Determining Square Roots of Rational Numbers

2.4

***MathLinks 9*, pages 72–81**

Suggested Timing
80–100 minutes

Materials
- ruler
- grid paper
- calculator

Blackline Masters
Master 2 Communication Peer Evaluation
Master 4 Number Lines
Master 8 Centimetre Grid Paper
Master 9 0.5 Centimetre Grid Paper
BLM 2–3 Chapter 2 Warm-Up
BLM 2–11 Section 2.4 Extra Practice
BLM 2–12 Section 2.4 Math Link

Mathematical Processes
- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

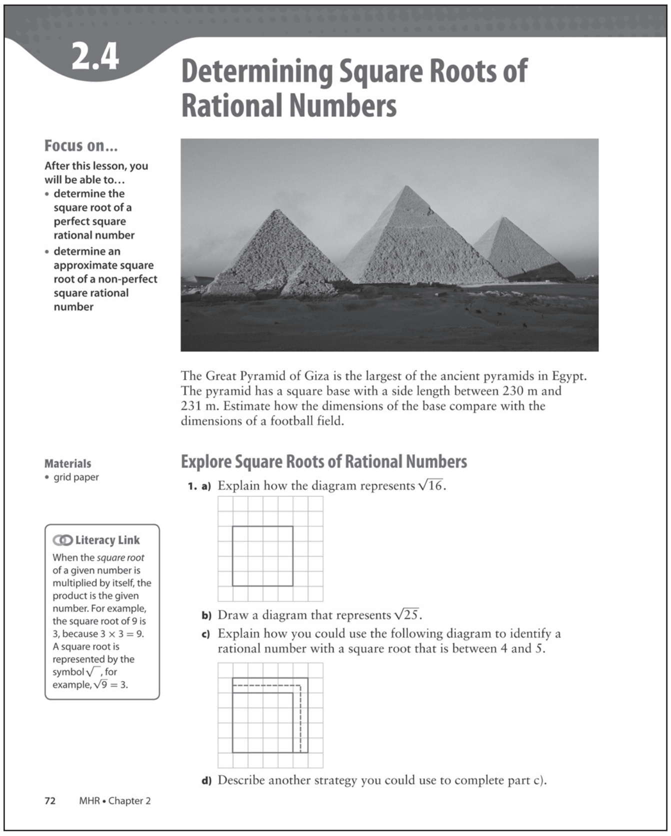
Specific Outcomes
N3 Demonstrate an understanding of rational numbers by:
- comparing and ordering rational numbers
- solving problems that involve arithmetic operations on rational numbers.

N5 Determine the square root of positive rational numbers that are perfect squares.

N6 Determine an approximate square root of positive rational numbers that are non-perfect squares.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–6, 7, 9, 11, 13, 15, 16, Math Link
Typical	#1–7, 9, 11, 13, 15, 16, two of 17–21, 26, 29, Math Link
Extension/Enrichment	#1–4, 17, 20, 23, 24, 29, 31–36

Planning Notes
Have students complete the warm-up questions on **BLM 2–3 Chapter 2 Warm-Up** to reinforce material learned in previous sections.


2.4
Determining Square Roots of Rational Numbers

Focus on...
After this lesson, you will be able to...
- determine the square root of a perfect square rational number
- determine an approximate square root of a non-perfect square rational number

The Great Pyramid of Giza is the largest of the ancient pyramids in Egypt. The pyramid has a square base with a side length between 230 m and 231 m. Estimate how the dimensions of the base compare with the dimensions of a football field.

Materials
- grid paper

Literacy Link
When the *square root* of a given number is multiplied by itself, the product is the given number. For example, the square root of 9 is 3, because $3 \times 3 = 9$. A square root is represented by the symbol $\sqrt{\ }$, for example, $\sqrt{9} = 3$.

Explore Square Roots of Rational Numbers

1. a) Explain how the diagram represents $\sqrt{16}$.
b) Draw a diagram that represents $\sqrt{25}$.
c) Explain how you could use the following diagram to identify a rational number with a square root that is between 4 and 5.
d) Describe another strategy you could use to complete part c).

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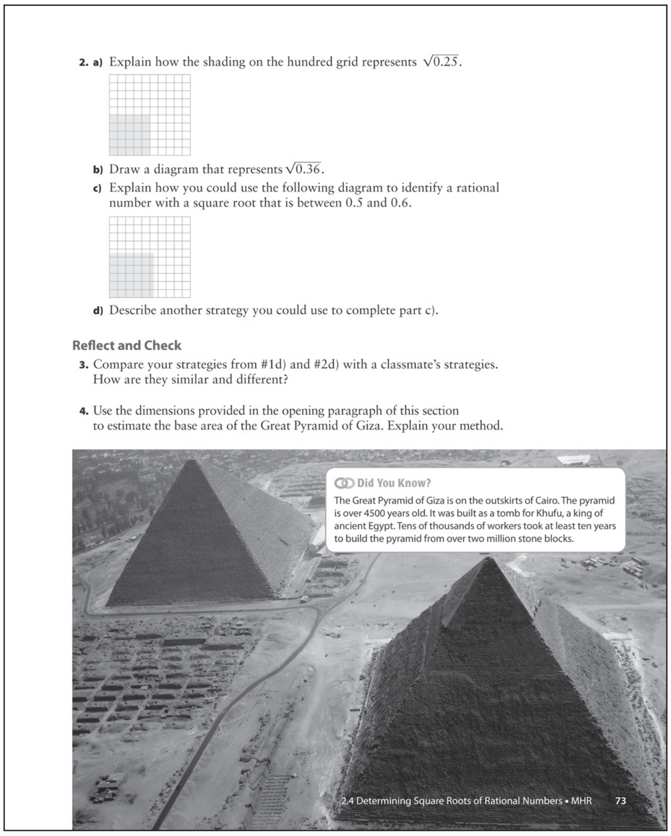
Refer students to the photographs on pages 72 and 73 that show the Great Pyramid of Giza in Egypt. The view from above on page 73 shows the square base of the pyramid. Draw students' attention to the Did You Know? with this photograph. The final sentence of the Did You Know? conveys a sense of the enormous size of the pyramid and the almost unimaginable toil required to build it.

To reinforce students' understanding of the scale of the Great Pyramid of Giza, refer to the opening paragraph of the section. You might have students work in groups to complete the estimate, in which they compare the dimensions of the base of the pyramid with the dimensions of a football field. You might present some coaching questions and have students discuss them in their groups:
- Will you include the end zones in the length of the field? Explain.
- What are the dimensions of a football field?
- Are all football fields the same size? Explain.
- Should you estimate the dimensions of a football field, or do you need exact values? Explain.



2. a) Explain how the shading on the hundred grid represents $\sqrt{0.25}$.

b) Draw a diagram that represents $\sqrt{0.36}$.

c) Explain how you could use the following diagram to identify a rational number with a square root that is between 0.5 and 0.6.

d) Describe another strategy you could use to complete part c).

Reflect and Check

3. Compare your strategies from #1d) and #2d) with a classmate's strategies. How are they similar and different?

4. Use the dimensions provided in the opening paragraph of this section to estimate the base area of the Great Pyramid of Giza. Explain your method.

Did You Know?

The Great Pyramid of Giza is on the outskirts of Cairo. The pyramid is over 4500 years old. It was built as a tomb for Khufu, a king of ancient Egypt. Tens of thousands of workers took at least ten years to build the pyramid from over two million stone blocks.

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After the groups have completed their comparison of the pyramid and a football field, discuss as a class any variations in their estimates and in the assumptions students made. Estimates will likely vary, but, if students' values for the dimensions of a field are reasonable, they might conclude that the Great Pyramid is about twice as long as a football field and about four times as wide. A regulation CFL field is about 137 m long, including the end zones, or about 101 m long without them. The width is about 59 m.

As a bridge to the Explore, which includes a question on the area of the base of the Great Pyramid, you may wish to ask students to use their estimates to compare the area of the base of the pyramid to the area of a football field. From the estimate in the previous paragraph, students might conclude that the base of the pyramid is about eight times the area of a football field. Some students may need to draw a diagram to make this comparison.

Explore Square Roots of Rational Numbers

Students have previously determined the square roots of whole numbers that are perfect or non-perfect squares. In this exploration, students will begin to extend their understanding to include the square roots of decimal numbers.

Literacy Link Before students begin the exploration, point out the Literacy Link to remind them of the meaning of *square root*. In preparation for #1 in the Explore, you might ask:

- How does the meaning of *square root* compare with the meaning of *square*?
- How could you use a diagram to show the relationship between the number 3 and its square?

Method 1 Have students work on the exploration in pairs or small groups and discuss their answers. When appropriate, ask coaching questions of individuals, groups, or the whole class. The following questions refer to the parts of #1. The same question formats could be used for #2, but with substitution of the appropriate decimal values from parts a) to c) of that question.

For #1 a), you might ask:

- How does the diagram represent 16?
- How does the diagram show the value of $\sqrt{16}$?

For #1b), you might ask:

- What shape should you draw in your diagram? Why?
- How does your diagram represent 25?
- How does your diagram show the value of $\sqrt{25}$?

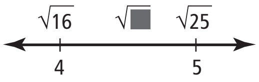
For #1c), you might ask:

- How does the diagram show a square root that is between 4 and 5?
- How does the diagram show the rational number that has this square root?
- How could you use the diagram to estimate the value of this rational number?
- How could you use the diagram to identify other rational numbers with a square root that is between 4 and 5?

In #1d), the following questions might open up students' thinking in different directions and provide support for students having difficulty:

- How is your answer to part c) related to the squares of 4 and 5? (One strategy is to square 4 and 5 and choose a rational number between the squares.)
- How can you calculate the square root of a non-perfect square? (A different strategy might be to use Estimate and Check, i.e., estimate the rational number and calculate its square root on a calculator until you obtain a square root that is between 4 and 5.)

- Does the following diagram suggest a strategy?



For #3, encourage open discussion. If each student group develops a strategy in #1d) and 2d), you might have different groups, rather than different individuals, compare their strategies. As well as comparing similarities and differences, you might ask students which method(s) they prefer, and why.

Estimates and methods will vary in #4. Any whole-number or decimal answer between, but not including 52 900 and 53 361, is acceptable. Some students may try to describe the range of possible values, rather than choosing a single value, but they may be unclear on the limits of the range. After students have completed the question, you might point out that they have identified a rational number with a square root between 230 and 231.

Method 2 Use number lines instead of grid paper in #1 and 2. For example, the number line included above in Method 1 could be the basis for answering the following question: How could you use the number line to identify a rational number with a square root that is between 4 and 5?

You might ask students to construct a similar number line labelled with the values given in #2. They can then describe how they could use the diagram to identify a rational number with a square root that is between 0.5 and 0.6. You may wish to provide students with **Master 4 Number Lines**.

Meeting Student Needs

- It may be better for your class to work through the Explore as a whole-class activity.
- Have students find Egypt on a map and discuss how far away it is from their own location. Direct students' attention to the photos in the student resource of the pyramids and discuss what these photos tell about Egypt (e.g., hypothesize what the climate is like). Discuss why the pyramids were built.
- Assist students in recalling their understanding of determining square roots.

ELL

- Use the images in the student resource to review what a pyramid is. As you use the terms in the opener, such as *base*, point to the part of the picture that you are referring to.
- Show students Egypt on a map and provide pictures of identifying features such as the pyramids in the student resource and also the Sphinx.
- Teach the following words in context: *dimensions* and *product*.
- Ensure that students are familiar with the square root sign. Students may have different ways of showing operations depending on what country they have moved from.

Gifted and Enrichment

- Discuss with students how some people have an ability to find particularly difficult mathematical solutions using mental math. Ask students to investigate the methods that these individuals use to perform tasks such as finding square roots to many decimal places, in their head. Students could report to the class some of the "tricks" they use.

WWW Web Link

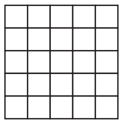
You may wish to have students research more information on the Great Pyramid of Giza. Go to www.mathlinks9.ca and follow the links.

Answers

Explore Square Roots of Rational Numbers

1. **a)** Example: It is a square with side lengths of 4 units. Its area is 16 square units.

b)

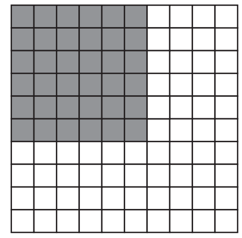


c) Example: The square of 4 is 16, and the square of 5 is 25. So, any number between 16 and 25 has a square root between 4 and 5.

d) Example: You could square any decimal number between 4 and 5. The value of this squared number would be between 16 and 25.

2. **a)** Example: If each square represents $\frac{1}{100}$, or 0.01, since the square of 0.5 is 0.25, there are 25 squares shaded on the hundred grid.

b)

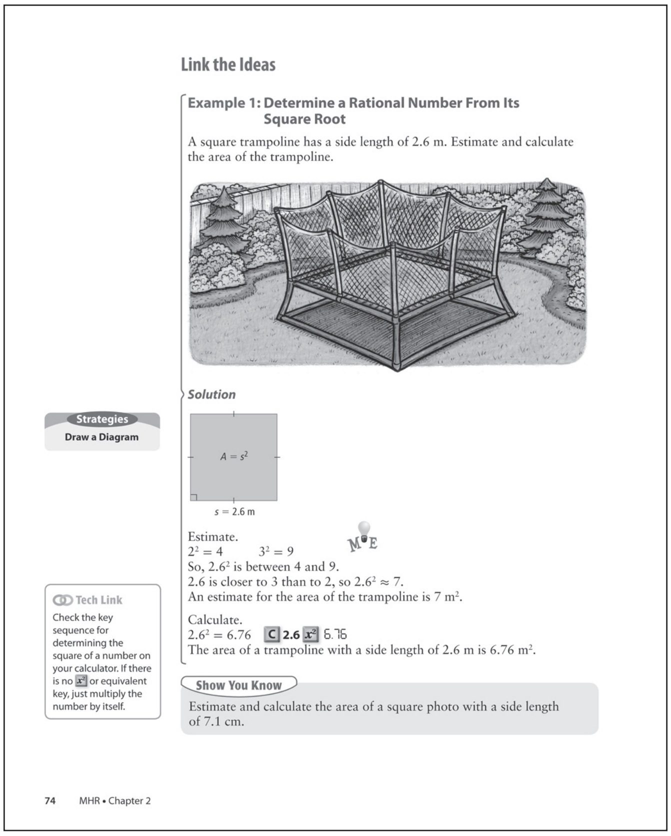


c) Example: Since the square of 0.5 is 0.25, and the square of 0.6 is 0.36, any number between 0.25 and 0.36 has a square root between 0.5 and 0.6.

d) Example: You could square any decimal number between 0.5 and 0.6. The value of this squared number would be between 0.25 and 0.36.

4. The area could be between 52 900 m^2 and 53 361 m^2. Example: A strategy is to square 230 and 231.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore.	• As a class, make a list of strategies that students used and compared in #1d) and 2d). Have students write the strategies and an example into their Foldable for future reference. • You may wish to provide students with **Master 8 Centimetre Grid Paper** and **Master 9 0.5 Centimetre Grid Paper** to use as they work on the Explore.



Link the Ideas

Example 1: Determine a Rational Number From Its Square Root

A square trampoline has a side length of 2.6 m. Estimate and calculate the area of the trampoline.

Solution

Strategies: Draw a Diagram

$A = s^2$

$s = 2.6$ m

Estimate.

$2^2 = 4$ $3^2 = 9$

So, 2.6^2 is between 4 and 9.

2.6 is closer to 3 than to 2, so $2.6^2 \approx 7$.

An estimate for the area of the trampoline is 7 m².

Calculate.

$2.6^2 = 6.76$ C 2.6 x^2 6.76

The area of a trampoline with a side length of 2.6 m is 6.76 m².

Tech Link: Check the key sequence for determining the square of a number on your calculator. If there is no x^2 or equivalent key, just multiply the number by itself.

Show You Know

Estimate and calculate the area of a square photo with a side length of 7.1 cm.

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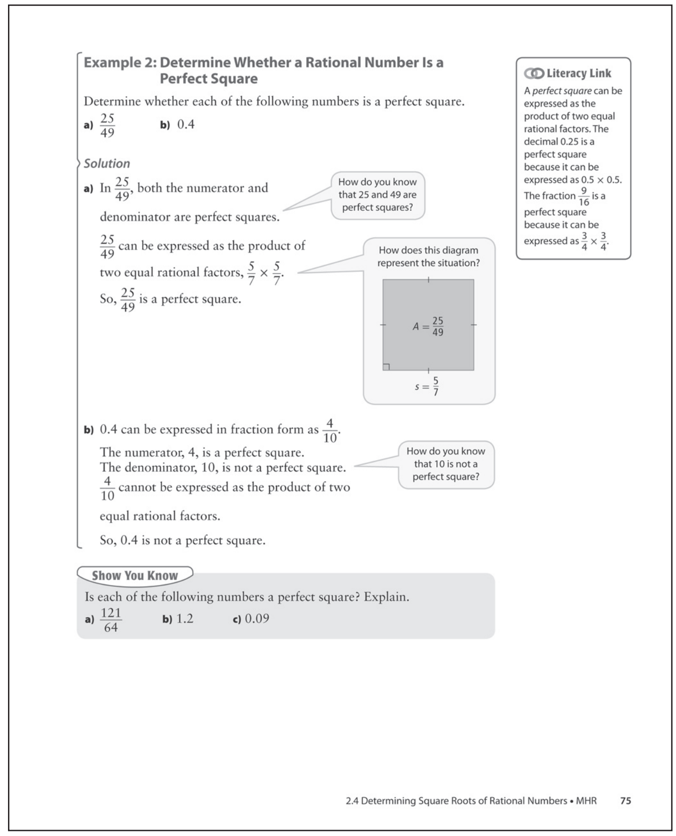

Example 2: Determine Whether a Rational Number Is a Perfect Square

Determine whether each of the following numbers is a perfect square.

a) $\frac{25}{49}$ **b)** 0.4

Solution

a) In $\frac{25}{49}$, both the numerator and denominator are perfect squares.

How do you know that 25 and 49 are perfect squares?

$\frac{25}{49}$ can be expressed as the product of two equal rational factors, $\frac{5}{7} \times \frac{5}{7}$.

How does this diagram represent the situation?

So, $\frac{25}{49}$ is a perfect square.

$A = \frac{25}{49}$

$s = \frac{5}{7}$

b) 0.4 can be expressed in fraction form as $\frac{4}{10}$.

The numerator, 4, is a perfect square.

The denominator, 10, is not a perfect square.

$\frac{4}{10}$ cannot be expressed as the product of two equal rational factors.

How do you know that 10 is not a perfect square?

So, 0.4 is not a perfect square.

Literacy Link: A *perfect square* can be expressed as the product of two equal rational factors. The decimal 0.25 is a perfect square because it can be expressed as 0.5 × 0.5. The fraction $\frac{9}{16}$ is a perfect square because it can be expressed as $\frac{3}{4} \times \frac{3}{4}$.

Show You Know

Is each of the following numbers a perfect square? Explain.

a) $\frac{121}{64}$ **b)** 1.2 **c)** 0.09

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Link the Ideas

Example 1

This example shows the estimation and calculation of the area of a square trampoline from its side length.

Direct students to the diagram in the solution. You might ask:

- How does the diagram represent the information given in the problem?
- What is the meaning of the formula $A = s^2$?

Next, have students examine the estimate and ask:

- How could you represent the statements $2^2 = 4$ and $3^2 = 9$ using grid paper or a number line?
- How could you use grid paper or a number line to show that 2.6^2 is between 4 and 9?
- Why is 7 a more reasonable estimate than 6?
- Why are the units of the estimate given as square metres?

Finally, have students examine the calculation. Ask:

- How would you calculate 2.6^2 if you did not have a calculator?
- How close is the calculated area to the estimate?

Emphasize the importance of the summary statement, including the units of measure.

Draw students' attention to the Tech Link. Have students check that they can obtain the correct answer to the calculation using their own calculator.

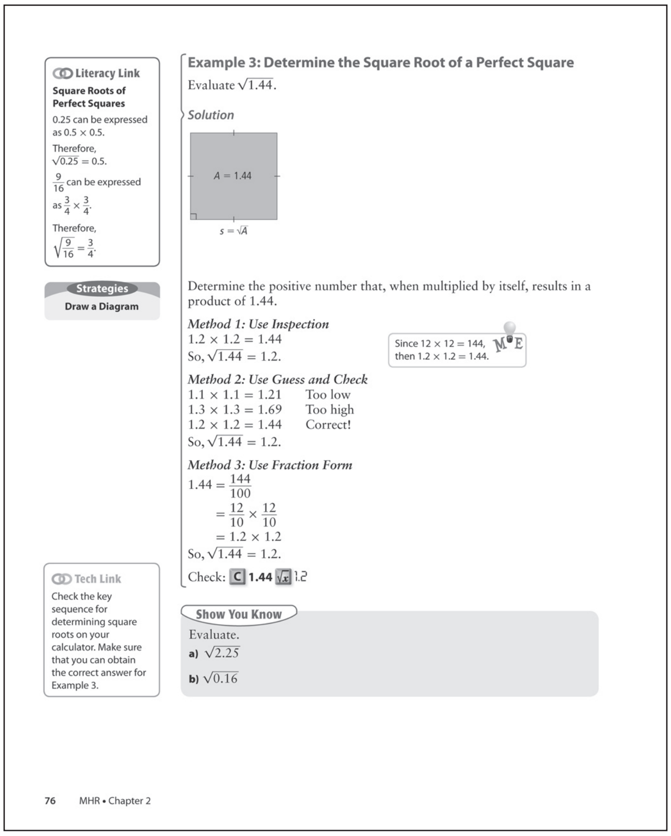
When students are working on the Show You Know, ask:

- What is a reasonable value for the estimate? How do you know?
- What are other reasonable values for the estimate?
- What are the units of the area of the photo?

Example 2

This example extends students' understanding of perfect squares (previously limited to whole numbers) to include fractions and decimals.

Literacy Link Direct students to the Literacy Link about perfect squares. Ask students to identify a whole-number perfect square. After students have seen the given perfect squares in decimal form and fraction form, ask them to provide an additional example of each. Ask students to explain why each of their examples is a perfect square.



Literacy Link

Square Roots of Perfect Squares

0.25 can be expressed as 0.5×0.5.

Therefore, $\sqrt{0.25} = 0.5$.

$\frac{9}{16}$ can be expressed as $\frac{3}{4} \times \frac{3}{4}$.

Therefore, $\sqrt{\frac{9}{16}} = \frac{3}{4}$.

Strategies

Draw a Diagram

Example 3: Determine the Square Root of a Perfect Square

Evaluate $\sqrt{1.44}$.

Solution

$A = 1.44$

$s = \sqrt{A}$

Determine the positive number that, when multiplied by itself, results in a product of 1.44.

Method 1: Use Inspection

$1.2 \times 1.2 = 1.44$

So, $\sqrt{1.44} = 1.2$.

Since $12 \times 12 = 144$, then $1.2 \times 1.2 = 1.44$.

Method 2: Use Guess and Check

$1.1 \times 1.1 = 1.21$ Too low

$1.3 \times 1.3 = 1.69$ Too high

$1.2 \times 1.2 = 1.44$ Correct!

So, $\sqrt{1.44} = 1.2$.

Method 3: Use Fraction Form

$$1.44 = \frac{144}{100} = \frac{12}{10} \times \frac{12}{10} = 1.2 \times 1.2$$

So, $\sqrt{1.44} = 1.2$.

Check: C 1.44 $\sqrt{x}$ 1.2

Tech Link

Check the key sequence for determining square roots on your calculator. Make sure that you can obtain the correct answer for Example 3.

Show You Know

Evaluate.

a) $\sqrt{2.25}$

b) $\sqrt{0.16}$

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The numerator and denominator in $\frac{25}{49}$ in part a) are perfect squares that students should recognize from their previous work. Draw students' attention to the thought bubbles in part a). Make sure that students can explain why 25 and 49 are perfect squares and how the given diagram represents $\frac{25}{49} = \frac{5}{7} \times \frac{5}{7}$. Reinforce the idea that the area of a square is the product of two equal side lengths, i.e., that the area is the square of the side length.

As students examine part b), point out the question in the thought bubble. If you wish to extend the example and challenge students, you might also ask:

- Can you express 0.4 as a fraction other than $\frac{4}{10}$? If so, give five examples. (Expect students to write equivalent fractions, such as $\frac{2}{5}$, $\frac{8}{20}$, $\frac{12}{30}$, $\frac{16}{40}$, and $\frac{40}{100}$.)
- Do any of your examples include a perfect square in both the numerator and the denominator? Explain. (Students will not be able to find an example that does.)
- Is $\frac{50}{98}$ a perfect square? (Students may be inclined to say no. In cases where neither the numerator nor the denominator of a fraction is a perfect square, it is important to check that the fraction is in lowest terms before concluding that it is not a perfect square.)

Have students complete the Show You Know in pairs or small groups and compare their solutions and explanations.

Example 3

This example extends students' understanding of square roots of perfect squares (previously limited to square roots of whole numbers) to include square roots of fractions and decimals.

Literacy Link Direct students to the Literacy Link about square roots of perfect squares. Ask them to identify the square root of a whole-number perfect square. After students have seen the given square roots of perfect squares in decimal form and fraction form, ask them to provide an additional example of each. Ask students to explain why each of their examples is the square root of a perfect square.

Point out the diagram and ask students how it represents the situation. Stress that, when students determine the square root of a number, they determine the side length of a square whose area equals the number.

As students examine Method 1, you might ask:

- Would you recognize by inspection that 1.44 is 1.2×1.2?
- If $12 \times 12 = 144$, how do you know that $1.2 \times 1.2 = 1.44$?

Point out that students are only likely to use Method 2 or Method 3 if they do not identify the square root by inspection. For Method 2, you might ask:

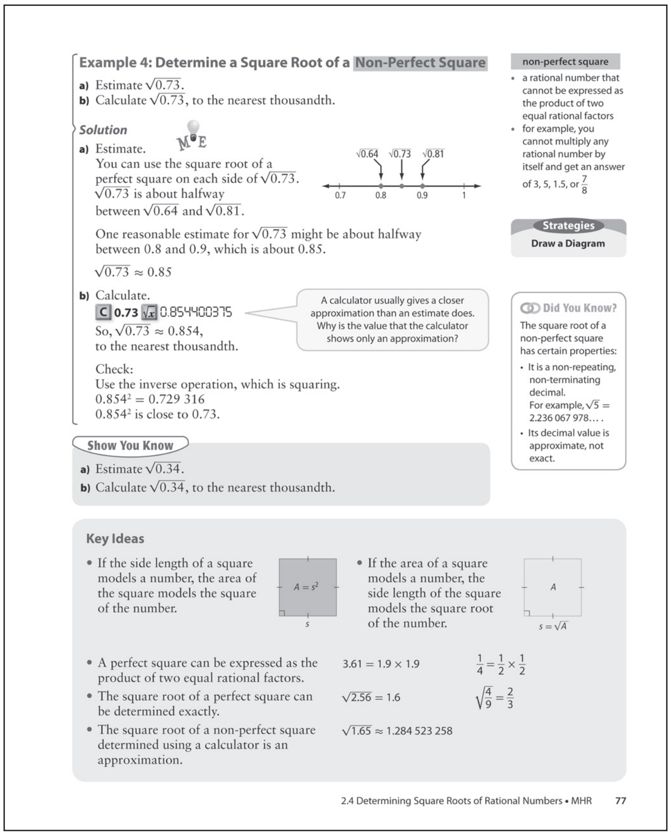
- Why might 1.1×1.1 be a reasonable first guess?
- After guessing 1.3×1.3, would it be reasonable to guess 1.4×1.4? Explain why or why not.

After students have consulted all three methods, you might ask which of the three methods they prefer and why.

Point out that the calculator-based check shown after Method 3 can be used for all three methods.

Point out the Tech Link to students. Have them record the keying sequence on their calculator if it is different from the one shown in the example.

Using a calculator could, of course, be a fourth possible solution method for the example. You might ask: If you determine the square root of a perfect square on a calculator, how can you check that the answer is correct?



Example 4: Determine a Square Root of a Non-Perfect Square

a) Estimate $\sqrt{0.73}$.
b) Calculate $\sqrt{0.73}$, to the nearest thousandth.

Solution

a) Estimate.
You can use the square root of a perfect square on each side of $\sqrt{0.73}$. $\sqrt{0.73}$ is about halfway between $\sqrt{0.64}$ and $\sqrt{0.81}$.

One reasonable estimate for $\sqrt{0.73}$ might be about halfway between 0.8 and 0.9, which is about 0.85.

$\sqrt{0.73} \approx 0.85$

b) Calculate.
C 0.73 $\sqrt{x}$ 0.854400375
So, $\sqrt{0.73} \approx 0.854$, to the nearest thousandth.

A calculator usually gives a closer approximation than an estimate does. Why is the value that the calculator shows only an approximation?

Check:
Use the inverse operation, which is squaring.
$0.854^2 = 0.729\ 316$
0.854^2 is close to 0.73.

non-perfect square
- a rational number that cannot be expressed as the product of two equal rational factors
- for example, you cannot multiply any rational number by itself and get an answer of 3, 5, 1.5, or $\frac{7}{8}$

Strategies
Draw a Diagram

Did You Know?
The square root of a non-perfect square has certain properties:
- It is a non-repeating, non-terminating decimal. For example, $\sqrt{5} = 2.236\ 067\ 978\ldots$.
- Its decimal value is approximate, not exact.

Show You Know

a) Estimate $\sqrt{0.34}$.
b) Calculate $\sqrt{0.34}$, to the nearest thousandth.

Key Ideas

- If the side length of a square models a number, the area of the square models the square of the number.

- If the area of a square models a number, the side length of the square models the square root of the number.

- A perfect square can be expressed as the product of two equal rational factors. $3.61 = 1.9 \times 1.9$ $\quad \frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$
- The square root of a perfect square can be determined exactly. $\sqrt{2.56} = 1.6$ $\quad \sqrt{\frac{4}{9}} = \frac{2}{3}$
- The square root of a non-perfect square determined using a calculator is an approximation. $\sqrt{1.65} \approx 1.284\ 523\ 258$

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Have students complete the Show You Know individually. Then, have them compare their solutions and explain why they chose to use a particular solution method.

Example 4

This example extends students' understanding of square roots of non-perfect squares (previously limited to square roots of whole numbers) to include square roots of decimals.

Draw students' attention to the definition of a non-perfect square. To check for understanding, you might ask: If you multiply a rational number by itself, is the product a perfect square or a non-perfect square? How do you know?

Point out the estimate in part a), which makes use of a number line to show the square roots of perfect squares on each side of $\sqrt{0.73}$. You might ask:

- Why do you think that $\sqrt{0.64}$ and $\sqrt{0.81}$ were used to make the estimate?
- Could you use $\sqrt{0.49}$ and $\sqrt{1}$ to make the estimate instead? Explain.
- What is a different kind of diagram you could use to represent the estimate? (Students may think of the side lengths of shaded areas on grids, which they worked with in the Explore.)
- Could you make the estimate without using a diagram?
- Do you find a diagram helpful for making the estimate? Explain.

When students examine the calculation in part b), draw their attention to the thought bubble and the question within it. Also, point out the Did You Know? beside the example. You may wish to use the term *irrational number* to describe a non-repeating, non-terminating decimal. You might mention that π is another example of an irrational number and that the value of π has been calculated to over a trillion decimal places, without the numbers repeating or terminating. If students have a calculator with a π key and a 10-digit display, you might ask them to determine the value of π to nine decimal places.

You could have students square 0.854400375 on a calculator with a 10-digit display. The result is 0.730000001. You might ask how this result confirms that the calculated square root of 0.73 is approximate.

Point out the Check in the solution and ask:

- What is the meaning of *inverse operation*?
- Why is squaring the inverse operation of taking the square root?
- What are two other examples of pairs of inverse operations?
- Why is 0.854^2 not exactly 0.73?

You might have students complete the Show You Know individually and then compare their solutions. Prompt partners to compare the perfect squares they used in completing the estimate and to ensure that their partner checked the calculated answer.

Key Ideas

The Key Ideas summarize concepts related to determining the square roots of positive rational numbers that are perfect squares or non-perfect squares. Draw attention to the first two statements, including the diagrams. To check understanding, you might ask students to give an example of an area that is a decimal and a perfect square, and then ask for the value of the side length. For the other three statements, you might ask students to make up their own examples. Students could prepare their own list of Key Ideas, with their own examples, and include it in their Foldable.

Meeting Student Needs

- It may be better for your class to work through the examples as a whole-class activity. Assign part of the Show You Know activities for Examples 2 and 3 as small-group or pair activities. Then, assign the rest of these Show You Know activities as individual student work. Assign the Show You Know activities for Examples 1 and 4 as small-group or pair activities. Then, assign similar Show You Know activities as individual student work.

ELL

- Teach the following terms in context: *trampoline*, *estimate*, *diagram*, *sequence*, *perfect square*, *numerator*, and *denominator*.
- Discuss the Key Ideas with students, rephrasing and restating the ideas using the diagrams in the student resource and others that you draw on the board.
- Students might not understand the word *models* in the first bullet of the Key Ideas. Restate this concept using terms such as *is the same as* and *is like*.

Common Errors

- Some students may misunderstand the word *between* as including end values, e.g., by identifying 0.25 or 0.36 as a rational number with a square root that is between 0.5 and 0.6 in Explore #2c).

R_x Explain the non-inclusiveness of *between* using simpler examples, e.g., 2 is between 1 and 3, but 1 is not between 1 and 3. Check for understanding by asking students to name all the whole numbers between 4 and 10, or all the integers between 3 and −4.

Answers

Example 1: Show You Know

Estimate: 49 cm^2. Calculate: 50.41 cm^2.

Example 2: Show You Know

a) Example: This number is a perfect square because both 121 and 64 are perfect squares.

b) Example: This number is not a perfect square because neither 12 nor 10 are perfect squares.

c) Example: This number is a perfect square because both 9 and 100 are perfect squares.

Example 3: Show You Know

a) 1.5 **b)** 0.4

Example 4: Show You Know

a) Example: 0.6. **b)** 0.583

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to model the example by drawing a diagram and labelling it accordingly. Coach them through showing one step of work with each part of the solution.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students it is important to look at both the numerator and the denominator in determining whether the rational number is a perfect square. • Point out to students that decimals may be easier to visualize if written as a fraction.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students use integer values initially. Have them describe the similarities and differences between working with the integer and working with the decimal. For their final answer, have them estimate the value first to facilitate the placement of the decimal.
Example 4 Have students do the Show You Know related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may find it helpful to use a number line.

Check Your Understanding

Communicate the Ideas

1. Max said that the square root of 6.4 is 3.2. Lynda claimed that the square root of 6.4 is 0.8. Jamila estimated that the square root of 6.4 should be about 2.5.
   a) Who is correct?
   b) What mistakes did the other two students make?

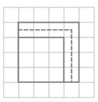
2. Without calculating any square roots, identify the square roots that have values between 4.5 and 5.5. Explain your reasoning.
   $\sqrt{21.3}$ $\sqrt{20.1}$ $\sqrt{31.7}$ $\sqrt{27.9}$ $\sqrt{30.5}$ $\sqrt{25.4}$ $\sqrt{30.2}$

3. Since $\sqrt{9}$ is less than 9, and $\sqrt{1.44}$ is less than 1.44, André predicted that $\sqrt{0.0625}$ would be less than 0.0625. Do you agree with his prediction? Explain.

4. a) Determine $\sqrt{2}$ using a scientific calculator. Record all of the digits that the calculator displays.
   b) Enter the decimal value of $\sqrt{2}$ from part a) into the calculator and square the value. Record the result.
   c) In part a), did the calculator display the exact value of $\sqrt{2}$? Explain how you know.

Practise

5. Use the diagram to identify a rational number with a square root between 3 and 4.



6. Identify a rational number with a square root between 0.7 and 0.8.

For help with #7 and #8, refer to Example 1 on page 74.

7. Estimate and calculate the number that has the given square root.
   a) 3.1 b) 12.5
   c) 0.62 d) 0.29

8. Estimate and calculate the area of each square, given its side length.
   a) 4.3 cm b) 0.035 km

For help with #9 and #10, refer to Example 2 on page 75.

9. Is each of the following rational numbers a perfect square? Explain.
   a) $\frac{1}{16}$ b) $\frac{5}{9}$
   c) 0.36 d) 0.9

10. Determine whether each rational number is a perfect square.
   a) $\frac{7}{12}$ b) $\frac{100}{49}$
   c) 0.1 d) 0.01

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For help with #11 and #12, refer to Example 3 on page 76.

11. Evaluate.
   a) $\sqrt{324}$ b) $\sqrt{2.89}$
   c) $\sqrt{0.0225}$ d) $\sqrt{2025}$

12. Calculate the side length of each square from its area.
   a) 169 m^2 b) 0.16 mm^2

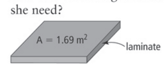
For help with #13 and #14, refer to Example 4 on page 77.

13. Estimate each square root. Then, calculate it to the specified number of decimal places.
   a) $\sqrt{39}$, to the nearest tenth
   b) $\sqrt{4.5}$, to the nearest hundredth
   c) $\sqrt{0.87}$, to the nearest thousandth
   d) $\sqrt{0.022}$, to the nearest thousandth

14. Given the area of each square, determine its side length. Express your answer to the nearest hundredth of a unit.
   a) 0.85 m^2 b) 60 cm^2

Apply

15. Kai needs to replace the strip of laminate that is glued to the vertical faces on a square tabletop. The tabletop has an area of 1.69 m^2. What length of laminate does she need?



16. a) The label on a 1-L can of paint states that the paint will cover an area of 10 m^2. What is the side length of the largest square area that the paint will cover? Express your answer to the nearest hundredth of a metre.
   b) What is the side length of the largest square area that a 3.79-L can of the same paint will cover? Express your answer to the nearest hundredth of a metre.
   c) Nadia is applying two coats of the paint to an area that is 4.6 m by 4.6 m. How much paint will she use if she applies the same amount of paint for each coat? Express your answer to the nearest tenth of a litre.

17. Some parks contain fenced gardens. Suppose that it costs $80 to build each metre of fence, including materials and labour.
   a) How much does it cost to enclose a square with an area of 120 m^2? Express your answer to the nearest dollar.
   b) Predict whether the total cost of enclosing two squares with an area of 60 m^2 each is the same as your answer to part a).
   c) Test your prediction from part b) and describe your findings.

18. A frame measures 30 cm by 20 cm. Can you mount a square picture with an area of 500 cm^2 in the frame? Explain.

19. A square picture with an area of 100 cm^2 is mounted on a square piece of matting. The matting has 2.5 times the area of the picture. If the picture is centred on the matting, what width of matting is visible around the outside of the picture? Give your answer to the nearest tenth of a centimetre.

20. Leon's rectangular living room is 8.2 m by 4.5 m. A square rug covers $\frac{2}{5}$ of the area of the floor. What is the side length of the rug, to the nearest tenth of a metre?

2.4 Determining Square Roots of Rational Numbers • MHR 79

Check Your Understanding

Communicate the Ideas

These questions allow students to communicate their understanding of the square roots of positive rational numbers that are perfect squares or non-perfect squares.

In #1, students examine common errors in the determination of the square root of a decimal number. After students have attempted the question, you might ask:

- How did you determine whose answer is correct?
- How can estimation help you to avoid making errors in determining square roots?
- How else can you check that a calculated square root is correct?

In #2, students can square 4.5 and 5.5 and then determine which of the given square roots are between $\sqrt{20.25}$ and $\sqrt{30.25}$. Students may or may not use a number line in making comparisons. Some students may use a grid to represent 4.5^2 and 5.5^2 as overlapping areas. Counting squares or calculating would establish their areas as 20.25 and 30.25 and would indicate which of the given square roots are between 4.5 and 5.5.

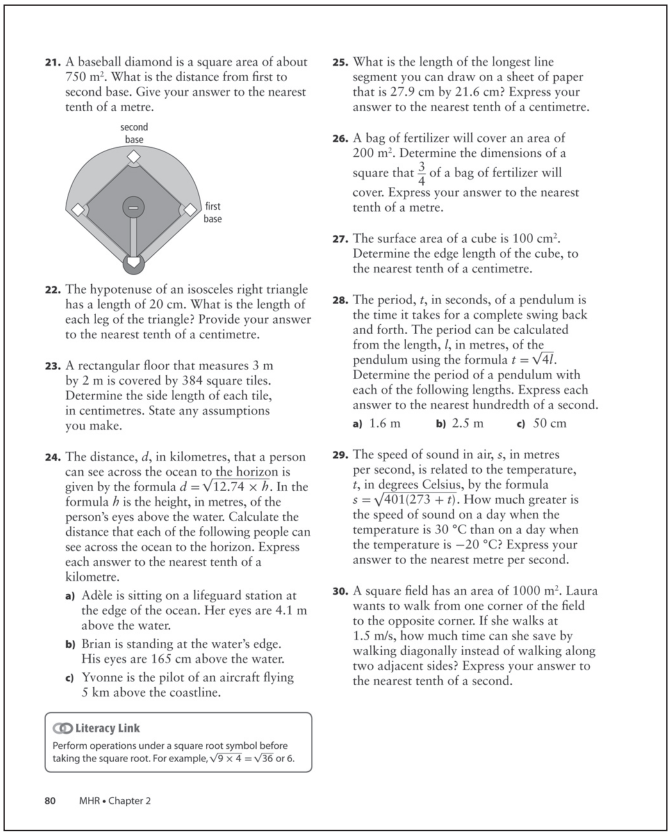
In #3, students consider a common misconception regarding the square roots of positive rational numbers that are less than 1. If students have difficulty in making a prediction, you might have them first consider the value of the square root of a simpler number, such as $\sqrt{0.25}$.

In #4, students consider the approximate value of the square root of a non-perfect square, as shown on a calculator. After students have completed #4, you might ask: Would you have expected the calculator to display the exact value of $\sqrt{2}$? Explain.

Practise

You may wish to have students work in pairs or small groups when completing the Practise questions. Alternatively, students might complete the Practise questions individually and share any difficulties with their group or the class. Continue to emphasize the importance of comparing calculated and estimated values.

In their answers to #8, 12, and 14, ensure that students include the appropriate units of measure.



21. A baseball diamond is a square area of about 750 m². What is the distance from first to second base. Give your answer to the nearest tenth of a metre.

second base

first base

22. The hypotenuse of an isosceles right triangle has a length of 20 cm. What is the length of each leg of the triangle? Provide your answer to the nearest tenth of a centimetre.

23. A rectangular floor that measures 3 m by 2 m is covered by 384 square tiles. Determine the side length of each tile, in centimetres. State any assumptions you make.

24. The distance, d, in kilometres, that a person can see across the ocean to the horizon is given by the formula $d = \sqrt{12.74 \times h}$. In the formula h is the height, in metres, of the person's eyes above the water. Calculate the distance that each of the following people can see across the ocean to the horizon. Express each answer to the nearest tenth of a kilometre.

a) Adèle is sitting on a lifeguard station at the edge of the ocean. Her eyes are 4.1 m above the water.

b) Brian is standing at the water's edge. His eyes are 165 cm above the water.

c) Yvonne is the pilot of an aircraft flying 5 km above the coastline.

Literacy Link

Perform operations under a square root symbol before taking the square root. For example, $\sqrt{9 \times 4} = \sqrt{36}$ or 6.

25. What is the length of the longest line segment you can draw on a sheet of paper that is 27.9 cm by 21.6 cm? Express your answer to the nearest tenth of a centimetre.

26. A bag of fertilizer will cover an area of 200 m². Determine the dimensions of a square that $\frac{3}{4}$ of a bag of fertilizer will cover. Express your answer to the nearest tenth of a metre.

27. The surface area of a cube is 100 cm². Determine the edge length of the cube, to the nearest tenth of a centimetre.

28. The period, t, in seconds, of a pendulum is the time it takes for a complete swing back and forth. The period can be calculated from the length, l, in metres, of the pendulum using the formula $t = \sqrt{4l}$. Determine the period of a pendulum with each of the following lengths. Express each answer to the nearest hundredth of a second.

a) 1.6 m **b)** 2.5 m **c)** 50 cm

29. The speed of sound in air, s, in metres per second, is related to the temperature, t, in degrees Celsius, by the formula $s = \sqrt{401(273 + t)}$. How much greater is the speed of sound on a day when the temperature is 30 °C than on a day when the temperature is −20 °C? Express your answer to the nearest metre per second.

30. A square field has an area of 1000 m². Laura wants to walk from one corner of the field to the opposite corner. If she walks at 1.5 m/s, how much time can she save by walking diagonally instead of walking along two adjacent sides? Express your answer to the nearest tenth of a second.

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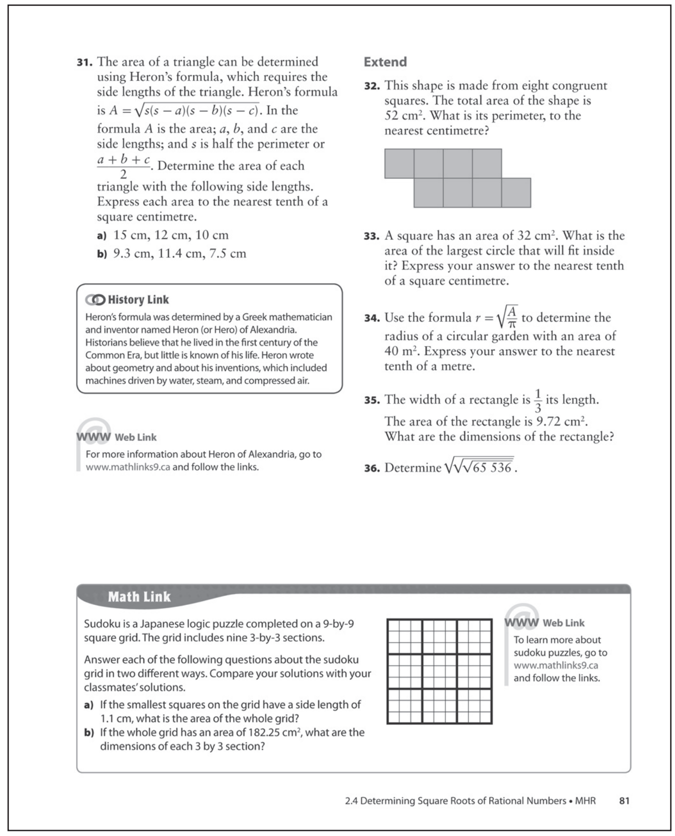

31. The area of a triangle can be determined using Heron's formula, which requires the side lengths of the triangle. Heron's formula is $A = \sqrt{s(s-a)(s-b)(s-c)}$. In the formula A is the area; a, b, and c are the side lengths; and s is half the perimeter or $\frac{a+b+c}{2}$. Determine the area of each triangle with the following side lengths. Express each area to the nearest tenth of a square centimetre.

a) 15 cm, 12 cm, 10 cm

b) 9.3 cm, 11.4 cm, 7.5 cm

History Link

Heron's formula was determined by a Greek mathematician and inventor named Heron (or Hero) of Alexandria. Historians believe that he lived in the first century of the Common Era, but little is known of his life. Heron wrote about geometry and about his inventions, which included machines driven by water, steam, and compressed air.

WWW Web Link

For more information about Heron of Alexandria, go to www.mathlinks9.ca and follow the links.

Extend

32. This shape is made from eight congruent squares. The total area of the shape is 52 cm². What is its perimeter, to the nearest centimetre?

33. A square has an area of 32 cm². What is the area of the largest circle that will fit inside it? Express your answer to the nearest tenth of a square centimetre.

34. Use the formula $r = \sqrt{\frac{A}{\pi}}$ to determine the radius of a circular garden with an area of 40 m². Express your answer to the nearest tenth of a metre.

35. The width of a rectangle is $\frac{1}{3}$ its length. The area of the rectangle is 9.72 cm². What are the dimensions of the rectangle?

36. Determine $\sqrt{\sqrt{\sqrt{65\,536}}}$.

Math Link

Sudoku is a Japanese logic puzzle completed on a 9-by-9 square grid. The grid includes nine 3-by-3 sections.

Answer each of the following questions about the sudoku grid in two different ways. Compare your solutions with your classmates' solutions.

a) If the smallest squares on the grid have a side length of 1.1 cm, what is the area of the whole grid?

b) If the whole grid has an area of 182.25 cm², what are the dimensions of each 3 by 3 section?

WWW Web Link

To learn more about sudoku puzzles, go to www.mathlinks9.ca and follow the links.

2.4 Determining Square Roots of Rational Numbers • MHR 81

Apply

As students work on the Apply problems, encourage them to compare and refine their solutions. Stress the importance of including the correct units of measure in the answers to applied problems.

In #22, 25, or 31, some students may have difficulty in recalling the Pythagorean relationship. You may wish to reactivate their knowledge by asking them to state the length of the hypotenuse in a triangle with legs of 3 cm and 4 cm and to explain their reasoning.

After students complete #23, you might ask if they prefer to convert the dimensions of the floor to centimetres at the outset, or to perform the calculations in metres before converting the answer to centimetres.

Literacy Link The solutions to #24, 28, 29, and 30 each involve substitution into a formula that includes a square root. Before students attempt any of these problems, draw attention to the Literacy Link that follows #24. To check students' understanding of the information and their ability to apply it to rational numbers, you might ask them to calculate $\sqrt{0.89 + 0.32}$ and $\sqrt{9 \div 4}$.

If students have difficulty in understanding #30, you might encourage them to begin by drawing a diagram.

Before students work on #31, you may wish to point out that it includes a 2000-year-old formula developed by a mathematician and inventor described in the Did You Know? that follows #31. Some students may be interested in learning more by exploring the Web Link that follows the Did You Know?

Extend

After students complete #32, you may wish to challenge them with the following problem: If there were nine congruent squares instead of eight, what arrangement of the squares would give the least value of the perimeter? As in #32, squares must be joined along entire sides.

If students have difficulty with #33, you might suggest that they begin by drawing a diagram. They will then see that the diameter of the circle equals the side length of the square.

Encourage students to use the π key on a calculator when solving #34.

Have students compare their solutions to #35. One approach is to write a formula that relates length to area by reasoning that $l \times \frac{l}{3} = A$, which results in the equation $\frac{l^2}{3} = 9.72$. An alternative is to relate width to area by reasoning that $w \times 3w = A$, which produces $3w^2 = 9.72$. The problem can also be solved without writing and solving an equation, by reasoning that the rectangle can be divided into three congruent squares, each with an area of 3.24 cm^2. The side length of each square is the width of the rectangle. You might ask students how this final solution relates to writing and solving the equation $3w^2 = 9.72$.

In #36, students may be surprised that their answer is quite small. You might ask how they could check that their answer is correct. Using the inverse operation would require squaring three times.

Literacy Link Have students develop a Frayer model showing what they already know about square roots at the beginning of this section. You may wish to have them revisit their Frayer models at the end of the section.

Math Link

In this Math Link, students apply their skills in determining the square roots of rational numbers in the context of a sudoku grid. Encourage students to discuss the advantages and disadvantages of different solution methods, and to state and explain their preferences.

Students will be asked to design their own game in the Wrap It Up! at the end of this chapter. They may choose to include a game board. Therefore, you might ask them to think about the sudoku grid as the basis for a game board. One possibility might be to use the board to tally the points scored in the rounds of a game. You might ask:

- If you were playing a game that involved rolling a die, how might you number the squares on a sudoku grid so that you could record the results?
- How might you build rewards and penalties into the grid (for example, scoring additional points for landing on particular squares, or losing points for landing on others)?

Another possible use of the grid would be as a way to generate positive and negative rational numbers. You might ask:

- How would you use the grid to generate numbers? (This might involve dropping a small object onto the grid and seeing where it lands.)
- How would you number the squares on the grid so that you could generate positive and negative rational numbers?
- How could you include operations on the resulting rational numbers in a game? (Each turn might involve dropping two objects on the grid and performing an operation on the two resulting numbers.)
- How would players score points in the game? (Points could be gained or lost for obtaining the greatest or least result from the preceding operation.)

Meeting Student Needs

- For #27, some students may benefit from using a model of a cube and from a reminder of what *surface area* means.
- Provide **BLM 2–11 Section 2.4 Extra Practice** to students who would benefit from more practice.

ELL

- For #1, clarify that *said*, *claimed*, and *estimated* are words used to describe how the students in the question gave their answers. Ensure that students understand what the word *correct* means. Draw a checkmark on the board as you say correct. If a student does not understand the word *mistake*, write the equation $4 + 2 = 5$ on the board and point to the 5, saying, "This is a mistake." After you correct the answer, put a checkmark and say, "Correct."
- Teach the following terms in context: *laminate*, *vertical faces*, *enclosing*, *fenced*, *mounted*, *matting*, and *lifeguard station*.
- Assign fewer Apply questions to English language learners as they will have to work through the language. Ensure that they have the vocabulary needed to understand the questions.

Common Errors

- Some students may take the square root of a decimal number incorrectly by either dividing by 2 or ignoring place value. For example, they may assume that $\sqrt{0.16}$ is 0.08 or 0.04, and that $\sqrt{0.9}$ is 0.45 or 0.3.

R_x Encourage students to check their answers by performing the inverse operation, i.e., by squaring the answer to make sure that they obtain the original number or, in the case of a non-perfect square, a value very close to it.

- In #19, some students may subtract the area of the picture from the area of the matting, to determine a difference of 150 cm^2, and then take the square root of this difference.

R_x Refer students to the diagram and point out that it contains only two squares, one with an area of 100 cm^2 and one with an area of 250 cm^2. There is no square with an area of 150 cm^2.

Answers

Communicate the Ideas

1. a) Jamila

b) Example: Max divided 6.4 by 2. Lynda found the square root of 64 and placed the decimal before the 8.

2. Example: Since the square of 4.5 is 20.25, and the square of 5.5 is 30.25, any number between 20.25 and 30.25 has a square root between 4.5 and 5.5. Therefore, the numbers are $\sqrt{21.3}$, $\sqrt{27.9}$, $\sqrt{25.4}$, $\sqrt{30.2}$.

3. Example: No, because the square root is 0.25

4. a) 1.414213562

b) 1.999999999

c) No. Example: 1.999999999 is not the same as 2.

Math Link

a) 98.01 cm^2 **b)** 4.5 cm

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • For students who find #1 challenging, spend time coaching them through square roots and ensure they can clearly state what a square root is. Have them do additional examples before moving forward. • Students having difficulty with #2 may benefit from a number line. Have them label the first six perfect squares on the number line. This should provide visual support in determining which square roots fall between 4.5 and 5.5. • For #3, have students change the values to integers to make the comparisons if the decimal values are a challenge. Once they have generalized their answer, have them return to the original question. Placing the suggested answers on a number line and comparing them to the position of the original square root may also clarify understanding. • For students having difficulty with #4, coach them through the question. Have them explain what each part is asking. Once they have completed the question, give them another value, for example, $\sqrt{5}$, and have them check the pattern for this value.
Assessment *for* Learning	
Practise and Apply Have students do #5–7, 9, 11, 13, 15, and 16. Students who have no problems with these questions can go on to the remaining Apply questions.	• Both #5 and 6 address the basic concept of the section, understanding square roots. Students should be able to complete the questions both algebraically and pictorially. If students are having difficulty with these two questions, have them review the Explore and model 4^2 and 5^2 on grid paper. Ask them to locate a value between these two values. • The same grid paper method, or a number line method, could be used for #7. Use #8 to check students' understanding, if necessary. • Some students may be confused by the fact the values are in fraction form in #9. Ask them how they would identify if a whole number was a perfect square. Point out that the same process is used for a fraction, only both the numerator and denominator must be checked. Use #10 to check students' understanding. • Students may require additional coaching for #11. Show students that some of the values in the question could be written as fractions. Students may benefit from writing the numbers from 1 to 25 in their Foldable, then listing the square of each number. It could be used as a quick reference. • Students will require a calculator for #13 but encourage them to estimate their values first. • For #15 and 16, encourage students to write out what they know and what they wish to solve. These questions may be completed with a partner. Encourage students to use diagrams for each part of #16. • Provide students who need it with grid paper to use as they work on the questions.
Literacy Link At the beginning of the section, have students work in pairs to develop a Frayer model on square roots.	• Use the students' Frayer models to identify their misconceptions about square roots. Address those misconceptions as you work through the section. • At the end of the section, have students revisit their Frayer models and make additions and improvements.
Assessment *as* Learning	
Math Link The Math Link on page 81 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 85.	• You may wish to provide students with grid paper so that they can copy parts of or all of the sudoku grid. • Students who need help getting started could use **BLM 2–12 Section 2.4 Math Link**, which provides scaffolding.
Math Learning Log Have students respond to the following prompts: • Two methods for determining the square root of a perfect square are ... The method I prefer is .... because ... • To estimate the square root of an imperfect square, I would...	• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. • If students have not generated their own examples and explanations of the various solution methods, have them write their responses to the Math Learning Log questions and place them into their Foldable.