the addition and subtraction of fractions and mixed numbers to include negative rational numbers.

When students consult part a), first direct their attention to the estimated difference. Students have previously estimated with positive proper fractions by considering whether the fractions are closest to 0, $\frac{1}{2}$, or 1. With the extension to include negative proper fractions, students need to consider whether a given fraction is closest to 1, $\frac{1}{2}$, 0, $-\frac{1}{2}$, or -1. You might ask:

- How do you know that $\frac{2}{5}$ is closest to $\frac{1}{2}$? (Some students may reason numerically, perhaps using equivalent fractions. For example, $\frac{2}{5}$ is $\frac{4}{10}$, which can be compared to 0 or $\frac{0}{10}$; $\frac{1}{2}$ or $\frac{5}{10}$; and 1 or $\frac{10}{10}$. Some students may represent $\frac{2}{5}$ with a diagram, e.g., on a number line, to show that it is closest to $\frac{1}{2}$.)
- How do you know that $-\frac{1}{10}$ is closest to 0? (Students may compare $-\frac{1}{10}$ to 0 or $-\frac{0}{10}$; $-\frac{1}{2}$ or $-\frac{5}{10}$; and -1 or $-\frac{10}{10}$. Some students may prefer to use a number line to establish that $-\frac{1}{10}$ is closest to 0.)

Next, point out the thought bubble that contains the number-line method. To activate students' knowledge and skills in adding positive fractions, you might ask students how they could calculate $\frac{2}{5} + \frac{1}{10}$ numerically.

When students examine the calculation method for the subtraction in part a), ask:

- Why do you need to use a common denominator?
- Is 10 the only common denominator you could use? Explain.
- Is there any advantage to using 10 as the common denominator? Explain.
- Why does $4 - (-1) = 5$?
- How can you rewrite $\frac{5}{10}$ as $\frac{1}{2}$?

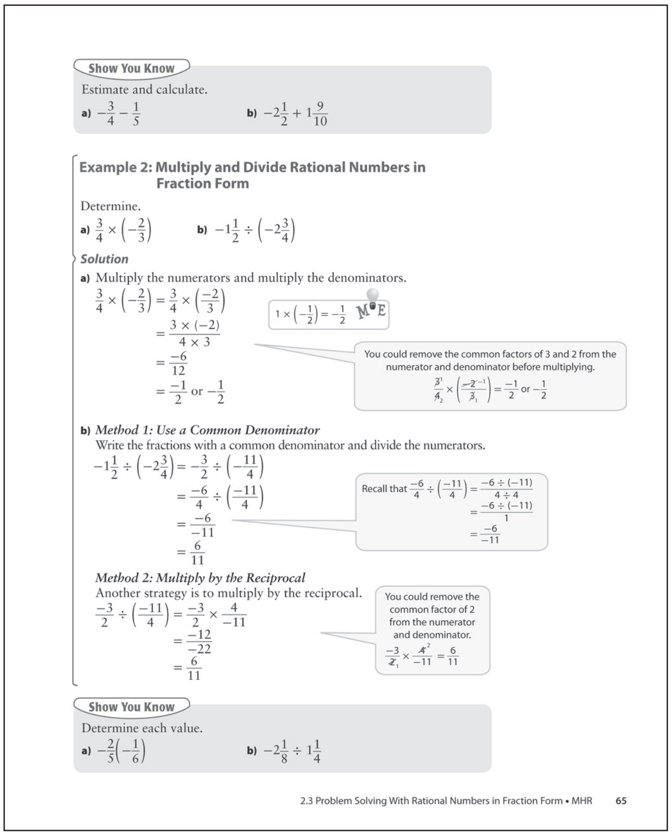
You might challenge students with this question:

- Why is $-\frac{1}{10}$ rewritten as $\frac{-1}{10}$, rather than as $\frac{1}{-10}$? (It is easier to use a positive common denominator. You might show students what would happen if you assigned the negative sign to the denominator and chose -10 as the common denominator. The subtraction would involve $\frac{-4 - 1}{-10} = \frac{-5}{-10}$. The correct answer would result, but there would be more negative signs to contend with in the calculation.)

Draw students' attention to the thought bubble beside the answer to reinforce the importance of comparing the calculated answer to the estimate. In this case, they happen to be equal.

When students consult part b), first direct their attention to the estimated difference. Students have previously estimated with positive mixed numbers by using the whole numbers closest to them. With the extension to include negative mixed numbers, students need to consider the closest integers when estimating. You might ask:

- How do you know that 4 is the closest integer to $3\frac{2}{3}$?
- How do you know that -2 is the closest integer to $-1\frac{3}{4}$?



Show You Know

Estimate and calculate.

a) $-\frac{3}{4} - \frac{1}{5}$ **b)** $-2\frac{1}{2} + 1\frac{9}{10}$

Example 2: Multiply and Divide Rational Numbers in Fraction Form

Determine.

a) $\frac{3}{4} \times \left(-\frac{2}{3}\right)$ **b)** $-1\frac{1}{2} \div \left(-2\frac{3}{4}\right)$

Solution

a) Multiply the numerators and multiply the denominators.

$$\begin{aligned}\frac{3}{4} \times \left(-\frac{2}{3}\right) &= \frac{3}{4} \times \left(\frac{-2}{3}\right)\\ &= \frac{3 \times (-2)}{4 \times 3}\\ &= \frac{-6}{12}\\ &= \frac{-1}{2} \text{ or } -\frac{1}{2}\end{aligned}$$

$1 \times \left(-\frac{1}{2}\right) = -\frac{1}{2}$ ME

You could remove the common factors of 3 and 2 from the numerator and denominator before multiplying.

$\frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \left(\frac{\cancel{-2}^{-1}}{\cancel{3}_{1}}\right) = \frac{-1}{2} \text{ or } -\frac{1}{2}$

b) *Method 1: Use a Common Denominator*

Write the fractions with a common denominator and divide the numerators.

$$\begin{aligned}-1\frac{1}{2} \div \left(-2\frac{3}{4}\right) &= -\frac{3}{2} \div \left(-\frac{11}{4}\right)\\ &= \frac{-6}{4} \div \left(\frac{-11}{4}\right)\\ &= \frac{-6}{-11}\\ &= \frac{6}{11}\end{aligned}$$

Recall that $\frac{-6}{4} \div \left(\frac{-11}{4}\right) = \frac{-6 \div (-11)}{4 \div 4} = \frac{-6 \div (-11)}{1} = \frac{-6}{-11}$

Method 2: Multiply by the Reciprocal

Another strategy is to multiply by the reciprocal.

$$\begin{aligned}\frac{-3}{2} \div \left(\frac{-11}{4}\right) &= \frac{-3}{2} \times \frac{4}{-11}\\ &= \frac{-12}{-22}\\ &= \frac{6}{11}\end{aligned}$$

You could remove the common factor of 2 from the numerator and denominator.

$\frac{-3}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{-11} = \frac{6}{11}$

Show You Know

Determine each value.

a) $-\frac{2}{5}\left(-\frac{1}{6}\right)$ **b)** $-2\frac{1}{8} \div 1\frac{1}{4}$

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There are two calculation methods in part b). When students consult Method 1, you might ask:

- How can you rewrite $3\frac{2}{3}$ as $\frac{11}{3}$, and $-1\frac{3}{4}$ as $-\frac{7}{4}$? (Students have not written negative mixed numbers as improper fractions before. Emphasize that, in a negative mixed number, such as $-1\frac{3}{4}$, both the integer and the fraction are negative. Therefore, $-1\frac{3}{4} = -1 + \left(-\frac{3}{4}\right)$ or $-\frac{4}{4} + \left(-\frac{3}{4}\right)$.)
- If you used a different common denominator, would you get the same answer? Explain.
- How can you rewrite $\frac{23}{12}$ as $1\frac{11}{12}$? (You may wish to point out that both of these ways of writing the answer are correct.)

Method 2 extends a method that students previously used to add positive mixed numbers by adding the whole numbers and adding the fractions. With the inclusion of negative mixed numbers, students now add integers and fractions. When students work through Method 2, ask:

- How do you know that $3 + \frac{2}{3} + (-1) + \left(-\frac{3}{4}\right) = 3 + (-1) + \frac{2}{3} + \left(-\frac{3}{4}\right)$?
- How close is the calculated answer close to the estimate?
- Do you prefer Method 1 or Method 2? Why?

When students complete the Show You Know, encourage them to check each other's answers, discuss their chosen methods, and make improvements to their solutions. The answers to both parts of the Show You Know are negative fractions. Students' answers may show the negative sign in different positions, for example, $-\frac{19}{20}$ or $\frac{-19}{20}$ for part a).

Example 2

This example extends students' previous work on the multiplication and division of fractions and mixed numbers to include negative rational numbers.

In part a), first direct students' attention to the estimated product. Remind students that the estimation technique used here involves deciding if each fraction is closest to 1, $\frac{1}{2}$, 0, $-\frac{1}{2}$, or -1. You might ask:

- How do you know that $-\frac{2}{3}$ is closest to $-\frac{1}{2}$?
- When you decide which value $\frac{3}{4}$ is closest to, is there more than one possibility? How do you know? (You might reassure students that using $\frac{1}{2}$, instead of 1, in estimating $\frac{3}{4}$ would not be "wrong.")

When students examine the calculations in part a), point out that two different methods are shown. The first method involves multiplying the numerators and multiplying the denominators, and then writing the product in lowest terms. The second method is probably less familiar to students at this stage. This method involves the division of the numerator and denominator by common factors before the resulting fractions are multiplied, so that the product obtained is already in lowest terms. To assist students in understanding the second method, you may wish to rewrite the initial expression to show the factors more clearly, i.e., $\frac{3}{4} \times \left(\frac{-2}{3}\right) = \frac{3\times1}{2\times2} \times \frac{2\times(-1)}{3\times1}$. Division of the numerator and denominator by the common factors 3 and 2 results in the expression $\frac{1}{2} \times \frac{-1}{1}$.

Example 3: Apply Operations With Rational Numbers in Fraction Form

At the start of a week, Maka had \$30 of her monthly allowance left. That week, she spent $\frac{1}{5}$ of the money on bus fares, another $\frac{1}{2}$ shopping, and $\frac{1}{4}$ on snacks. How much did she have left at the end of the week?

Solution

You can represent the \$30 Maka had at the beginning of the week by 30.

You can represent the fractions of the money spent by $-\frac{1}{5}$, $-\frac{1}{2}$, and $-\frac{1}{4}$.

Why would you represent the \$30 by a positive rational number?

Why would you represent the fractions of money spent by negative rational numbers?

Calculate each dollar amount spent.

For bus fares:

$$-\frac{1}{5} \times 30 = \frac{-1}{5} \times \frac{30}{1} = \frac{-30}{5} = -6$$

For shopping:

$$-\frac{1}{2} \times 30 = \frac{-1}{2} \times \frac{30}{1} = \frac{-30}{2} = -15$$

For snacks:

$$-\frac{1}{4} \times 30 = \frac{-1}{4} \times \frac{30}{1} = \frac{-30}{4} = -\frac{15}{2} \text{ or } -7.5$$

Determine the total dollar amount spent.
$-6 + (-15) + (-7.5) = -28.5$

You could also calculate the total by adding the three fractions $-\frac{1}{5}$, $-\frac{1}{2}$, and $-\frac{1}{4}$ and multiplying their sum by 30. Why does this strategy work? Which strategy do you prefer?

Determine how much Maka had left.
$30 + (-28.5) = 1.5$

Maka had \$1.50 left at the end of the week.

Show You Know

Stefano had \$46 in a bank account that he was not using. Each month for three months, the bank withdrew $\frac{1}{4}$ of this amount as a service fee. How much was left in the account after the last withdrawal?

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To encourage students to compare the two methods, you might ask:

- By what common factor do you divide the numerator and denominator to write $\frac{-6}{12}$ in lowest terms as $\frac{-1}{2}$?
- Why does dividing the numerator and denominator by common factors of 3 and 2 before multiplying give the same result?
- Which of the two methods do you prefer? Why?

You may wish to point out that assigning the negative sign to the numerator in part a) is arbitrary. To reinforce this idea, you might have students use the expression $\frac{3}{4} \times \left(\frac{2}{-3}\right)$ to complete the calculation by both methods.

Encourage students to check that the calculated product is reasonable by comparison with the estimate.

Part b) includes two division methods that students developed and applied in grade 8 with positive fractions and mixed numbers. Students may not recall them, especially the method that involves writing the fractions with a common denominator and dividing the numerators. If so, you might use a diagram to demonstrate that $\frac{3}{5} \div \frac{1}{5} = 3$, that $\frac{3}{4} \div \frac{3}{8} = 2$ (i.e., $\frac{6}{8} \div \frac{3}{8} = 2$), and that $\frac{3}{2} \div \frac{3}{10} = 5$ (i.e., $\frac{15}{10} \div \frac{3}{10} = 5$). Also, direct students' attention to the thought bubble, which shows how it is possible to divide out the denominator 4. If students do not recall the method involving multiplication by the reciprocal, you might have them check that multiplying $\frac{3}{5} \times \frac{5}{1}$, $\frac{3}{4} \times \frac{8}{3}$, and $\frac{3}{2} \times \frac{10}{3}$ gives the same answers as you demonstrated with diagrams.

When students examine Method 1 in part b), you might ask:

- Could you use a different common denominator? Explain.
- Is there an advantage to using 4 as the common denominator? Explain.
- How do you know $\frac{-6}{4} \div \left(\frac{-11}{4}\right)$ is equal to $\frac{-6}{11}$?
- Is the calculated answer close to the estimate? Explain.
- Is there a different estimate you could use? If so, what is it?

When students examine Method 2 in part b), draw their attention to the thought bubble. Stress that the division of the numerator and denominator by common factors is carried out at the multiplication stage of the solution. Reiterate that students may or may not choose to divide by common factors in this way. Ask:

- What is a reciprocal? Explain using examples.
- By what common factor do you divide the numerator and denominator to express $\frac{-12}{-22}$ in lowest terms as $\frac{6}{11}$?
- Do you prefer Method 1 or Method 2? Explain.

Before students complete the Show You Know, ask them to explain the meaning of the parentheses in part a).

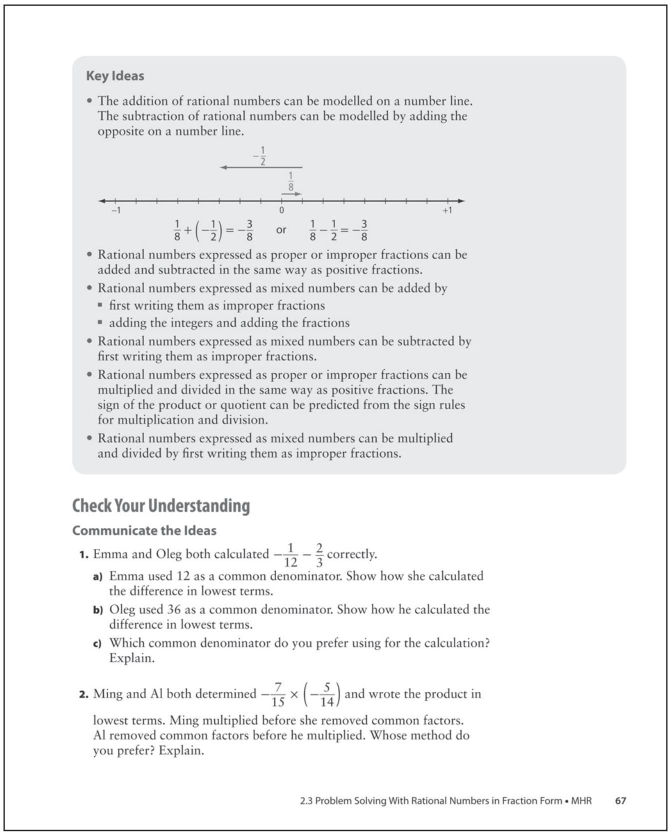
Example 3

This example presents an application of operations with rational numbers in fraction form and extends students' thinking to include problems that involve more than one operation.

Point out the first thought bubble in the solution and ask students to explain why the positive and negative rational numbers are used to represent the quantities given in the problem.

As students examine the solution, you might ask:

- Why is multiplication used to determine each dollar amount spent?
- Why is addition used to determine the total dollar amount spent?



Key Ideas

- The addition of rational numbers can be modelled on a number line. The subtraction of rational numbers can be modelled by adding the opposite on a number line.

$\frac{1}{8} + \left(-\frac{1}{2}\right) = -\frac{3}{8}$ or $\frac{1}{8} - \frac{1}{2} = -\frac{3}{8}$

- Rational numbers expressed as proper or improper fractions can be added and subtracted in the same way as positive fractions.
- Rational numbers expressed as mixed numbers can be added by
  - first writing them as improper fractions
  - adding the integers and adding the fractions
- Rational numbers expressed as mixed numbers can be subtracted by first writing them as improper fractions.
- Rational numbers expressed as proper or improper fractions can be multiplied and divided in the same way as positive fractions. The sign of the product or quotient can be predicted from the sign rules for multiplication and division.
- Rational numbers expressed as mixed numbers can be multiplied and divided by first writing them as improper fractions.

Check Your Understanding

Communicate the Ideas

1. Emma and Oleg both calculated $-\frac{1}{12} - \frac{2}{3}$ correctly.
   a) Emma used 12 as a common denominator. Show how she calculated the difference in lowest terms.
   b) Oleg used 36 as a common denominator. Show how he calculated the difference in lowest terms.
   c) Which common denominator do you prefer using for the calculation? Explain.

2. Ming and Al both determined $-\frac{7}{15} \times \left(-\frac{5}{14}\right)$ and wrote the product in lowest terms. Ming multiplied before she removed common factors. Al removed common factors before he multiplied. Whose method do you prefer? Explain.

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- How could you represent the addition on a number line?
- Why is addition used to determine how much Maka had left? (If students think that the amount Maka had left is represented by 30 − (−28.5), you might ask whether the result of this calculation would make sense in the context of the problem. It clearly would not, in that Maka would appear to have more money than before she started spending.)
- Could you use subtraction to determine how much Maka had left? Explain.
- Would estimation be useful in this problem? Explain.
- Does the summary statement correctly represent the positive numerical answer of 1.5? Explain.

Have students compare the wording of the summary statement to the wording of the original question to demonstrate the function of the summary statement.

Point out the second thought bubble in the solution and have students consider the alternative solution method. You might ask students to write the alternative solution in full, discuss the stages with a partner or in a group, decide which solution they prefer, and explain why. You might challenge students to think of yet another solution. (A third possibility is to add the three negative fractions, add the total to 1 to determine the fraction of the money that was left, and multiply this fraction by 30.)

Example 3 does not include the order of operations, but writing a single expression to represent the solution might be a suitable challenge for some students. You might encourage these students to work together on the expression, or to work individually and then compare their expressions. Emphasize that writing such expressions when solving problems can be difficult and that students are free to solve problems in stages, as shown in the example. A single expression that represents the given solution is $30 + \left(-\frac{1}{5}\right) \times 30 + \left(-\frac{1}{2}\right) \times 30 + \left(-\frac{1}{4}\right) \times 30$.

For the Show You Know, some students may model their solution after the given solution to Example 3 by calculating $-\frac{1}{4} \times 46$ as $-11\frac{1}{2}$, determining the total of the withdrawals as $-11\frac{1}{2} + \left(-11\frac{1}{2}\right) + \left(-11\frac{1}{2}\right)$, and then adding the resulting total to 46. However, some students may realize that, since all three withdrawals are the same, the total of the withdrawals equals $3 \times \left(-11\frac{1}{2}\right)$. Still others might first add $-\frac{1}{4} + \left(-\frac{1}{4}\right) + \left(-\frac{1}{4}\right)$, or multiply $3 \times \left(-\frac{1}{4}\right)$ and then use the resulting fraction either to calculate the total amount withdrawn or to determine the fraction of the funds remaining after the withdrawals. Encourage students to use the method they prefer.

Key Ideas

The Key Ideas summarize operations with rational numbers in fraction form. To check students' understanding, you might ask how the number line models the given examples of addition and subtraction. You might also ask students to make up a worked example for each of the five categories that follow the number line in the Key Ideas. Encourage students to include at least one negative fraction or mixed number in each example. Students could prepare their own list of Key Ideas, with their own examples, and include it in their Foldable.

Meeting Student Needs

- It may be better for your class to work through the examples as a whole-class activity. Assign part of the Show You Know activities for Examples 1 and 2 as small-group or pair activities. Then, assign the rest of these Show You Know activities as individual student work. Assign the Show You Know activity for Example 3 as small-group or pair activities. Then, assign a similar activity to this Show You Know as individual student work.

ELL

- Ensure that students know what *common denominator* means. It may be necessary to clarify by demonstrating on the board how to find a common denominator.
- Read through the Key Ideas with the class. Restate the Key Ideas in different words and using pictures to ensure that students understand all of the concepts. On the board, give examples of rational numbers expressed as proper and improper fractions and as integers.

Gifted and Enrichment

- Ask students to investigate the difficulty in finding lowest common denominators of rational numbers in fraction form. Have them create a list of types of numbers that are especially challenging or particularly easy. As a prompt, you may ask:
  - Why are larger numbers not necessarily more difficult to work with than smaller ones?
  - What are the factors that make finding the lowest common denominator of a pair of numbers easy? Difficult?

  Ask students to explain their reasoning.

Common Errors

- When students estimate the sums and differences of positive and negative proper fractions, some students may be unsure whether each fraction is closest to 1, $\frac{1}{2}$, 0, $-\frac{1}{2}$, or -1.

R$_x$ Encourage students to model fractions on a number line, both to carry out particular estimates and to help develop their number sense.

- Some students may convert negative mixed numbers to improper fractions incorrectly.

R$_x$ Emphasize that a negative mixed number consists of a negative integer and a negative proper fraction. Give an example, such as $-2\frac{1}{3}$, and ask students if this number lies to the left or right of -2 on a horizontal number line. Having established that $-2\frac{1}{3}$ is $-2 + \left(-\frac{1}{3}\right)$, show that it can be written as $-1 + (-1) + \left(-\frac{1}{3}\right)$ and hence as $-\frac{3}{3} + \left(-\frac{3}{3}\right) + \left(-\frac{1}{3}\right)$, resulting in the improper fraction $-\frac{7}{3}$. Ask students to verify that $-2\frac{1}{3}$ and $-\frac{7}{3}$ can be represented by the same point on a number line. Finally, show students a common error, namely treating $-2\frac{1}{3}$ as $-2 + \left(+\frac{1}{3}\right)$. The result of the conversion would then be $-\frac{5}{3}$. Ask students to show that $-2\frac{1}{3}$ and $-\frac{5}{3}$ are represented by different points on a number line.

WWW Web Link

To practise adding and subtracting rational numbers in fraction form, go to www.mathlinks9.ca and follow the links.

To practise multiplying and dividing rational numbers in fraction form, go to www.mathlinks9.ca and follow the links.

Answers

Example 1: Show You Know

a) -1, $-\frac{19}{20}$ **b)** -1, $-\frac{3}{5}$

Example 2: Show You Know

a) $\frac{1}{15}$ **b)** $-1\frac{7}{10}$

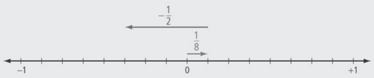
Example 3: Show You Know

$11.50

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to solve the questions in as many ways as possible to ensure that they have additional strategies to draw from. • Provide number lines for students who would benefit from this approach. • For visual learners, you may wish to use basic paper folding to assist them with addition and subtraction of fractions.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • For students who struggle with steps and processes, do not introduce the removal of common factors to them. It is another step that may cause further confusion. • You may need to coach students in changing mixed fractions to improper fractions.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students may have difficulty in deciding what operation needs to be performed. Assist them in recalling from their work with percents in grade 8 that to find a percent of a number, you need to multiply by the percent. Similarly, to find a fraction of a number, you need to multiply by the fraction. • Encourage students to complete one step of the question at a time and to identify verbally the values given and what they mean in the problem.

Key Ideas

- The addition of rational numbers can be modelled on a number line. The subtraction of rational numbers can be modelled by adding the opposite on a number line.



$\frac{1}{8} + \left(-\frac{1}{2}\right) = -\frac{3}{8}$ or $\frac{1}{8} - \frac{1}{2} = -\frac{3}{8}$

- Rational numbers expressed as proper or improper fractions can be added and subtracted in the same way as positive fractions.
- Rational numbers expressed as mixed numbers can be added by
  - first writing them as improper fractions
  - adding the integers and adding the fractions
- Rational numbers expressed as mixed numbers can be subtracted by first writing them as improper fractions.
- Rational numbers expressed as proper or improper fractions can be multiplied and divided in the same way as positive fractions. The sign of the product or quotient can be predicted from the sign rules for multiplication and division.
- Rational numbers expressed as mixed numbers can be multiplied and divided by first writing them as improper fractions.

Check Your Understanding

Communicate the Ideas

1. Emma and Oleg both calculated $-\frac{1}{12} - \frac{2}{3}$ correctly.
   a) Emma used 12 as a common denominator. Show how she calculated the difference in lowest terms.
   b) Oleg used 36 as a common denominator. Show how he calculated the difference in lowest terms.
   c) Which common denominator do you prefer using for the calculation? Explain.

2. Ming and Al both determined $-\frac{7}{15} \times \left(-\frac{5}{14}\right)$ and wrote the product in lowest terms. Ming multiplied before she removed common factors. Al removed common factors before he multiplied. Whose method do you prefer? Explain.

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3. a) Calculate $\frac{-9}{12} \div \frac{3}{8}$ by multiplying by the reciprocal.
   b) Calculate $\frac{-9}{12} \div \frac{3}{8}$ by writing the fractions with a common denominator and dividing the numerators.
   c) Which method do you prefer for this calculation? Explain.

4. Joshua suggested his own method for multiplying and dividing rational numbers in fraction form. For $-\frac{4}{5} \times \frac{2}{3}$, he calculated $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$. Then, he reasoned that the product must be negative because $-\frac{4}{5}$ and $\frac{2}{3}$ have different signs. He gave the answer as $-\frac{8}{15}$. Describe an advantage and a disadvantage of Joshua's method.

Practise

For help with #5 and #6, refer to Example 1 on page 64. Write your answers in lowest terms.

5. Estimate and calculate.
   a) $\frac{3}{10} + \frac{1}{5}$ b) $2\frac{1}{3} + \left(-1\frac{1}{4}\right)$
   c) $-\frac{5}{12} - \frac{5}{12}$ d) $-2\frac{1}{2} - \left(-3\frac{1}{3}\right)$
   e) $-\frac{5}{6} + \frac{1}{3}$ f) $\frac{3}{8} - \left(-\frac{1}{4}\right)$

6. Estimate and calculate.
   a) $\frac{2}{3} - \frac{3}{4}$ b) $-\frac{2}{9} + \left(-\frac{1}{3}\right)$
   c) $-\frac{1}{4} + \left(-\frac{3}{5}\right)$ d) $-\frac{3}{4} - \left(-\frac{5}{8}\right)$
   e) $1\frac{1}{2} - 2\frac{1}{4}$ f) $1\frac{2}{5} + \left(-1\frac{3}{4}\right)$

For help with #7 and #8, refer to Example 2 on page 65. Write your answers in lowest terms.

7. Estimate and calculate.
   a) $\frac{4}{5} \div \frac{5}{6}$ b) $3\frac{1}{3}\left(1\frac{3}{4}\right)$
   c) $\frac{1}{8} \times \left(-\frac{2}{5}\right)$ d) $-\frac{9}{10} \div \left(-\frac{4}{5}\right)$
   e) $-\frac{3}{8} \times 5\frac{1}{3}$ f) $\frac{1}{10} \div \left(-\frac{3}{8}\right)$

8. Estimate and calculate.
   a) $-\frac{3}{4} \times \left(-\frac{1}{9}\right)$ b) $1\frac{1}{3} \div 1\frac{1}{4}$
   c) $-\frac{3}{8} \div \frac{7}{10}$ d) $-2\frac{1}{8} \div 1\frac{1}{4}$
   e) $\frac{7}{9}\left(-\frac{6}{11}\right)$ f) $-1\frac{1}{2} \div \left(-2\frac{1}{2}\right)$

For help with #9 and #10, refer to Example 3 on page 66.

9. Lori owed her mother \$39. Lori paid back $\frac{1}{3}$ of this debt and then paid back $\frac{1}{4}$ of the remaining debt.
   How much does Lori still owe her mother?

10. A carpenter has 64 m of baseboard. He installs $\frac{1}{2}$ of the baseboard in one room. He installs another $\frac{3}{5}$ of the original amount of baseboard in another room. How much baseboard does he have left?

Apply

11. In everyday speech, *in a jiffy* means in a very short time. In science, a specific value sometimes assigned to a jiffy is $\frac{1}{100}$ s. Naima can type at 50 words/min. On average, how many jiffies does she take to type each word?

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Check Your Understanding

Communicate the Ideas

These questions allow students to explain aspects of operations on rational numbers in fraction form.

In #1, students observe that different common denominators can be used in the subtraction of fractions, and decide whether they see any benefit in using the lowest common denominator.

In #2, students decide whether or not they prefer to divide the numerator and denominator by common factors before multiplying fractions. Encourage students to determine the product both ways before they decide which method they prefer.

In #3, students decide which method for dividing fractions they prefer.

In #4, students consider a different method for multiplying and dividing rational numbers. Encourage students to discuss the advantages and disadvantages of the method.

Practise

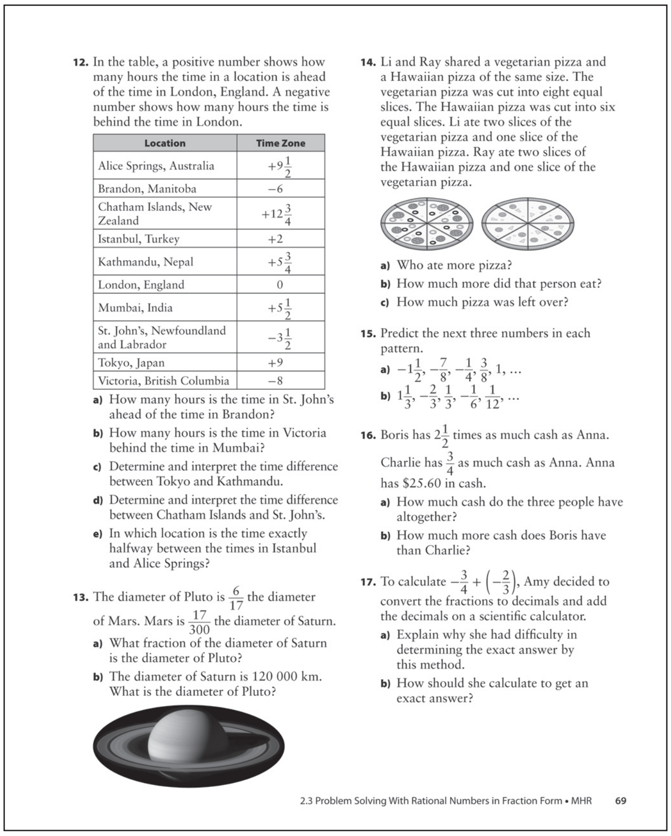
You may wish to have students work in pairs or small groups when completing the Practise questions. Encourage students to compare their chosen solution methods and discuss their advantages and disadvantages. Alternatively, students might complete the Practise questions individually and share any difficulties with their group or the class. You might remind students of the importance of comparing their calculated and estimated values to check that their answers are reasonable.

For #9 and 10, remind students of the need for a summary statement at the end of the solution and the need to check that their answer is reasonable in the context of the problem. For example, an answer greater than \$39 in #9, or greater than 64 m in #10, would clearly not be reasonable.

Apply

Encourage students to think carefully about the operation(s) they need to use to solve word problems. Encourage students to compare their solutions and discuss the advantages and disadvantages of different strategies.

For #14, some students can be expected to do all calculations in stages. They may, for example, calculate $2 \times \frac{1}{8}$ to determine the fraction of a vegetarian pizza that Li ate, and then add $\frac{1}{6}$ of the Hawaiian pizza to calculate Li's total consumption.


12. In the table, a positive number shows how many hours the time in a location is ahead of the time in London, England. A negative number shows how many hours the time is behind the time in London.

Location	Time Zone
Alice Springs, Australia	$+9\frac{1}{2}$
Brandon, Manitoba	-6
Chatham Islands, New Zealand	$+12\frac{3}{4}$
Istanbul, Turkey	$+2$
Kathmandu, Nepal	$+5\frac{3}{4}$
London, England	0
Mumbai, India	$+5\frac{1}{2}$
St. John's, Newfoundland and Labrador	$-3\frac{1}{2}$
Tokyo, Japan	$+9$
Victoria, British Columbia	-8

a) How many hours is the time in St. John's ahead of the time in Brandon?
b) How many hours is the time in Victoria behind the time in Mumbai?
c) Determine and interpret the time difference between Tokyo and Kathmandu.
d) Determine and interpret the time difference between Chatham Islands and St. John's.
e) In which location is the time exactly halfway between the times in Istanbul and Alice Springs?

13. The diameter of Pluto is $\frac{6}{17}$ the diameter of Mars. Mars is $\frac{17}{300}$ the diameter of Saturn.
a) What fraction of the diameter of Saturn is the diameter of Pluto?
b) The diameter of Saturn is 120 000 km. What is the diameter of Pluto?

14. Li and Ray shared a vegetarian pizza and a Hawaiian pizza of the same size. The vegetarian pizza was cut into eight equal slices. The Hawaiian pizza was cut into six equal slices. Li ate two slices of the vegetarian pizza and one slice of the Hawaiian pizza. Ray ate two slices of the Hawaiian pizza and one slice of the vegetarian pizza.
a) Who ate more pizza?
b) How much more did that person eat?
c) How much pizza was left over?

15. Predict the next three numbers in each pattern.
a) $-1\frac{1}{2}, -\frac{7}{8}, -\frac{1}{4}, \frac{3}{8}, 1, \ldots$
b) $1\frac{1}{3}, -\frac{2}{3}, \frac{1}{3}, -\frac{1}{6}, \frac{1}{12}, \ldots$

16. Boris has $2\frac{1}{2}$ times as much cash as Anna. Charlie has $\frac{3}{4}$ as much cash as Anna. Anna has $25.60 in cash.
a) How much cash do the three people have altogether?
b) How much more cash does Boris have than Charlie?

17. To calculate $-\frac{3}{4} + \left(-\frac{2}{3}\right)$, Amy decided to convert the fractions to decimals and add the decimals on a scientific calculator.
a) Explain why she had difficulty in determining the exact answer by this method.
b) How should she calculate to get an exact answer?

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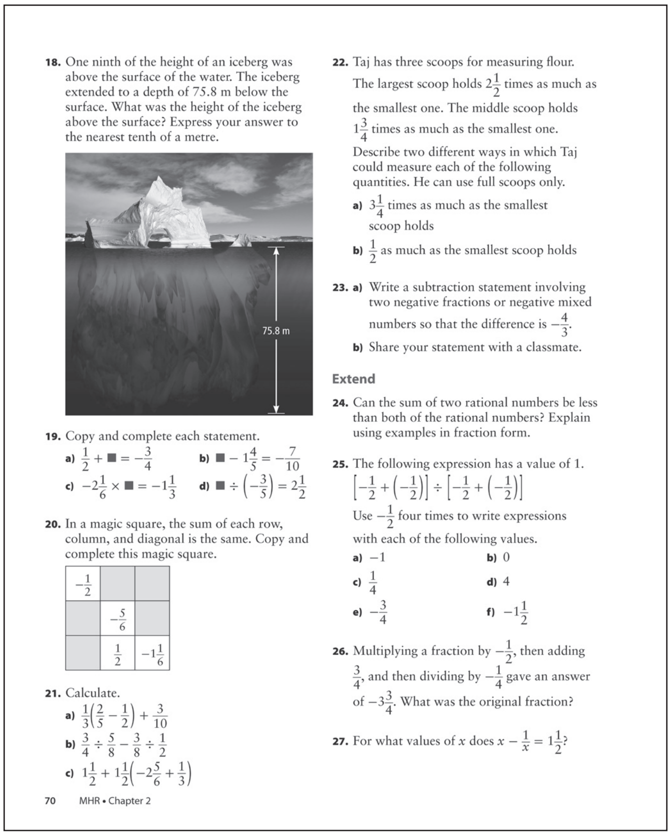
18. One ninth of the height of an iceberg was above the surface of the water. The iceberg extended to a depth of 75.8 m below the surface. What was the height of the iceberg above the surface? Express your answer to the nearest tenth of a metre.

75.8 m

19. Copy and complete each statement.
a) $\frac{1}{2} + ■ = -\frac{3}{4}$
b) $■ - 1\frac{4}{5} = -\frac{7}{10}$
c) $-2\frac{1}{6} \times ■ = -1\frac{1}{3}$
d) $■ \div \left(-\frac{3}{5}\right) = 2\frac{1}{2}$

20. In a magic square, the sum of each row, column, and diagonal is the same. Copy and complete this magic square.

$-\frac{1}{2}$		
	$-\frac{5}{6}$	
	$\frac{1}{2}$	$-1\frac{1}{6}$

21. Calculate.
a) $\frac{1}{3}\left(\frac{2}{5} - \frac{1}{2}\right) + \frac{3}{10}$
b) $\frac{3}{4} \div \frac{5}{8} - \frac{3}{8} \div \frac{1}{2}$
c) $1\frac{1}{2} + 1\frac{1}{2}\left(-2\frac{5}{6} + \frac{1}{3}\right)$

22. Taj has three scoops for measuring flour. The largest scoop holds $2\frac{1}{2}$ times as much as the smallest one. The middle scoop holds $1\frac{3}{4}$ times as much as the smallest one. Describe two different ways in which Taj could measure each of the following quantities. He can use full scoops only.
a) $3\frac{1}{4}$ times as much as the smallest scoop holds
b) $\frac{1}{2}$ as much as the smallest scoop holds

23. a) Write a subtraction statement involving two negative fractions or negative mixed numbers so that the difference is $-\frac{4}{3}$.
b) Share your statement with a classmate.

Extend

24. Can the sum of two rational numbers be less than both of the rational numbers? Explain using examples in fraction form.

25. The following expression has a value of 1.
$\left[-\frac{1}{2} + \left(-\frac{1}{2}\right)\right] \div \left[-\frac{1}{2} + \left(-\frac{1}{2}\right)\right]$
Use $-\frac{1}{2}$ four times to write expressions with each of the following values.
a) -1
b) 0
c) $\frac{1}{4}$
d) 4
e) $-\frac{3}{4}$
f) $-1\frac{1}{2}$

26. Multiplying a fraction by $-\frac{1}{2}$, then adding $\frac{3}{4}$, and then dividing by $-\frac{1}{4}$ gave an answer of $-3\frac{3}{4}$. What was the original fraction?

27. For what values of x does $x - \frac{1}{x} = 1\frac{1}{2}$?

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However, some students may realize that they can write a single expression, $2 \times \frac{1}{8} + \frac{1}{6}$, for this part of the solution and evaluate it using the order of operations. You might ask students which of these approaches they prefer, and why.

In #15, the correct answer can be found by reasoning in different ways. In part a), students are most likely to add $\frac{5}{8}$ to determine a term from the previous term. You might ask if there is an alternative. (Subtracting $-\frac{5}{8}$ would also work.) In part b), a term can be calculated from the previous term by multiplying by $-\frac{1}{2}$ or dividing by -2. You might ask why these different possibilities exist in each part. (You can subtract by adding the opposite. You can divide by multiplying by the reciprocal.)

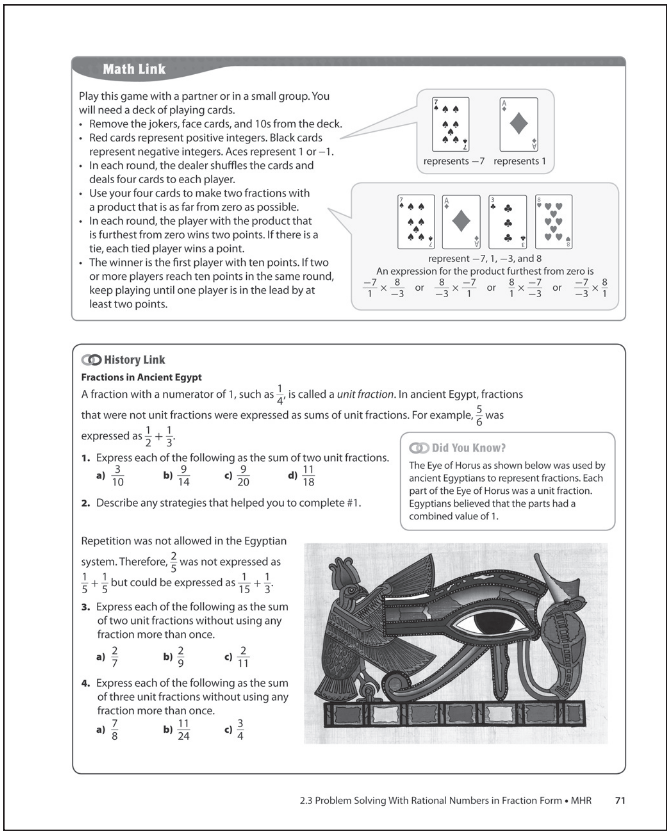
The given data in #16 include two numbers in fraction form and one number in decimal form. You might ask students what form(s) they used in the calculations, and why. Some may prefer to perform operations on the numbers in their given forms, e.g., by multiplying $\frac{3}{4} \times 25.60$ to determine the cash that Charlie has. Other students may prefer to work entirely in decimal form, e.g., by multiplying 0.75×25.60. Opting to work entirely in fraction form, e.g., by multiplying $\frac{3}{4} \times 25\frac{60}{100}$, is a less likely choice, even if the mixed number is expressed in lowest terms (i.e., $\frac{3}{4} \times 25\frac{3}{5}$).

In #17, students will see an advantage of doing some calculations in fraction form, rather than in decimal form. You might ask:

- Would you experience the same difficulty if you added $-\frac{3}{5} + \left(-\frac{1}{8}\right)$ in decimal form? Explain.
- Would you prefer to work in fraction form or decimal form in this case? Explain.

In #20, students need to evaluate the sum of three numbers in fraction form to determine the magic number. For students who have difficulty with this extension, you might suggest that they add only two fractions at a time.

There are many possible answers to #22, and you may wish to challenge students to provide more than two answers for each part. When students are examining alternatives, they may work with the mixed numbers $2\frac{1}{2}$ and $1\frac{3}{4}$, while overlooking the smallest scoop, to which they can assign a value of 1.



Math Link

Play this game with a partner or in a small group. You will need a deck of playing cards.

- Remove the jokers, face cards, and 10s from the deck.
- Red cards represent positive integers. Black cards represent negative integers. Aces represent 1 or −1.
- In each round, the dealer shuffles the cards and deals four cards to each player.
- Use your four cards to make two fractions with a product that is as far from zero as possible.
- In each round, the player with the product that is furthest from zero wins two points. If there is a tie, each tied player wins a point.
- The winner is the first player with ten points. If two or more players reach ten points in the same round, keep playing until one player is in the lead by at least two points.

represents −7 represents 1

represent −7, 1, −3, and 8

An expression for the product furthest from zero is

$\frac{-7}{1} \times \frac{8}{-3}$ or $\frac{8}{-3} \times \frac{-7}{1}$ or $\frac{8}{1} \times \frac{-7}{-3}$ or $\frac{-7}{-3} \times \frac{8}{1}$

History Link

Fractions in Ancient Egypt

A fraction with a numerator of 1, such as $\frac{1}{4}$, is called a *unit fraction*. In ancient Egypt, fractions that were not unit fractions were expressed as sums of unit fractions. For example, $\frac{5}{6}$ was expressed as $\frac{1}{2} + \frac{1}{3}$.

1. Express each of the following as the sum of two unit fractions.
   a) $\frac{3}{10}$ **b)** $\frac{9}{14}$ **c)** $\frac{9}{20}$ **d)** $\frac{11}{18}$
2. Describe any strategies that helped you to complete #1.

Repetition was not allowed in the Egyptian system. Therefore, $\frac{2}{5}$ was not expressed as $\frac{1}{5} + \frac{1}{5}$ but could be expressed as $\frac{1}{15} + \frac{1}{3}$.

3. Express each of the following as the sum of two unit fractions without using any fraction more than once.
   a) $\frac{2}{7}$ **b)** $\frac{2}{9}$ **c)** $\frac{2}{11}$
4. Express each of the following as the sum of three unit fractions without using any fraction more than once.
   a) $\frac{7}{8}$ **b)** $\frac{11}{24}$ **c)** $\frac{3}{4}$

Did You Know?

The Eye of Horus as shown below was used by ancient Egyptians to represent fractions. Each part of the Eye of Horus was a unit fraction. Egyptians believed that the parts had a combined value of 1.

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If students have difficulty with #23, you might suggest they work with integers first. For example, you might ask students to write a subtraction statement involving two negative integers so that the difference is −3.

Extend

In #24, have students compare their examples and reach a generalization about two rational numbers with a sum that is less than both of them.

If students enjoy the challenge of #25, have them also try to write expressions that equal $\frac{1}{16}$, $\frac{1}{3}$, $\frac{3}{8}$, and $\frac{1}{2}$.

In #26, students apply the Work Backward strategy to a problem involving rational numbers.

In #27, students will probably determine $x = 2$ fairly quickly, but they may have more difficulty in determining $x = -\frac{1}{2}$. To challenge students further, you might replace $1\frac{1}{2}$ in the equation by $2\frac{2}{3}$, $3\frac{3}{4}$, and $4\frac{4}{5}$. Then, have students look for patterns, predict the values of x for $x - \frac{1}{x} = 9\frac{9}{10}$, and check their predictions.

Literacy Link Have students show what they already know about fractions at the beginning of this section. You may wish to have them revisit their Frayer models at the end of the section.

Math Link

In this Math Link, students apply their skills in multiplying rational numbers as they play a card game. Before they begin playing, point out the examples shown pictorially in the Math Link. Suggest that students have a few trial runs so that they are clear on the rational numbers that are generated. You might ask students to describe a strategy they can use to identify an expression for the product furthest from zero.

Students will be asked to design their own game in the Wrap It Up! at the end of this chapter. Therefore, you might ask them to think about the design of this card game and about ways that the game might be modified. You might ask:

- Why are the jokers and face cards removed from the deck?
- Is there an alternative to removing these cards? Explain.
- Suppose the 10s were not removed from the deck. Could you still play the game? Would it be any harder or easier than before? Explain.
- Suppose you rolled a red die to create positive integers and a black die to create negative integers, instead of using red and black cards. Would the game be harder or easier than before? Explain.
- Could you use spinners to create positive and negative integers? If so, what would your spinners look like? What integers could they create?
- Suggest other ways of creating positive and negative integers to be used in the game.
- If points were awarded for the sum that is furthest from zero, instead of the product, would the game be any harder or easier? Explain.
- Would you change the way that points are awarded in the game? Explain.

This History Link is an extension exploration set in the context of the Ancient Egyptian mathematical system. Refer students to the visual of the Eye of Horus. You may wish to show how the parts are represented by unit fractions (see *MathLinks 7*, page 228). Have students work in pairs or small groups, so that they can compare their answers and their strategies.

Many students will initially approach #1 by working forward and using the Guess and Check strategy. Encourage them to look for other strategies. If students have difficulty, you might assist them with some coaching questions. For example, in part b), students may realize that $\frac{9}{14}$ is a little more than $\frac{1}{2}$. They may, therefore, try $\frac{1}{2}$ as one of the unit fractions and determine that $\frac{1}{7}$ is the other. After students have obtained this answer, you might ask:

- How can you write $\frac{9}{14}$ so that the numerator is the sum of two natural numbers? (The possibilities are $\frac{8+1}{14}$, $\frac{7+2}{14}$, $\frac{6+3}{14}$, and $\frac{5+4}{14}$.)
- How can you use each of your answers to write $\frac{9}{14}$ as the sum of two fractions with natural-number numerators? (The possibilities are $\frac{8}{14} + \frac{1}{14}$, $\frac{7}{14} + \frac{2}{14}$, $\frac{6}{14} + \frac{3}{14}$, and $\frac{5}{14} + \frac{4}{14}$.)
- What are some sums in which you can express both fractions as unit fractions? ($\frac{7}{14} + \frac{2}{14}$ can be expressed as $\frac{1}{2} + \frac{1}{7}$ in lowest terms.)
- Why is it that you can express both fractions as unit fractions? (Both 7 and 2 are factors of 14.)
- Does your previous answer suggest a strategy for expressing a fraction as the sum of two unit fractions? (Express the numerator of the fraction as the sum of two factors of the denominator.)

After students have developed a strategy in #1 and described it in #2, you might have them test it on some other fractions, such as $\frac{4}{9}$, $\frac{11}{30}$, $\frac{8}{15}$, and $\frac{10}{21}$. You might also ask if the strategy gives more than one answer in any of these cases. (Because 11 can be expressed as $10 + 1$ and as $6 + 5$, $\frac{11}{30}$ can be expressed as $\frac{1}{3} + \frac{1}{30}$ or as $\frac{1}{5} + \frac{1}{6}$.)

You might challenge students to return to #1a) and find a different way to express $\frac{3}{10}$ as the sum of two unit fractions. They will likely have determined $\frac{1}{5} + \frac{1}{10}$ already. Expressing $\frac{3}{10}$ as the equivalent fraction $\frac{6}{20}$ results in $\frac{5}{20} + \frac{1}{20}$ or $\frac{1}{4} + \frac{1}{20}$. If you assigned $\frac{8}{15}$ (as suggested in the previous paragraph), students most likely expressed it as $\frac{1}{3} + \frac{1}{5}$. You might ask for a different way to express it. Expressing $\frac{8}{15}$ as $\frac{16}{30}$ also results in $\frac{1}{30} + \frac{1}{2}$.

In #3, to avoid repetition, students will need to work with equivalent fractions. You might begin by asking students to describe the process they would use to write $\frac{2}{5}$ as $\frac{1}{15} + \frac{1}{3}$. The presence of $\frac{1}{15}$ in the sum is a clue that students can first express $\frac{2}{5}$ as $\frac{6}{15}$, then $\frac{6}{15} = \frac{1}{15} + \frac{5}{15}$ or $\frac{1}{15} + \frac{1}{3}$. Note that multiples of 15 could also be used as the denominator; for example, $\frac{12}{30} = \frac{2}{30} + \frac{10}{30}$ or $\frac{1}{15} + \frac{1}{3}$.

In #3a), expressing $\frac{2}{7}$ as $\frac{8}{28}$ gives $\frac{7}{28} + \frac{1}{28}$ or $\frac{1}{4} + \frac{1}{28}$. Note that students may give different answers. For example, in #3b), using $\frac{4}{18}$ gives $\frac{3}{18} + \frac{1}{18}$ or $\frac{1}{6} + \frac{1}{18}$, but using $\frac{10}{45}$ results in $\frac{9}{45} + \frac{1}{45}$ or $\frac{1}{5} + \frac{1}{45}$.

In #4, students extend their learning to include sums of three unit fractions. Note that students may again give different answers. For example, in #4a), working with $\frac{7}{8}$ results in $\frac{1}{8} + \frac{1}{4} + \frac{1}{2}$, whereas using the equivalent fraction $\frac{21}{24}$ reveals $\frac{1}{24} + \frac{1}{3} + \frac{1}{2}$. If you wish to assign more fractions in #4, students might also try $\frac{3}{7}$, $\frac{5}{9}$, and $\frac{8}{13}$.

Meeting Student Needs

- Provide **BLM 2–9 Section 2.3 Extra Practice** to students who would benefit from more practice.

ELL

- Ensure that students know what the terms *difference*, *lowest terms*, and *calculation* mean.
- For #4, give examples of advantages and disadvantages of an activity students would be familar with (e.g., bringing lunch to school, doing homework before engaging in recreational activities).
- For #9, students might not understand the terms *owed*, *paid back*, and *debt*. Provide an example to demonstrate these terms, using the names of some students in the class. For example, pretend to give a student $10 and then indicate that the student owes money to you and that it needs to be paid back.

Gifted and Enrichment

- Encourage students to complete the History Link.

Common Errors

- Some students may have difficulty dividing the numerator and denominator by common factors before multiplying fractions.

Rx Have students practise this technique by beginning with expressions that contain only positive fractions and that require the division of only one number in the numerator and one number in the denominator by a common factor, e.g., $\frac{3}{8} \times \frac{4}{5}$. Then, move on to expressions with positive fractions that require the division of both numbers in the numerator and both numbers in the denominator by common factors, e.g., $\frac{5}{6} \times \frac{9}{10}$. When students have developed some facility, begin to introduce negative fractions. If division by common factors remains a stumbling block, remind students that they can choose to multiply without first dividing by common factors.

Answers

Communicate the Ideas

1. a) $\frac{-1}{12} + \frac{-8}{12} = \frac{-9}{12} = \frac{-3}{4}$ **b)** $\frac{-3}{36} + \frac{-24}{36} = \frac{-27}{36} = \frac{-3}{4}$

c) Example: 12 is the preferred common denominator because the numbers involved in the computation are smaller.

2. Example: Al's method is preferred because with this method there is no need to rewrite the answer in lowest terms.

3. a) $\frac{-9}{12} \times \frac{8}{3} = -2$ **b)** $\frac{-18}{24} \div \frac{9}{24} = -2$

c) Example: Multiplying by the reciprocal is preferred because it involves fewer computations.

4. Example: An advantage is there are no signs to deal with when doing the computation. A disadvantage is you may forget to place the correct sign into the final answer.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to model their preferred methods for #1 to 3 in their Foldable for future reference. Challenge them to be comfortable in solving correctly with their least preferred method. • Consider handing out copies of **Master 2 Communication Peer Evaluation** for students to assess each other's answers to one or more of the questions.
Assessment *for* Learning	
Practise and Apply Have students do #5, 7, 9, 12, and 13. Students who have no problems with these questions can go on to the remaining Apply questions.	• Students may find it helpful to use **Master 4 Number Lines** for some of the Practise questions. • For #5, remind students that subtracting a negative is the same as adding a positive. • If students are struggling with the process in #5, encourage them to refer to their Foldable and/or Example 1 to identify the steps needed to solve. Encourage them to write out each step. • For students who can solve in more than one way, encourage them to attempt #7 by completing the left column of questions using one method and the right using another. • For #9 and 13, students should again be encouraged to write out the important information given and complete the work in steps. Encourage them to verbalize what they know and are trying to find. Be sure they show all their calculations. • For #13, have students write out the mathematical statement before solving it.
Math Link The Math Link on page 71 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 85.	• Students who need help getting started could use **BLM 2–10 Section 2.3 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Literacy Link At the beginning of this section, have students work in pairs to develop a Frayer model on fractions.	• Use students' Frayer models to identify their misconceptions about fractions. • At the end of the section, have students revisit their Frayer models and make additions and improvements.
Math Learning Log Have students respond to the following prompts: • Two ways to add rational numbers that are mixed numbers are ... The method I prefer is ... because ... • How do you complete the following subtraction by adding the opposite: $\frac{1}{2} - \frac{2}{3}$? Explain your thinking.	• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. • If students have not generated their own examples and explanations of the various solution methods, have them write their responses to the Math Learning Log questions and place them into their Foldable.