Using Exponents to Describe Numbers

MathLinks 9, pages 92–98

Suggested Timing

45-60 minutes

Materials

calculator

Blackline Masters

Master 2 Communication Peer Evaluation BLM 3–3 Chapter 3 Warm-Up BLM 3–5 Section 3.1 Extra Practice BLM 3–6 Section 3.1 Math Link

Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Math and Estimation (ME)
- Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)
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Specific Outcomes

- **N1** Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:
- representing repeated multiplication using powers
- using patterns to show that a power with an exponent of zero is equal to one
- solving problems involving powers.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 6, 8, 11, 12, 14, Math Link
Typical	# 1–3, 5, 6, 8, 11, 12, 14, 16, Math Link
Extension/Enrichment	#1-3, 12, 16-23

Planning Notes

In this section, students represent repeated multiplication with exponents and explore how powers represent repeated multiplication.

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce material learned in previous sections.

As a class, read through the introduction. To get students thinking about repeated multiplication, ask:

- How might you estimate how many pieces of cake Alice will need to eat?
- What methods did you use to estimate?

3.1	Using Exponents to Describe Numbers
Focus on After this lesson, you will be able to • represent repeated multiplication with exponents • describe how powers multiplication	700 m
	In the story Allee <i>m</i> wonderhand, Allee Could Change net Size dramatically by earing cask. If she needed to triple her height, she would eat a piece of cake. Imagine that she is currently 1 m tall. She needs to increase her height to 700 m in order to see over a hill. How many pieces of cake do you think she will need to eat?

Explore Repeated Multiplication

 Create a table that shows how Alice's height changes after eating one, two, and three pieces of cake. Describe any patterns you see in the table

- a) How many pieces of cake does Alice need to eat to become at least 700 m tall? Show how you arrived at your answer.
 b) What is Alice's height after eating the number of pieces of cake in part a)?
 - c) How many factors of 3 do you need to multiply to obtain your answer to part b)?

 Explore how you could use a calculator to determine Alice's height after eating eight pieces of cake. Share your method(s) with your classmates. Record the methods that work for your calculator.

Explore Repeated Multiplication

MHR • Chapter 3

In this Explore, the story *Alice in Wonderland* is given a twist in order to develop the concept of powers and their usefulness in abbreviating expressions with repeated multiplication.

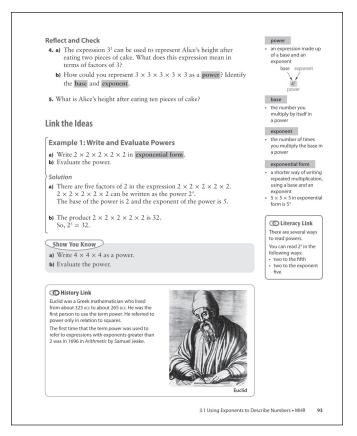
Read and discuss the introductory paragraph with the class.

Method 1 Have students work individually or in pairs to answer #1 to 5. As a class, go over the answers and discuss how a calculator can be used.

Method 2 As a class, discuss the tables students' create in #1. Have students describe patterns that they see. Ask students to describe what the next row in the table would look like. Then, have students work in pairs to answer the remaining questions.

As you circulate, watch how students are trying to solve #2a). Are they extending their table from #1? How are they keeping track of their attempts?

For #3, observe whether students are trying to use powers to solve the question or whether they are repeatedly multiplying by 3.



For #1, some students may need assistance developing the table. Suggest possible headings: Pieces of Cake Eaten, Height as a Product of Factors of Three, and Alice's Height (m).

For #5, try to get students to explain how they arrived at their answer for Alice's height after eating ten pieces of cake. Do not allow students simply to record their answer without showing their method. For some students, that may mean describing their method to you orally at their desk.

Literacy Link With the class, go through the different ways to describe a power that are listed in the margin on page 93. Provide practice to students by writing a few powers in symbolic form on the board and having students read them.

Meeting Student Needs

- Read the appropriate section from the book *Alice in Wonderland*. Discuss how it relates to the section opener and Explore.
- It might be better for your students to work through the Explore as a whole-class activity.
- Encourage students to use the power key, as opposed to repeated multiplication, on their calculator.
- For this section, you may want to invite in a person from the Aboriginal community who does beading. Have the person discuss how to bead a medallion. For example, you start with the centre bead, then, the next row is 8 beads, then, 16 beads, and so on. Lead a class discussion about the beading design: how many rows, how many colors, how many in each row, etc. Ask students to write the numbers exponentially.

ELL

• Teach the following words in context: *dramatically*, *triple*, and *currently*.

Common Errors

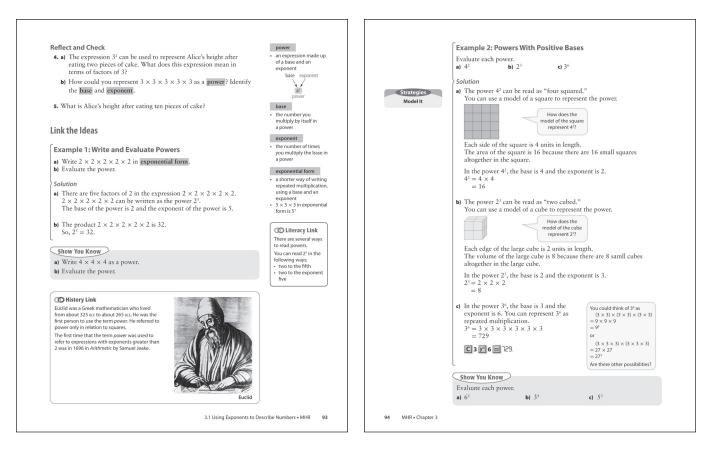
- Some students may not be able to solve #2a) by extending the pattern.
- R_x Ensure that students are organizing their findings in a way that is conducive to finding the answer, either by using a table or by using another effective strategy.

Answers

Explore Repeated Multiplication

- **1.** Example: Three is multiplied by itself the number of times that corresponds to the number of pieces of cake that are eaten.
- 2. a) Multiply 3 by itself six times to get a result of 729. So, she would need to eat six pieces.
 b) 729 m c) 6
- **3.** Example: Multiply 729 by 3 two more times, or multiply 729 by 9, or use the exponent key on the calculator.
- **4.** a) There are two factors of three. b) 3⁵. Base: 3; exponent: 5.
- **5.** $3^{10} = 59\ 049\ m$

Assessment	Supporting Learning
Assessment as Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Observe how they use the terms <i>power</i> , <i>base</i> , and <i>exponent</i> . Check that they are using them correctly.	 As a class, discuss the connections between the terms <i>power</i>, <i>base</i>, and <i>exponent</i>. If students seem hesitant about the terms, give them further examples such as 4 × 4 × 4 × 4 and 7 × 7 × 7 to check for understanding. Ensure that students have a good understanding of the difference between the expanded form and the elements of the exponential form. It is suggested that students use a calculator to answer #5.



Link the Ideas

Example 1

This example focuses on the concept of a power and how it relates to repeated multiplication, or factored form. Coach students by asking:

- The answer 2⁵ has an exponent of 5. Where does the 5 come from?
- When evaluating powers, what other methods do you use to find the answer?

Example 2

The first two parts of this example provide a visual model for understanding powers with exponents of 2 and 3. Ask:

- What part of each diagram in a) and b) represents the base?
- What part represents the exponent?

Example 3

This example introduces negative bases and the use of parentheses to indicate that the exponent should be applied to the negative sign in the base. You may wish to ask:

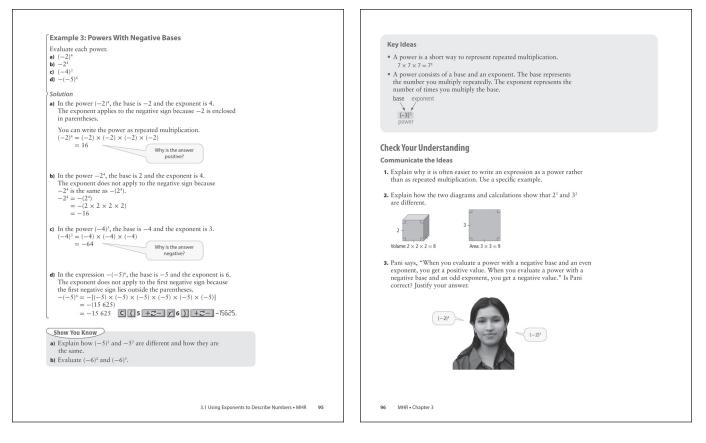
- What is the base and what is the coefficient for the powers in parts a) and b)?
- Why is the base negative in part a) but positive in part b)?

Key Ideas

The Key Ideas provide a concise explanation for the concept of a power and why it is useful in math. Coach students by asking:

- Where does the 3 come from in the power 7^3 ?
- What is the coefficient of the power $(-3)^5$?

Have students record the Key Ideas in their Foldable. Encourage them to use their own wording and examples.



Meeting Student Needs

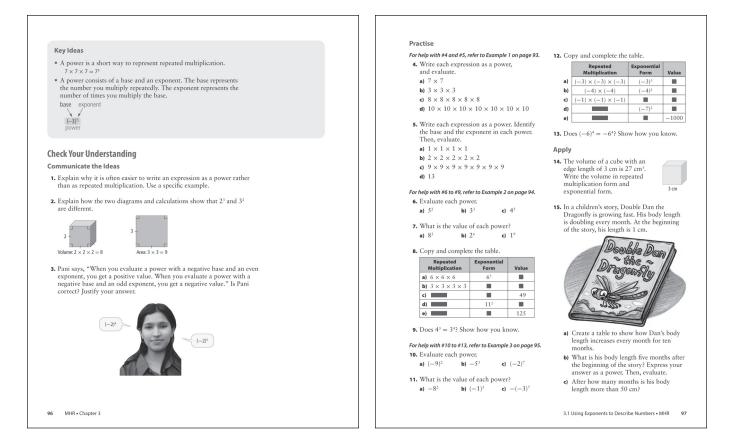
• Students may benefit from doing the examples as a whole-class activity. They might then complete the first part of the Show You Know as a small-group or paired activity and the second part as individual student work.

Common Errors

- Some students may evaluate the power incorrectly by multiplying the base by the exponent.
- $\mathbf{R}_{\mathbf{x}}$ Have students continue to write out the powers as repeated multiplication to help them understand the meaning of the exponent.
- In Example 3, some students may not understand why parentheses are necessary for a negative base such as $(-2)^4$.
- $\mathbf{R}_{\mathbf{x}}$ Help clarify for students that the parentheses ensure that the negative sign is applied (base multiplied by -1) before the base is multiplied by itself repeatedly.

Answers	
Example 1: Show You Know	Example 3: Show You Know
a) 4 ³ b) 64	a) Example: In the expression $(-5)^2$, the base is -5 , so $-5 \times (-5) = 25$. In the expression -5^2 , the base is 5 and the exponent does not apply
Example 2: Show You Know	to the negative sign, so $-(5 \times 5) = -25$.
a) 36 b) 81 c) 125	b) 36, -7776

Assessment	Supporting Learning	
Assessment <i>for</i> Learning		
Example 1 Have students do the Show You Know related to Example 1.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Ensure students are not reversing the base and the exponent. You may wish to ask a question such as, "How does 5² differ from 2⁵?" Have students identify the elements of the power by name. You made need to clarify or review the meaning of the word <i>evaluate</i>. 	
Example 2 Have students do the Show You Know related to Example 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Encourage students who do not understanding the connection between the power and the diagram to draw the diagram and label the dimensions. Although not explicitly stated in the question, ensure that students can model and determine the answers in more than one way. The use of the calculator is only one tool for this methodology. 	
Example 3 Have students do the Show You Know related to Example 3.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Present students with a power that has a negative base and an exponent too large for the calculator to handle. Ask students if the power evaluates to a positive or negative number. Students must think about whether the exponent is odd or even. Present students with a similar question as above but use a negative base with and without brackets and with even and odd exponents. Again, it should be too large for the calculator to handle. Ensure students can rationalize whether the answer is positive or negative. 	



Check Your Understanding

Communicate the Ideas

The first question addresses the advantage of writing repeated multiplication as a power. The second question helps students develop the concept of powers through a visual representation. You may wish to ask:

- For #1, what would 6¹² look like written as repeated multiplication?
- In #2, which diagram represents 2³?

Practise

Note that #4 and 5 are very similar. If students are assigned only one of these questions, have them do #5. In #5d), students need to be able to write a single factor as a power with an exponent of 1.

Questions #6 and 7 are very similar to Example 2. Students should be able to evaluate each of these without a calculator. Students may verify their solution with a calculator for practice with technology. In #8, the heading of the third column is Value. You may wish to clarify that this is the evaluated form of the power. Ask students if there are any parts of this question that have more than one possible multiplication and exponential form that yield the same answer.

In #9, students see that interchanging the base and exponent usually results in two distinct values. An exception is 2^4 and 4^2 .

Although #10 and 11 are similar, ensure that students are assigned #11 because #11a) is a type of question that most commonly results in mistakes.

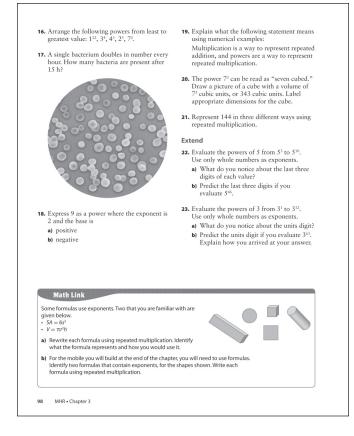
Question #13 emphasizes the importance of parentheses with negative bases when the exponent is an even number.

Apply

For #14, some students may need to be reminded that each dimension (height, width, and depth) will be 3 cm.

Note that #15 relates well to the Explore. It also provides some practice for a similar type of question students will encounter in section 3.4.

For #17, check whether students are able to solve a contextual problem using an exponential expression.



For #21, students may need to determine the prime factorization of 144 in order to find the answer. Remind students that the factors can be arranged/grouped in any order.

Extend

Questions #22 and 23 are patterning questions. Encourage students to find similar patterns with other whole-number bases.

Literacy Link After section 3.1, work with students to start the upper left leg of the spider map, entitled Using Exponents. Brainstorm and discuss as a class the information needed to begin this leg.

Math Link

Later in the chapter, students use formulas to simplify exponential expressions. This Math Link provides students with practice in this skill and also helps prepare them for the Wrap It Up! project they will do at the end of the chapter. Encourage students to look for some unique three-dimensional shapes that they might use in their mobile for the Wrap It Up! Coach students by asking:

- If you wished to find any kind of formula, where might you look?
- What are some names of three-dimensional shapes?

Meeting Student Needs

• Provide **BLM 3–5 Section 3.1 Extra Practice** to students who would benefit from more practice.

ELL

- Teach the following words in context: *bacterium*, *doubles*, *least*, *greatest*, and *predict*.
- Allow new English-language students to write their answers in their first language initially and then to do their best to translate their answers into English. This strategy gives students the opportunity to think through their answers first without being impeded by language.

Gifted and Enrichment

• In #9, students determine that when the base and exponent of a power are interchanged, the results are usually two distinct values. Have students determine when the results are the same value.

Common Errors

- Some students may struggle with answering #3.
- **R**_x Provide students with appropriate sample powers to help them get started.
- Some students may not remember some of the relevant formulas.
- **R**_x Help students to recall these formulas and discuss resources that would contain diagrams and formulas (e.g., agenda, student resources, the Internet).

Answers

Communicate the Ideas

Math Link

- **1.** Example: If the exponent is large, it takes less time and space to write as a power. For example, $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$.
- **2.** Example: The cube shows finding the volume by multiplying 2 by itself three times. The square shows finding the area by multiplying 3 by itself.
- **3.** Yes, Pani is correct. Example: $(-7)^2 = 49$ and $(-7)^3 = -343$.

b) Example: The formula for the area of a square is $s^2 = s \times s$. The formula for the volume of a cube is $s^3 = s \times s \times s$.

Assessment	Supporting Learning	
Assessment <i>as</i> Learning		
Communicate the Ideas Have all students complete #1 to 3.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Have students use Master 2 Communication Peer Evaluation to assess another student's answer to one of #1 to 3. Question 3 is similar to the Show You Know in Example 3. Students should have little difficulty answering this question after Example 3. Encourage visual students to write the exponential expression and label it. 	
Assessment for Learning		
Practise and Apply Have students do #5, 6, 8, 11, 12, and 14. Students who have no problems with these questions can go on to the remaining Apply questions.	 Encourage students to use their calculator to verify their answers for #6, 8, and 11. Check that students are using the exponent button. Students having difficulty with #5 should refer back to how they completed the Show You Know for Example 1. Students having difficulty with # 11 may need some extra coaching on Example 3. Reviewing the suggested approaches for Communicate the Ideas #3 may also assist students in making necessary links. 	
Math Link The Math Link on page 98 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 123.	 Students who need help getting started could use BLM 3–6 Section 3.1 Math Link, which provides scaffolding. For students who are uncertain as to the types of shapes they would like to use in their mobile, have them generate the formulas for all the shapes shown, starting with matching the two given in the Math Link to the shapes shown. 	
Assessment <i>as</i> Learning		
Literacy Link At the end of section 3.1, have students work in pairs to complete the upper left leg of the spider map, entitled Using Exponents.	• Have students list all the rules and terms they learned that are associated with powers. The terms that students record might include <i>repeated multiplication, exponent, base</i> , etc. The rules that students add might include the sign rules or the effect of parentheses on negative bases.	
 Math Learning Log Have students respond to the following prompts: The difference between repeated multiplication and exponents is Evaluate -2⁴ and (-2)⁴. Explain why the answers are opposites of each other. 	 Students could write responses to these questions into their Foldable in the appropriate location. Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. 	

<sup>a) SA = 6 × s × s. This is the formula for the surface area of a cube, where s represents the measure of the edge of the cube.
V = × r × r × h. This is the formula for the volume of a right cylinder, where r represents the radius and h represents the height.</sup>