Exponent Laws

3.2

***MathLinks 9*, pages 99–107**

Suggested Timing
50–60 minutes

Materials
- calculator

Blackline Masters
Master 2 Communication Peer Evaluation
BLM 3–3 Chapter 3 Warm-Up
BLM 3–7 Section 3.2 Extra Practice

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

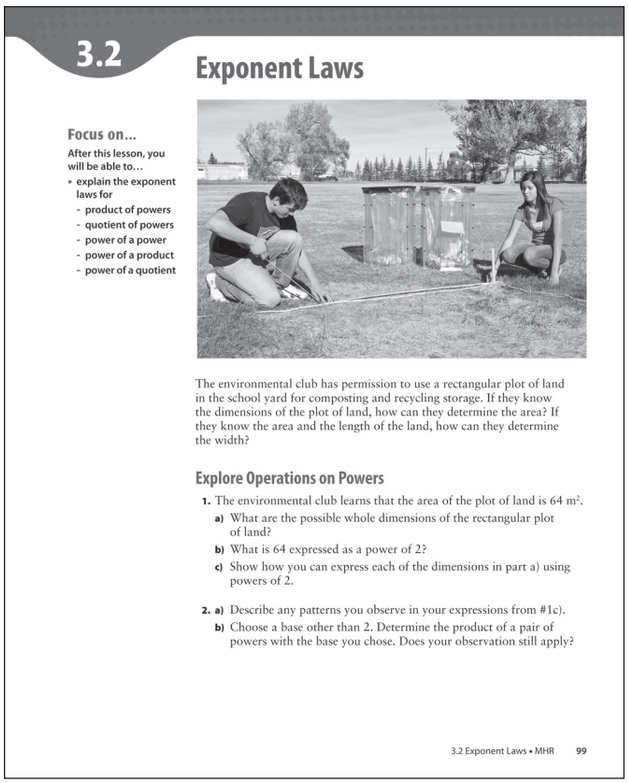
Specific Outcomes
N1 Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:
- representing repeated multiplication using powers
- using patterns to show that a power with an exponent of zero is equal to one
- solving problems involving powers.

N2 Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6, 7, 9, 11, 14, 17, 19
Typical	#1–4, 6, 7, 8, 9, 11, 14, 17, 19, 20, 21
Extension/Enrichment	#1–4, 13, 15, 17, 20–23, 25, 26

Planning Notes

In this section, students explore the exponent laws. Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce material learned in previous sections.



3.2

Exponent Laws

Focus on...

After this lesson, you will be able to...
- explain the exponent laws for
  - product of powers
  - quotient of powers
  - power of a power
  - power of a product
  - power of a quotient

The environmental club has permission to use a rectangular plot of land in the school yard for composting and recycling storage. If they know the dimensions of the plot of land, how can they determine the area? If they know the area and the length of the land, how can they determine the width?

Explore Operations on Powers

1. The environmental club learns that the area of the plot of land is 64 m^2.
   a) What are the possible whole dimensions of the rectangular plot of land?
   b) What is 64 expressed as a power of 2?
   c) Show how you can express each of the dimensions in part a) using powers of 2.

2. a) Describe any patterns you observe in your expressions from #1c).
   b) Choose a base other than 2. Determine the product of a pair of powers with the base you chose. Does your observation still apply?

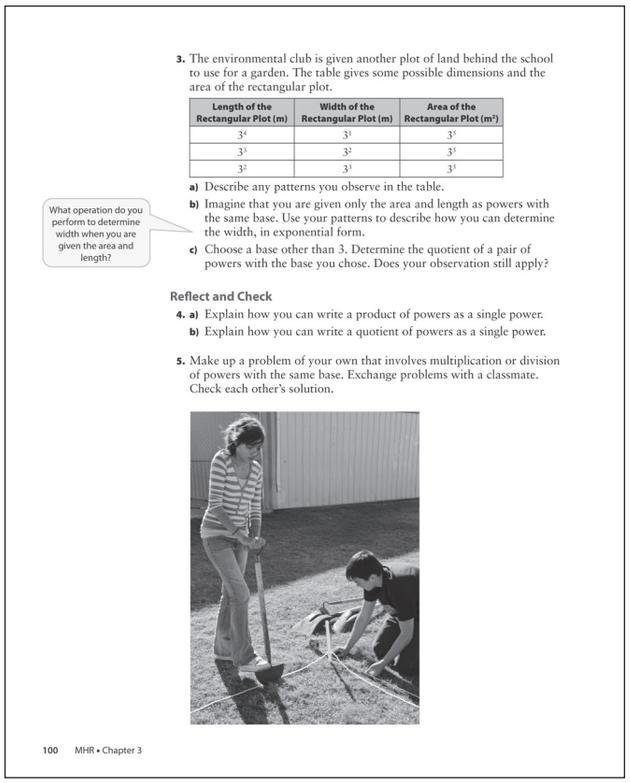
3.2 Exponent Laws • MHR 99

Explore Operations on Powers

In this Explore, students investigate how can they can use exponents to determine the area of a plot of land if the dimensions are known, and how they can use exponents to determine the width of a plot of land if they know the area and the length.

Method 1 Have students work individually through #1. As you circulate, ask students what is meant by *whole dimensions* for the rectangular plot. Are students including the 8 m by 8 m square as a possible rectangular plot? If not, remind students that squares are rectangles. Check that students who include a rectangle with dimensions of 1 m by 64 m can successfully express 1 as a power of 2. Some students might need to be encouraged to sketch a picture of each rectangle. You may wish to ask questions such as the following:
- What are some ways you can organize your findings?
- Compare your findings with a classmate. Did you find the same set of dimensions as your classmate?

Then, have students work through the remaining questions with a classmate.



3. The environmental club is given another plot of land behind the school to use for a garden. The table gives some possible dimensions and the area of the rectangular plot.

Length of the Rectangular Plot (m)	Width of the Rectangular Plot (m)	Area of the Rectangular Plot (m^2)
3^4	3^1	3^5
3^3	3^2	3^5
3^2	3^3	3^5

a) Describe any patterns you observe in the table.

b) Imagine that you are given only the area and length as powers with the same base. Use your patterns to describe how you can determine the width, in exponential form.

What operation do you perform to determine width when you are given the area and length?

c) Choose a base other than 3. Determine the quotient of a pair of powers with the base you chose. Does your observation still apply?

Reflect and Check

4. a) Explain how you can write a product of powers as a single power.
b) Explain how you can write a quotient of powers as a single power.

5. Make up a problem of your own that involves multiplication or division of powers with the same base. Exchange problems with a classmate. Check each other's solution.

100 MHR • Chapter 3

Method 2 Students work in pairs to brainstorm whole dimensions of rectangles that provide an area of 64 m^2. Take up the answers as a class and ensure that everyone has the complete set of dimensions. You may wish to ask:

- What do these sets of dimensions have in common? (powers of 2)
- What about 1? Can it be expressed as a power of 2?

Let students complete the rest of the questions with their partner. Guide students by asking the following types of questions:

- How are you organizing your data?
- Why might you include diagrams in your findings?
- How could you label the rectangles that you have drawn in order to identify them?

Meeting Student Needs

- It might be better for your students to work through the Explore as a whole-class activity.

ELL

- Teach the following terms in context: *environmental club*, *permission*, *plot of land*, *courtyard*, *recycling*, and *dimensions*.

Common Errors

- Some students may not fully explore all possible dimensions of rectangles.

R_x Remind students that 1 is a factor of 64. Encourage students to determine the prime factorization of 64 in order to find some missing factor pairs.

Answers

Explore Operations on Powers

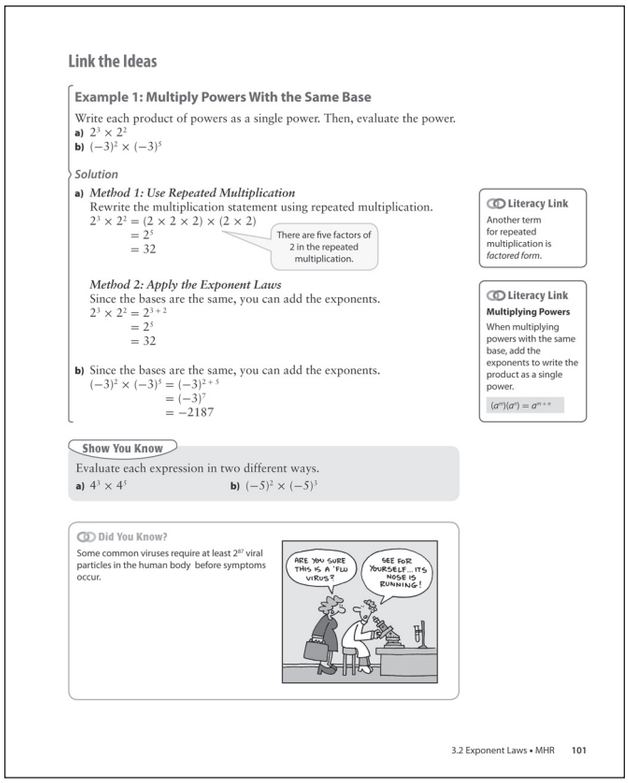
1. a) 1 and 64, 2 and 32, 4 and 16, 8 and 8
b) 2^6 **c)** $2^0 \times 2^6$, $2^1 \times 2^5$, $2^2 \times 2^4$, $2^3 \times 2^3$

2. a) Example: The sum of the exponents on any pair of dimensions when written in exponential from is 6.
b) Yes. Example: 81: $3^0 \times 3^4$, $3^1 \times 3^3$, $3^2 \times 3^2$

3. a) Example: All bases are 3. The sum of the exponents on the exponential forms of the length and width is 5.
b) Example: Subtract the exponent on the length written in exponential form from the exponent on the area written in exponential from.
c) Yes. Example: $4^5 \div 4^3 = 4^2$

4. a) Example: The product of two exponential expressions with the same base is equal to an expression with the same base and an exponent that is the sum of the exponents of the factors.
b) Example: The quotient of two exponential expressions with the same base is equal to an expression with the same base and an exponent that is the difference of the exponents of the factors.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Check that students develop a strategy for writing a product of powers as a single power. Check whether they are using the exponent law or describing a method of expansion and simplification. Ask them whether they think that they can do both.	• If students are focusing entirely on one method, check that they are capable of using the second method through questioning: How could you solve it in a different way from what you have written for #4a)? • For struggling learners, provide a basic area question such as: – If the length is 4 and the width is 3, find the area. – How did you find the area? – How is multiplying 3 by 4 the same as the process in the table? • Note that #3b) may be difficult for non-visual learners to understand. Encourage them to draw a picture or show expanded form and set the division up accordingly. Alternatively, you could continue with the area question above and ask: – If you know the area and the length is 4, how could you find the width? – How does this process link to #3b)? How does it link to #4b)?



Link the Ideas

Example 1: Multiply Powers With the Same Base

Write each product of powers as a single power. Then, evaluate the power.

a) $2^3 \times 2^2$

b) $(-3)^2 \times (-3)^5$

Solution

a) *Method 1: Use Repeated Multiplication*

Rewrite the multiplication statement using repeated multiplication.

$2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2)$

$= 2^5$

$= 32$

There are five factors of 2 in the repeated multiplication.

Method 2: Apply the Exponent Laws

Since the bases are the same, you can add the exponents.

$2^3 \times 2^2 = 2^{3+2}$

$= 2^5$

$= 32$

b) Since the bases are the same, you can add the exponents.

$(-3)^2 \times (-3)^5 = (-3)^{2+5}$

$= (-3)^7$

$= -2187$

Literacy Link

Another term for repeated multiplication is *factored form*.

Literacy Link

Multiplying Powers

When multiplying powers with the same base, add the exponents to write the product as a single power.

$(a^m)(a^n) = a^{m+n}$

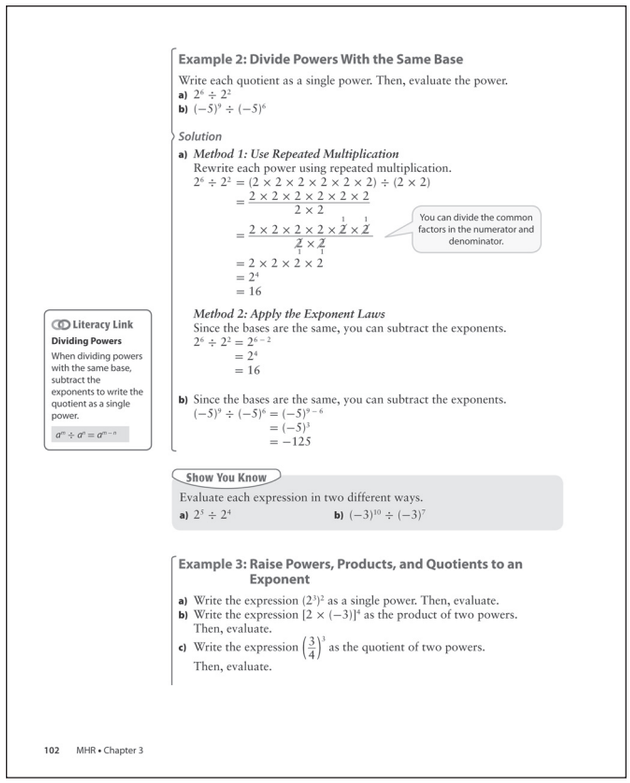
Show You Know

Evaluate each expression in two different ways.

a) $4^3 \times 4^3$ **b)** $(-5)^2 \times (-5)^3$

Did You Know?

Some common viruses require at least 2^{10} viral particles in the human body before symptoms occur.



Example 2: Divide Powers With the Same Base

Write each quotient as a single power. Then, evaluate the power.

a) $2^6 \div 2^2$

b) $(-5)^9 \div (-5)^6$

Solution

a) *Method 1: Use Repeated Multiplication*

Rewrite each power using repeated multiplication.

$2^6 \div 2^2 = (2 \times 2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2)$

$= \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2}$

$= \frac{2 \times 2 \times 2 \times 2 \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}}}$

$= 2 \times 2 \times 2 \times 2$

$= 2^4$

$= 16$

You can divide the common factors in the numerator and denominator.

Method 2: Apply the Exponent Laws

Since the bases are the same, you can subtract the exponents.

$2^6 \div 2^2 = 2^{6-2}$

$= 2^4$

$= 16$

b) Since the bases are the same, you can subtract the exponents.

$(-5)^9 \div (-5)^6 = (-5)^{9-6}$

$= (-5)^3$

$= -125$

Literacy Link

Dividing Powers

When dividing powers with the same base, subtract the exponents to write the quotient as a single power.

$a^m \div a^n = a^{m-n}$

Show You Know

Evaluate each expression in two different ways.

a) $2^5 \div 2^4$ **b)** $(-3)^{10} \div (-3)^7$

Example 3: Raise Powers, Products, and Quotients to an Exponent

a) Write the expression $(2^3)^2$ as a single power. Then, evaluate.

b) Write the expression $[2 \times (-3)]^4$ as the product of two powers. Then, evaluate.

c) Write the expression $\left(\frac{3}{4}\right)^3$ as the quotient of two powers. Then, evaluate.

Link the Ideas

Example 1

In this example, students multiply powers with a common base. Ensure students practise multiplying powers by expanding first before relying solely on the exponent law. Students who jump too quickly to adding exponents to solve will not get a feel for the operations with powers. For students who solve multiplication of powers with the appropriate exponent law, check their understanding of solving through expansion as you circulate by asking: How else could you solve this problem?

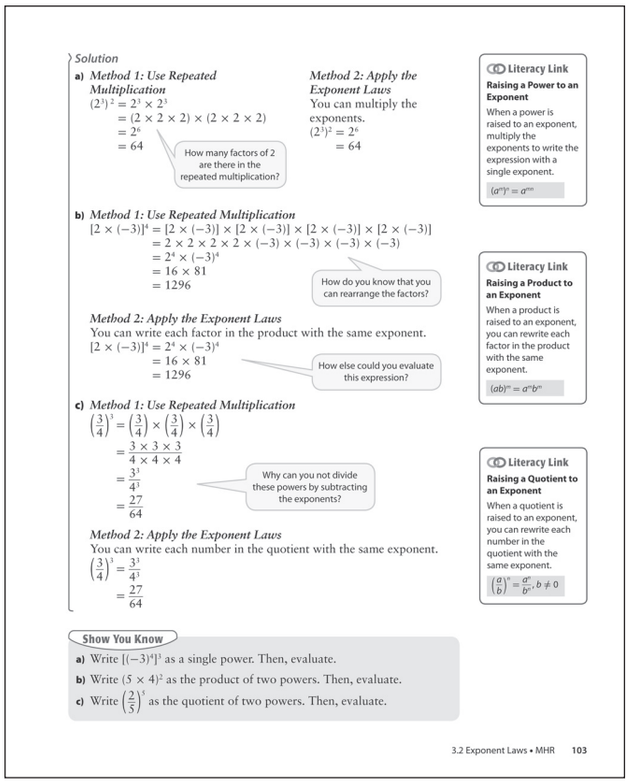
Literacy Link Point out the Literacy Link on page 101, which explains that factored form is another way of saying repeated multiplication form. Use both terms throughout the chapter to help reinforce their meaning. After Example 1, focus students' attention on the second Literacy Link on page 101 about multiplying powers. Ask students what numbers in part a) of Example 1 correspond to a, m, and n in the Literacy Link. Explain that the Literacy Link shows a way to represent the general rule.

As students multiply powers with negative bases, encourage them to use parentheses around the base to reinforce the fact that the negative sign is part of the base and therefore the exponent also applies to the negative sign.

Example 2

In this example, students divide powers with a common base. As with Example 1, ensure students understand the reasoning behind the exponent law and that they understand how to use parentheses.

Literacy Link After Example 2, direct students' attention to the Literacy Link on page 102 about dividing powers. Ask students what numbers in part b) of Example 2 correspond to a, m, and n in the Literacy Link.



Solution

a) *Method 1: Use Repeated Multiplication*

$(2^3)^2 = 2^3 \times 2^3$
$= (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
$= 2^6$
$= 64$

How many factors of 2 are there in the repeated multiplication?

Method 2: Apply the Exponent Laws

You can multiply the exponents.

$(2^3)^2 = 2^6$
$= 64$

Literacy Link
Raising a Power to an Exponent
When a power is raised to an exponent, multiply the exponents to write the expression with a single exponent.
$(a^m)^n = a^{mn}$

b) *Method 1: Use Repeated Multiplication*

$[2 \times (-3)]^4 = [2 \times (-3)] \times [2 \times (-3)] \times [2 \times (-3)] \times [2 \times (-3)]$
$= 2 \times 2 \times 2 \times 2 \times (-3) \times (-3) \times (-3) \times (-3)$
$= 2^4 \times (-3)^4$
$= 16 \times 81$
$= 1296$

How do you know that you can rearrange the factors?

Method 2: Apply the Exponent Laws

You can write each factor in the product with the same exponent.

$[2 \times (-3)]^4 = 2^4 \times (-3)^4$
$= 16 \times 81$
$= 1296$

How else could you evaluate this expression?

Literacy Link
Raising a Product to an Exponent
When a product is raised to an exponent, you can rewrite each factor in the product with the same exponent.
$(ab)^m = a^m b^m$

c) *Method 1: Use Repeated Multiplication*

$\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)$
$= \frac{3 \times 3 \times 3}{4 \times 4 \times 4}$
$= \frac{3^3}{4^3}$
$= \frac{27}{64}$

Why can you not divide these powers by subtracting the exponents?

Method 2: Apply the Exponent Laws

You can write each number in the quotient with the same exponent.

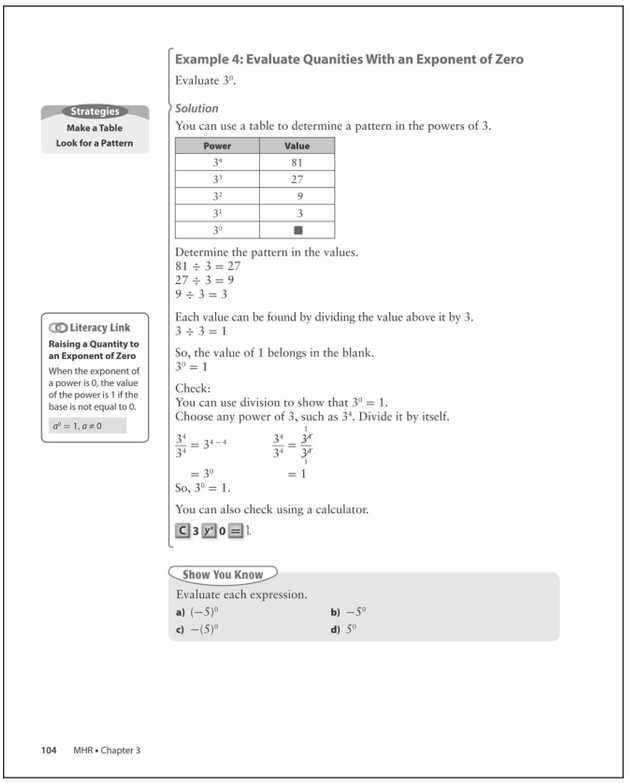
$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3}$
$= \frac{27}{64}$

Literacy Link
Raising a Quotient to an Exponent
When a quotient is raised to an exponent, you can rewrite each number in the quotient with the same exponent.
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

Show You Know

a) Write $[(-3)^4]^3$ as a single power. Then, evaluate.
b) Write $(5 \times 4)^2$ as the product of two powers. Then, evaluate.
c) Write $\left(\frac{2}{5}\right)^4$ as the quotient of two powers. Then, evaluate.

3.2 Exponent Laws • MHR 103



Example 4: Evaluate Quanities With an Exponent of Zero

Evaluate 3^0.

Strategies
Make a Table
Look for a Pattern

Solution

You can use a table to determine a pattern in the powers of 3.

Power	Value
3^4	81
3^3	27
3^2	9
3^1	3
3^0	■

Determine the pattern in the values.
$81 \div 3 = 27$
$27 \div 3 = 9$
$9 \div 3 = 3$

Each value can be found by dividing the value above it by 3.
$3 \div 3 = 1$

So, the value of 1 belongs in the blank.
$3^0 = 1$

Check:
You can use division to show that $3^0 = 1$.
Choose any power of 3, such as 3^4. Divide it by itself.

$\frac{3^4}{3^4} = 3^{4-4}$
$= 3^0$

$\frac{3^4}{3^4} = \frac{\cancel{3}^{\cancel{4}}}{\cancel{3}^{\cancel{4}}}$
$= 1$

So, $3^0 = 1$.

You can also check using a calculator.

C 3 y^x 0 = 1.

Literacy Link
Raising a Quantity to an Exponent of Zero
When the exponent of a power is 0, the value of the power is 1 if the base is not equal to 0.
$a^0 = 1, a \neq 0$

Show You Know

Evaluate each expression.
a) $(-5)^0$
b) -5^0
c) $-(5)^0$
d) 5^0

104 MHR • Chapter 3

Example 3

In this example, students have an opportunity to apply exponent rules to powers of powers, powers of products, and powers of quotients. For students who appear to rely on using the exponent laws to solve, ask them how to solve the problem through repeated multiplication. Conversely, check that students are able to generalize to the exponent rule and do not rely solely on repeated multiplication as their solution method.

Literacy Link After Example 3 part a), direct students to read the Literacy Link on page 103 related to raising a power to an exponent. Ask students what numbers in part a) of Example 3 correspond to *a*, *m*, and *n* in part a) of Example 3.

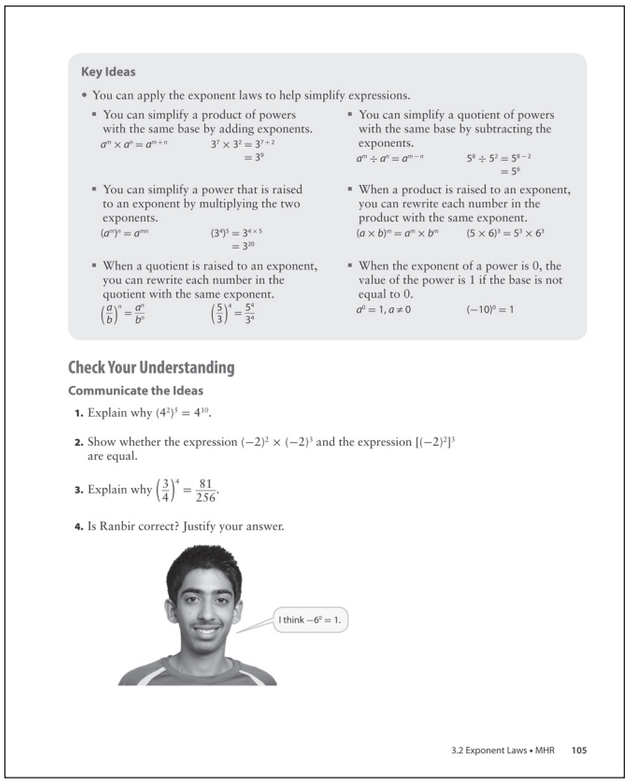
After Example 3 part b), have students read the Literacy Link about raising a product to an exponent. Ask students what numbers in part b) of Example 3 correspond to *a*, *b*, and *m*.

After Example 3 part c), ask students to turn their attention to the Literacy Link related to raising a quotient to an exponent. Ask students what numbers in part c) of Example 3 correspond to *a*, *b*, and *n*.

Example 4

Through a patterning exercise, students develop an understanding of the concept that a non-negative base with an exponent of 0 is equivalent to 1. Verify this finding by using the division rule of multiplication where a power is divided by itself. Ensure that students understand that a fraction with an equal numerator and denominator has a value of 1.

Literacy Link After Example 4, focus students' attention on the Literacy Link on page 104 related to a power with an exponent of zero. Explain that the base cannot equal zero because the power 0^0 is not defined. You may wish to have students try evaluating this power with a calculator. The calculator should return some type of error.



Key Ideas

- You can apply the exponent laws to help simplify expressions.
  - You can simplify a product of powers with the same base by adding exponents.
    $a^m \times a^n = a^{m+n}$ $3^7 \times 3^2 = 3^{7+2} = 3^9$
  - You can simplify a quotient of powers with the same base by subtracting the exponents.
    $a^m \div a^n = a^{m-n}$ $5^8 \div 5^2 = 5^{8-2} = 5^6$
  - You can simplify a power that is raised to an exponent by multiplying the two exponents.
    $(a^m)^n = a^{mn}$ $(3^4)^5 = 3^{4 \times 5} = 3^{20}$
  - When a product is raised to an exponent, you can rewrite each number in the product with the same exponent.
    $(a \times b)^m = a^m \times b^m$ $(5 \times 6)^3 = 5^3 \times 6^3$
  - When a quotient is raised to an exponent, you can rewrite each number in the quotient with the same exponent.
    $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\left(\frac{5}{3}\right)^4 = \frac{5^4}{3^4}$
  - When the exponent of a power is 0, the value of the power is 1 if the base is not equal to 0.
    $a^0 = 1, a \neq 0$ $(-10)^0 = 1$

Check Your Understanding

Communicate the Ideas

1. Explain why $(4^2)^5 = 4^{10}$.
2. Show whether the expression $(-2)^2 \times (-2)^3$ and the expression $[(-2)^2]^3$ are equal.
3. Explain why $\left(\frac{3}{4}\right)^4 = \frac{81}{256}$.
4. Is Ranbir correct? Justify your answer.

3.2 Exponent Laws • MHR 105

Key Ideas

The Key Ideas summarize the exponent laws developed in this section. Have students verify each of the examples given by expanding and simplifying.

There is a great deal of conceptual understanding that occurs in this section of the chapter. Help students by asking them to choose the two or three exponent rules that are most challenging to them. Consider re-teaching the exponent rules that most students choose.

Have students write their own summary of the Key Ideas, along with examples, and add them to their Foldable.

Meeting Student Needs

- Students may benefit from doing the examples as a whole-class activity. They might then complete the first part of the Show You Know as a small-group or paired activity and the second part as individual student work.
- Some students may benefit from continuing to solve problems using expansion before using the exponent laws.
- Discuss the idea of laws with students: what the word *law* means, what the purposes of laws are, etc. Brainstorm laws that are relevant to students' lives and discuss what impact they have. You may wish to discuss the fact that Aboriginal communities have traditional laws. These laws help to keep balance within the lives of many First Nations' people. Invite in an Elder from a nearby community to discuss traditional laws and protocol with students.

ELL

- Teach the following words in context: *evaluate*, *quotient*, and *product*.

Common Errors

- Some students may reverse the exponent rules for multiplying and dividing powers.

R$_x$ Solve additional problems using expansion and then simplification. Through discovery, students will understand that when multiplying powers, the exponent in the final power is the sum of the initial exponents.

- Some students may add exponents when a power is raised to another exponent.

R$_x$ Solve additional problems using expansion and then simplification.

- Some students may evaluate powers with exponents of 0 as being equal to 0.

R$_x$ Use the law of division to show that when a power is divided by itself, the answer is 1 and the power will have 0 as its exponent. Have students verify this finding with a calculator.

Answers

Example 1: Show You Know

a) $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 65\,536$; $4^{3+5} = 4^8 = 65\,536$

b) $(-5) \times (-5) \times (-5) \times (-5) \times (-5) = -3125$;
$(-5)^{2+3} = (-5)^5 = -3125$

Example 2: Show You Know

a) $\frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} = 2$; $2^{5-4} = 2^1 = 2$

b) $\frac{(-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3)}{(-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3)}$
$= -27$; $(-3)^{10-7} = (-3)^3 = -27$

Example 3: Show You Know

a) $(-3)^{12} = 531\,441$

b) $5^2 \times 4^2 = 400$

c) $\frac{2^5}{5^5} = \frac{32}{3125}$

Example 4: Show You Know

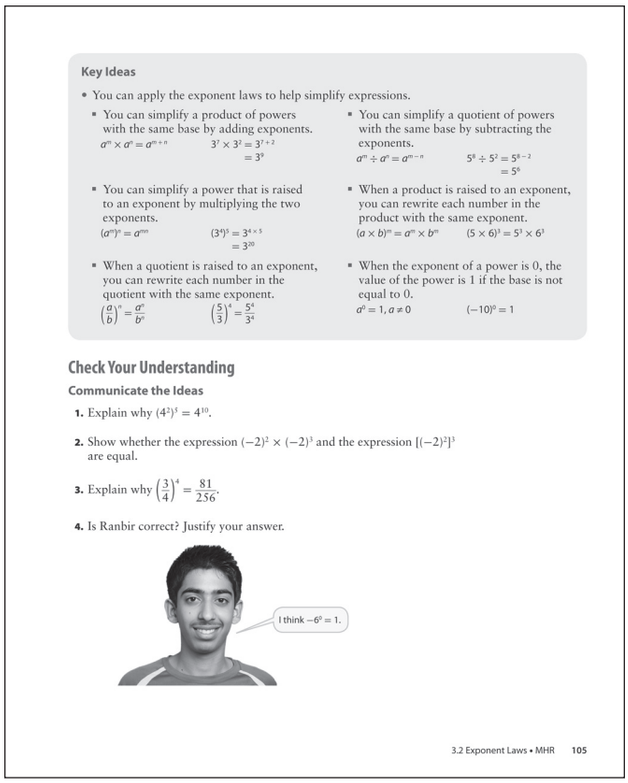
a) 1

b) -1

c) -1

d) 1

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Ensure that students are using parentheses around negative bases. • Suggest that students create two columns on their page labelled Method 1 and Method 2 and solve each of the Show You Know questions modelling the methods in the example. • Remind students to keep the parentheses until the final answer for questions that have brackets around a negative number in the base. Remind them that removing the brackets may create a different question all together.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Ensure that students are using parentheses around negative bases.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Check whether students are solving by expanding and then simplifying, or by using the exponent laws. • Each part of Example 3 shows students a way to apply the exponent rules to three unique styles of questions. Some students may approach the questions by simplifying what is in the brackets first and then applying repeated multiplication or a power law. This process is acceptable but students should be encouraged to make use of the rules applying to exponents. Ask students who are simplifying first to show the solution in at least two ways. • Ensure students understand the differences in the applications of each of the questions in Example 3 before moving on. Provide additional sample problems for students to complete if necessary.
Example 4 Have students do the Show You Know related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Make sure that students are clear on the difference between part a) and part b) of the Show You Know in terms of order of operations.



Key Ideas

- You can apply the exponent laws to help simplify expressions.
  - You can simplify a product of powers with the same base by adding exponents.
    $a^m \times a^n = a^{m+n}$ $\quad 3^2 \times 3^2 = 3^{2+2} = 3^4$
  - You can simplify a quotient of powers with the same base by subtracting the exponents.
    $a^m \div a^n = a^{m-n}$ $\quad 5^8 \div 5^2 = 5^{8-2} = 5^6$
  - You can simplify a power that is raised to an exponent by multiplying the two exponents.
    $(a^m)^n = a^{mn}$ $\quad (3^4)^5 = 3^{4 \times 5} = 3^{20}$
  - When a product is raised to an exponent, you can rewrite each number in the product with the same exponent.
    $(a \times b)^m = a^m \times b^m$ $\quad (5 \times 6)^3 = 5^3 \times 6^3$
  - When a quotient is raised to an exponent, you can rewrite each number in the quotient with the same exponent.
    $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\quad \left(\frac{5}{3}\right)^4 = \frac{5^4}{3^4}$
  - When the exponent of a power is 0, the value of the power is 1 if the base is not equal to 0.
    $a^0 = 1, a \neq 0$ $\quad (-10)^0 = 1$

Check Your Understanding

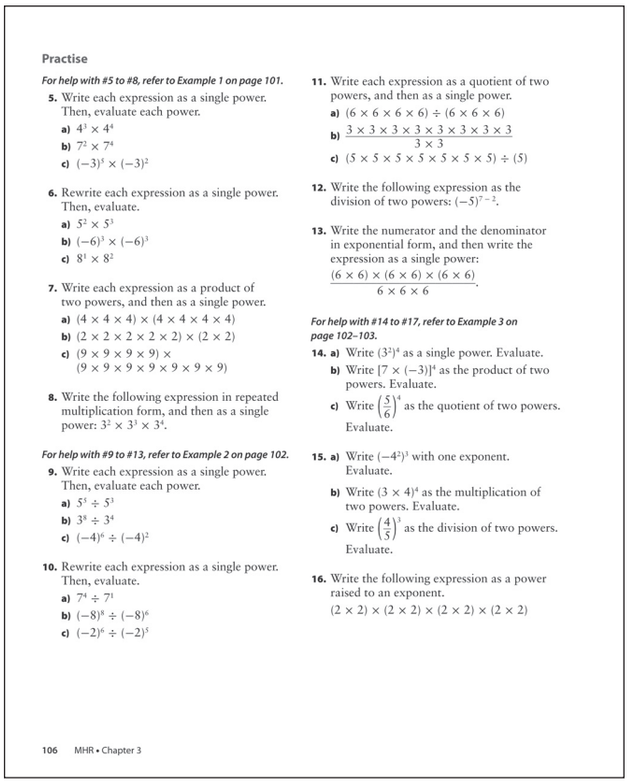
Communicate the Ideas

1. Explain why $(4^2)^5 = 4^{10}$.

2. Show whether the expression $(-2)^2 \times (-2)^3$ and the expression $[(-2)^2]^3$ are equal.

3. Explain why $\left(\frac{3}{4}\right)^4 = \frac{81}{256}$.

4. Is Ranbir correct? Justify your answer.



Practise

For help with #5 to #8, refer to Example 1 on page 101.

5. Write each expression as a single power. Then, evaluate each power.
- **a)** $4^3 \times 4^4$
- **b)** $7^2 \times 7^4$
- **c)** $(-3)^5 \times (-3)^2$

6. Rewrite each expression as a single power. Then, evaluate.
- **a)** $5^2 \times 5^3$
- **b)** $(-6)^3 \times (-6)^3$
- **c)** $8^1 \times 8^2$

7. Write each expression as a product of two powers, and then as a single power.
- **a)** $(4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4)$
- **b)** $(2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2)$
- **c)** $(9 \times 9 \times 9 \times 9) \times (9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9)$

8. Write the following expression in repeated multiplication form, and then as a single power: $3^2 \times 3^3 \times 3^4$.

For help with #9 to #13, refer to Example 2 on page 102.

9. Write each expression as a single power. Then, evaluate each power.
- **a)** $5^5 \div 5^3$
- **b)** $3^8 \div 3^4$
- **c)** $(-4)^6 \div (-4)^2$

10. Rewrite each expression as a single power. Then, evaluate.
- **a)** $7^4 \div 7^1$
- **b)** $(-8)^8 \div (-8)^6$
- **c)** $(-2)^6 \div (-2)^5$

11. Write each expression as a quotient of two powers, and then as a single power.
- **a)** $(6 \times 6 \times 6 \times 6) \div (6 \times 6 \times 6)$
- **b)** $\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$
- **c)** $(5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5) \div (5)$

12. Write the following expression as the division of two powers: $(-5)^{7-2}$.

13. Write the numerator and the denominator in exponential form, and then write the expression as a single power: $\frac{(6 \times 6) \times (6 \times 6) \times (6 \times 6)}{6 \times 6 \times 6}$.

For help with #14 to #17, refer to Example 3 on page 102–103.

14.
- **a)** Write $(3^2)^4$ as a single power. Evaluate.
- **b)** Write $[7 \times (-3)]^4$ as the product of two powers. Evaluate.
- **c)** Write $\left(\frac{5}{6}\right)^4$ as the quotient of two powers. Evaluate.

15.
- **a)** Write $(-4^2)^3$ with one exponent. Evaluate.
- **b)** Write $(3 \times 4)^4$ as the multiplication of two powers. Evaluate.
- **c)** Write $\left(\frac{4}{3}\right)^3$ as the division of two powers. Evaluate.

16. Write the following expression as a power raised to an exponent.
$(2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$

Check Your Understanding

Communicate the Ideas

These questions allow students to apply their understanding of the exponent laws. Have students work individually and then check their answers with a partner.

For #1, check whether students are relying on the exponent law. Ensure students can verify the equation with repeated multiplication. Conversely, for students who are relying only on repeated multiplication to solve these types of problems, help them generalize their findings into the exponent rules described in the Key Ideas.

Practise

These questions require students to simplify operations with powers using exponent laws and through expansion. Ensure that students get sufficient practice with these basic questions.

For #5, some students may benefit from writing both powers in repeated multiplication form first. Assign #7 and 11 because they model expansion of the factors in a power.

For #8, students may need to write each power in repeated multiplication form before expressing their answer as a single power.

Many students may find #12 challenging. It may be beneficial to write the quotient of the two powers with a base of -5 and boxes for the exponents. Then, ask students what numbers should go in the boxes.

For #15a), check whether students have the correct sign for their answer. Students who have an incorrect sign will need to review order of operations.

For #16, you may need to ask some students about the significance of the parentheses. It may be beneficial to model an example for some students prior to having them complete #16.

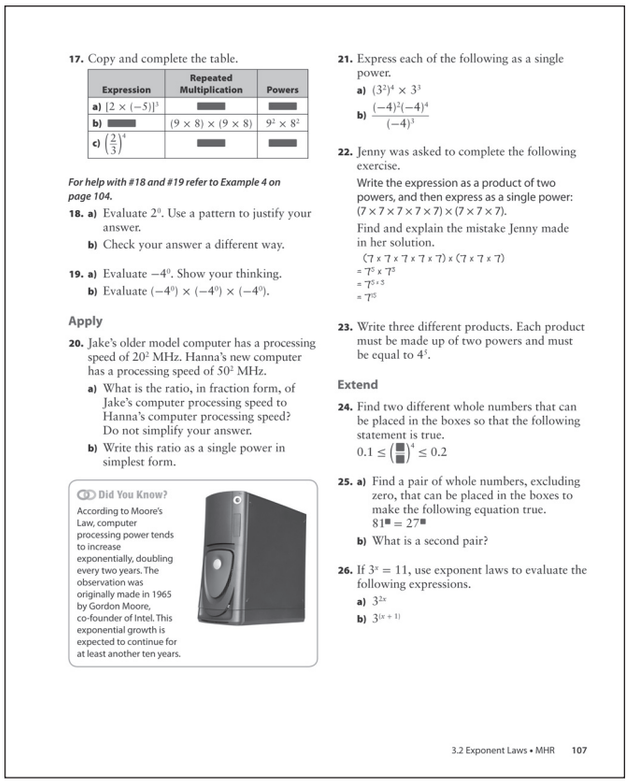
In #17, the first column and last column both contain powers. However, the last column will contain more than one power in each expression.

For #18, students may need to be prompted to check their answer as instructed in part b). Suggest that students use the quotient of powers with the same base. For example, $8 \div 8 = 2^3 \div 2^3 = 2^{3-3} = 2^0 = 1$.

Apply

These questions provide an opportunity for students to apply more than one exponent law in an expression.

For #20, students may be interested in reading about Moore's Law. Refer to the Did You Know? that follows.



17. Copy and complete the table.

Expression	Repeated Multiplication	Powers
a) $[2 \times (-5)]^3$		
b)	$(9 \times 8) \times (9 \times 8)$	$9^2 \times 8^2$
c) $\left(\frac{2}{3}\right)^4$		

For help with #18 and #19 refer to Example 4 on page 104.

18. a) Evaluate 2^0. Use a pattern to justify your answer.
b) Check your answer a different way.

19. a) Evaluate -4^0. Show your thinking.
b) Evaluate $(-4^0) \times (-4^0) \times (-4^0)$.

Apply

20. Jake's older model computer has a processing speed of 20^2 MHz. Hanna's new computer has a processing speed of 50^2 MHz.
a) What is the ratio, in fraction form, of Jake's computer processing speed to Hanna's computer processing speed? Do not simplify your answer.
b) Write this ratio as a single power in simplest form.

Did You Know?
According to Moore's Law, computer processing power tends to increase exponentially, doubling every two years. The observation was originally made in 1965 by Gordon Moore, co-founder of Intel. This exponential growth is expected to continue for at least another ten years.

21. Express each of the following as a single power.
a) $(3^2)^4 \times 3^3$
b) $\frac{(-4)^2(-4)^4}{(-4)^3}$

22. Jenny was asked to complete the following exercise.
Write the expression as a product of two powers, and then express as a single power: $(7 \times 7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7)$.
Find and explain the mistake Jenny made in her solution.
$(7 \times 7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7)$
$= 7^5 \times 7^3$
$= 7^{5 \times 3}$
$= 7^{15}$

23. Write three different products. Each product must be made up of two powers and must be equal to 4^2.

Extend

24. Find two different whole numbers that can be placed in the boxes so that the following statement is true.
$0.1 \leq \left(\frac{\blacksquare}{\blacksquare}\right)^4 \leq 0.2$

25. a) Find a pair of whole numbers, excluding zero, that can be placed in the boxes to make the following equation true.
$81^{\blacksquare} = 27^{\blacksquare}$
b) What is a second pair?

26. If $3^x = 11$, use exponent laws to evaluate the following expressions.
a) 3^{2x}
b) $3^{(x+1)}$

3.2 Exponent Laws • MHR 107

In #22, students are required to perform error analysis. These questions are useful for students to identify and focus on some of the common errors when working with powers.

Extend

For #24, students may wish to convert the two boundary values into fractions when they begin. Students will likely experiment with different numbers in the search for a valid solution.

For #25, some students may need a hint: How could each base in the equality be written as a power with a common base?

Literacy Link After section 3.2, work with students to start the upper right leg of the spider map, entitled Exponents Law. Brainstorm and discuss as a class the information needed to begin this leg.

Meeting Student Needs

- Provide **BLM 3–7 Section 3.2 Extra Practice** to students who would benefit from more practice.

ELL

- Teach the following terms in context: *justify*, and *whole numbers*.
- Consider allowing students not to explain or justify their answers.

Common Errors

- In #4, some students may struggle with how to show that Ranbir is correct.

R_x Prompt students to use patterning or the division rule for powers.

- Some students may perform the wrong operation with exponents as they try to apply the exponent laws.

R_x Have students solve the problems using repeated multiplication and then simplification.

Answers

Communicate the Ideas

1. Example: $(4^2)^5 = 4^2 \times 4^2 \times 4^2 \times 4^2 \times 4^2$
$= 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^{10}$

2. The expressions are not equal.
$(-2)^2 \times (-2)^3 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = (-2)^5$
$[(-2)^2]^3 = (-2)^2 \times (-2)^2 \times (-2)^2$
$= (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = (-2)^6$

3. $\left(\frac{3}{4}\right)^4 = \frac{3^4}{4^4} = \frac{81}{256}$

4. Yes. Example: $\frac{-6^2}{-6^2} = \frac{36}{36} = 1$, and $\frac{-6^2}{-6^2} = -6^{2-2} = -6^0$.
Therefore, $-6^0 = 1$.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to 4. Have students share their responses to #2 and 3 with a partner and listen to each other's explanations. Use student responses to assess their understanding of the laws of power of products and quotients. Have students share their response to #4 with the class and discuss the resulting approaches.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Consider having students assess each other's responses to #2 and/or 3 using **Master 2 Communication Peer Evaluation**. • Ensure students have a good understanding and are able to justify their thinking before moving onto the Practise section.
Assessment *for* Learning	
Practise Have students do #6, 7, 9, 11, 14, 17, 19. Students who have no problems with these questions can go on to the Apply questions.	• Note that #5 and 6 are similar questions relating to multiplication and #9 and 10 are similar questions relating to division. Consider assigning only one of each set and using the remainder for remediation if needed. • Note that #7 and 11 are opposite in operation but basic to understanding exponents. Ensure students are able to correctly complete the expressions as single powers and apply the appropriate operations. Provide additional questions of support if needed. Consider scaffolding the questions by creating three deliberate parts such as: – Write each expansion as a power with an exponent. – Write a multiplication/division statement that reflects what is shown. – Write the result as a single power. • In #14, a variety of powers are covered. Use the appropriate parts from #15 if remediation is required. • In #17, students are provided with an opportunity to move between the three different presentations of powers taught in this section. Extend the chart and provide additional questions if needed. Students should demonstrate that they are comfortable moving among each of the columns.
Assessment *as* Learning	
Literacy Link At the end of section 3.2, have students work in pairs to complete the upper right leg of the spider map, entitled Exponent Laws.	• Have students list all the rules and terms they learned that are associated with the exponent laws. • Students may want to record each of the exponent laws on their spider map. Encourage them to explain the laws using wording and/or symbols of their choice and to include their own examples.
Math Learning Log Have students respond to the following prompts: • Explain a common error that occurs in simplifying expressions with powers. Use an example in your explanation. • The power rule I find the most confusing is ...	• Some students may have a difficult time knowing what a common error is in exponents, particularly if students are making that error. You may wish to provide them with examples of common errors and have them identify the error and correct it. Some samples are: $2^4 \times 2^5 = 2^{20}$ $(2^3)^6 = 2^9$ $\frac{2^8}{2^4} = 2^2$ $[(-5)^2]^4 = -5^8$