3.3 Order of Operations

***MathLinks 9*, pages 108–113**

Suggested Timing
50–60 minutes

Materials
- calculator
- ruler (optional)

Blackline Masters
Master 2 Communication Peer Evaluation
BLM 3–3 Chapter 3 Warm-Up
BLM 3–8 Section 3.3 Extra Practice
BLM 3–9 Section 3.3 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☐ Visualization (V)

Specific Outcomes
N1 Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:
- representing repeated multiplication using powers
- using patterns to show that a power with an exponent of zero is equal to one
- solving problems involving powers.

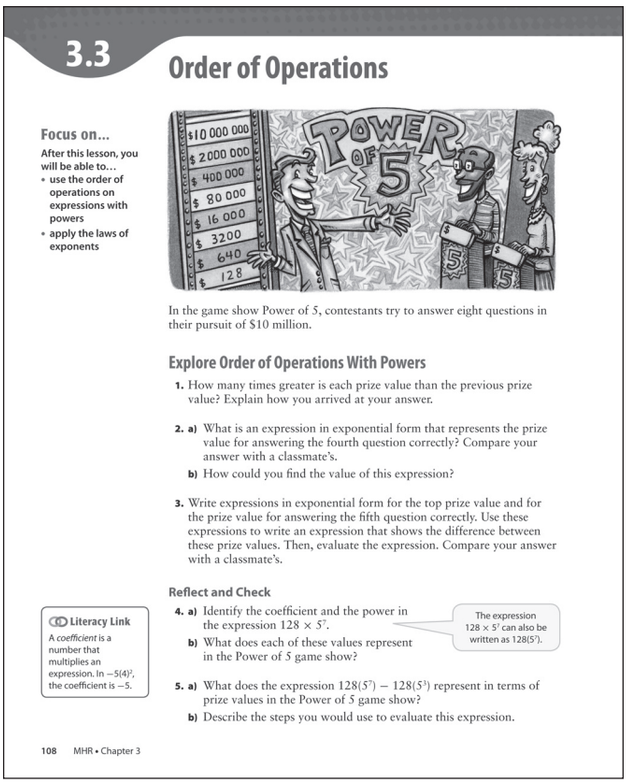
N2 Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.
N4 Explain and apply the order of operations, including exponents, with and without technology.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–8, 10, 11, 15, Math Link
Typical	#1–8, 10, 11, 14, 15, 18, Math Link
Extension/Enrichment	#1–4, 11, 12, 17–18

Planning Notes

In this section, students use the order of operations on expressions with powers and apply the laws of exponents.

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce material learned in previous sections.



3.3 Order of Operations

Focus on...
After this lesson, you will be able to...
- use the order of operations on expressions with powers
- apply the laws of exponents

In the game show Power of 5, contestants try to answer eight questions in their pursuit of $10 million.

Explore Order of Operations With Powers

1. How many times greater is each prize value than the previous prize value? Explain how you arrived at your answer.

2. a) What is an expression in exponential form that represents the prize value for answering the fourth question correctly? Compare your answer with a classmate's.
   b) How could you find the value of this expression?

3. Write expressions in exponential form for the top prize value and for the prize value for answering the fifth question correctly. Use these expressions to write an expression that shows the difference between these prize values. Then, evaluate the expression. Compare your answer with a classmate's.

Reflect and Check

Literacy Link
A *coefficient* is a number that multiplies an expression. In $-5(4)^2$, the coefficient is -5.

4. a) Identify the coefficient and the power in the expression 128×5^7.
   b) What does each of these values represent in the Power of 5 game show?

The expression 128×5^7 can also be written as $128(5^7)$.

5. a) What does the expression $128(5^7) - 128(5^1)$ represent in terms of prize values in the Power of 5 game show?
   b) Describe the steps you would use to evaluate this expression.

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Explore Order of Operations With Powers

In this Explore, students have an opportunity to create and assess exponential expressions. The focus is on the product of a power and its coefficient and the application of the proper order of operations.

Method 1 Introduce the Explore to the class by discussing current game shows on television (e.g., Are You Smarter Than a Fifth Grader? and Who Wants to Be a Millionaire?) where there is a hierarchical set of monetary prizes that the contestant is trying to claim based on the number of questions answered successfully. Then, have students answer the questions in pairs.

Method 2 On the board, write the prize amounts for the game. Ask students to rewrite each prize using prime factorization. Then, ask students to answer #1. Have students work in pairs to answer the remaining questions using the prime factorizations listed on the board. You may wish to ask questions such as the following:
- What is the difference between the coefficient and the base of a power?

- Should you multiply the base and the coefficient prior to assessing the power?

Literacy Link Direct students to read the Literacy Link on page 108 related to the word *coefficient*. Emphasize that the power must be evaluated before the coefficient is multiplied, according to proper order of operations.

Meeting Student Needs

- It might be better for your students to work through the Explore as a whole-class activity.

ELL

- Teach the following terms in context: *game show*, *contestants*, *pursuit*, *exponential form*, and *fourth*.
- English language learners may have the math skills to answer the question, "How many times greater is each prize value than the previous prize value?" However, they may get stuck on the language. Use a simple equation on the board to demonstrate what *how many times greater* means.

Answers

Explore Order of Operations With Power

1. Five times greater. Example: Divide two consecutive prize values.

2. a) Examples: 128×5^3, $2^7 \times 5^3$

b) Example: Find the value of 5^3, which is 12, and multiply it by the value of 2^7, which is 128.

3. The top prize is $128 \times 5^7 = 128 \times 78\,125 = \$10\,000\,000$. The fifth place prize is $128 \times 5^4 = 128 \times 625 = \$80\,000$. The difference between these prize values is $(128 \times 5^7) - (128 \times 5^4)$ $= 10\,000\,000 - 80\,000 = \$9\,920\,000$.

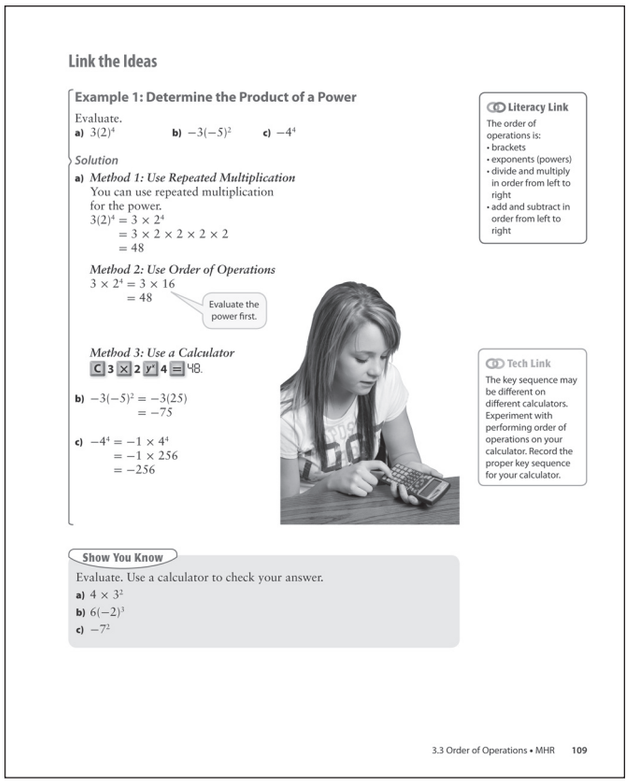
4. a) The coefficient is 128, and the power is 5^7.

b) Example: 128 represents the value of the first question. 5^7 represents how many times greater the top prize is compared to the prize for the first question.

5. a) The difference in prize values between the top prize and the prize for the fourth question.

b) Example: Find the value of 5^7, find the value of 5^3, subtract the two values, and multiply the difference by 128.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Ensure that students understand the term *coefficient*. Check that they are ordering the operations properly.	Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Ensure that students understand the difference between the coefficient and the power. • When evaluating the expression, watch that students complete the power first. • Students who are struggling with the concept may benefit from seeing the multiplication written first as repeated or expanded multiplication.



Link the Ideas

Example 1: Determine the Product of a Power

Evaluate.
a) $3(2)^4$ **b)** $-3(-5)^2$ **c)** -4^4

Solution

a) *Method 1: Use Repeated Multiplication*
You can use repeated multiplication for the power.
$3(2)^4 = 3 \times 2^4$
$= 3 \times 2 \times 2 \times 2 \times 2$
$= 48$

Method 2: Use Order of Operations
$3 \times 2^4 = 3 \times 16$
$= 48$
Evaluate the power first.

Method 3: Use a Calculator
C 3 × 2 y^x 4 = 48.

b) $-3(-5)^2 = -3(25)$
$= -75$

c) $-4^4 = -1 \times 4^4$
$= -1 \times 256$
$= -256$

Literacy Link
The order of operations is:
- brackets
- exponents (powers)
- divide and multiply in order from left to right
- add and subtract in order from left to right

Tech Link
The key sequence may be different on different calculators. Experiment with performing order of operations on your calculator. Record the proper key sequence for your calculator.

Show You Know
Evaluate. Use a calculator to check your answer.
a) 4×3^2
b) $6(-2)^3$
c) -7^2

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Example 2: Evaluate Expressions With Powers

Evaluate.
a) $4^2 - 8 \div 2 + (-3^2)$ **b)** $-2(-15 - 4^2) + 4(2 + 3)^3$

Solution

a) *Method 1: Use Order of Operations*
$4^2 - 8 \div 2 + (-3^2)$
$= 16 - 8 \div 2 + (-9)$
$= 16 - 4 + (-9)$
$= 12 + (-9)$
$= 3$

Method 2: Use a Calculator
C 4 y^x 2 − 8 ÷ 2 + 3 +/− y^x 2 = 3.
You may need to use a different key sequence on your calculator.

b) *Method 1: Use Order of Operations*
$-2(-15 - 4^2) + 4(2 + 3)^3$
$= -2(-15 - 16) + 4(5)^3$
$= -2(-31) + 4(5)^3$
$= -2(-31) + 4(125)$
$= 62 + 4(125)$
$= 62 + 500$
$= 562$

Method 2: Use a Calculator
C 2 +/− × (15 +/− − 4 x^2) + 4 ×
(2 + 3) y^x 3 = 562.

Show You Know
Evaluate.
a) $4^2 + (-4^2)$ **b)** $8(5 + 2)^2 - 12 \div 2^2$

Key Ideas
- Expressions with powers can have a numerical coefficient. Evaluate the power, and then multiply by the coefficient.
- Evaluate expressions with powers using the proper order of operations.
  - brackets
  - exponents
  - divide and multiply in order from left to right
  - add and subtract in order from left to right

Expression	Coefficient	Power	Repeated Multiplication	Value
$5(4^2)$	5	4^2	$5 \times 4 \times 4$	80
$(-2)^4$	1	$(-2)^4$	$(-2)(-2)(-2)(-2)$	16
-3^4	−1	3^4	$-1 \times 3 \times 3 \times 3 \times 3$	−81

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Link the Ideas

Example 1

Example 1 provides an opportunity to simplify powers with coefficients other than 1. The focus is the importance of the placement of parentheses. For part c), ask students how they know that the exponent does not apply to the negative sign.

Literacy Link To help some students with the concept of proper order of operations, refer to the Literacy Link in the right margin. You may wish to remind them of the mnemonic BEDMAS to remember this order. Check that students can sequence the operations correctly on a calculator to verify their solution.

Example 2

Students who are struggling with the concept of proper order of operations will need to work on these skills before they can be successful with this example. To help them to identify the operations, it may help students to write them in a different colour in the expression when they record the question in their notebook. Encourage students to share their answers. Ensure that students have checked their calculators for the proper keying sequence for the order of operations. Some calculators require additional bracketing for the correct answer. Suggest to students that they record the correct keying sequence for their calculator.

Key Ideas

The table in the Key Ideas provides a visual description of the coefficient and power in an exponential expression. The second half of the Key Ideas reiterates the proper order of operations for simplifying expressions.

Have students add notes to their Foldable to summarize the Key Ideas using their own words and examples.

Meeting Student Needs

- Students may benefit from doing the examples as a whole-class activity. They might then complete the first part of the Show You Know as a small-group or paired activity and the second part as individual student work.
- Some students may benefit from colour-coding the operations and counting the number of operations prior to ordering them.

ELL

- Teach the following terms in context: *key sequence*, *numerical coefficient*, and *visible*.

Common Errors

- Some students may include a negative sign with a base instead of realizing that the negative sign refers to a coefficient of -1.

R$_x$ Show students that -3^2 is the same as $-1(3^2)$.

Answers

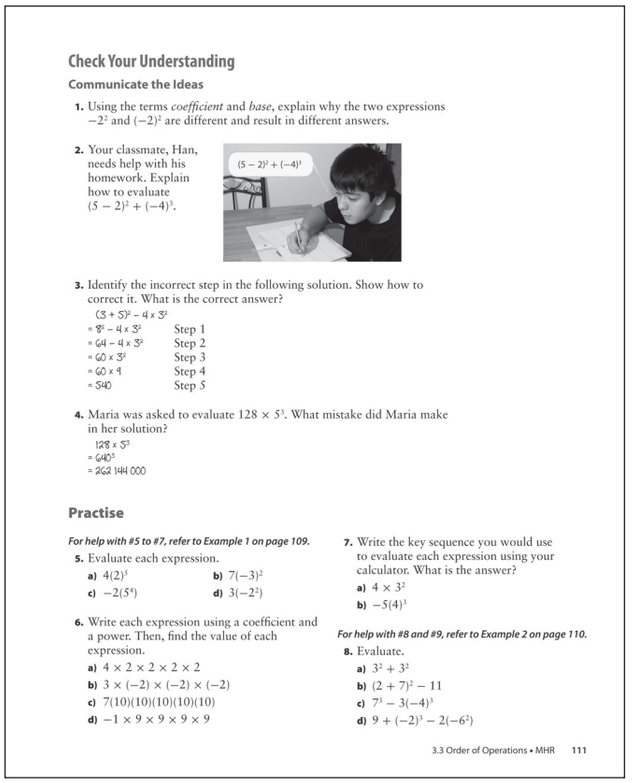
Example 1: Show You Know

a) 36 **b)** -48 **c)** -49

Example 2: Show You Know

a) 0 **b)** 389

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Check that students understand that parentheses affect the order of operations. • Some students may benefit from writing the acronym BEDMAS into their Foldable and using it a s a guide in solving questions involving order of operations. Alternatively, some students may find the memory phrase **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally (parentheses, exponents, multiplication and division, addition and subtraction) an easier method to remember the order. • Encourage students to complete only one step at a time and show their work. Model the V shape that the work should appear in as one more operation is completed in each step.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Check that students are applying the order of operations properly. • Ensure that students have checked their calculators for the proper keying sequence for the order of operations. Some calculators require additional bracketing for the correct answer.



Check Your Understanding

Communicate the Ideas

1. Using the terms *coefficient* and *base*, explain why the two expressions -2^2 and $(-2)^2$ are different and result in different answers.

2. Your classmate, Han, needs help with his homework. Explain how to evaluate $(5-2)^2+(-4)^3$.

3. Identify the incorrect step in the following solution. Show how to correct it. What is the correct answer?

$(5+5)^2-4\times3^2$
$=8^2-4\times3^2$ Step 1
$=64-4\times3^2$ Step 2
$=60\times3^2$ Step 3
$=60\times9$ Step 4
$=540$ Step 5

4. Maria was asked to evaluate 128×5^3. What mistake did Maria make in her solution?

128×5^3
$=640^3$
$=262\,144\,000$

Practise

For help with #5 to #7, refer to Example 1 on page 109.

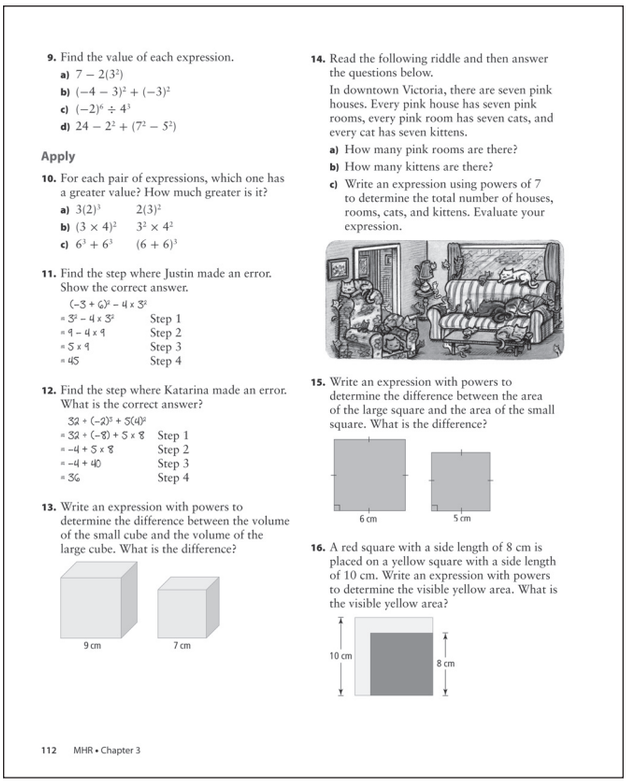
5. Evaluate each expression.
   a) $4(2)^5$
   b) $7(-3)^2$
   c) $-2(5^4)$
   d) $3(-2^2)$

6. Write each expression using a coefficient and a power. Then, find the value of each expression.
   a) $4\times2\times2\times2\times2$
   b) $3\times(-2)\times(-2)\times(-2)$
   c) $7(10)(10)(10)(10)(10)$
   d) $-1\times9\times9\times9\times9$

7. Write the key sequence you would use to evaluate each expression using your calculator. What is the answer?
   a) 4×3^2
   b) $-5(4)^3$

For help with #8 and #9, refer to Example 2 on page 110.

8. Evaluate.
   a) 3^2+3^2
   b) $(2+7)^2-11$
   c) $7^3-3(-4)^3$
   d) $9+(-2)^3-2(-6^2)$



9. Find the value of each expression.
   a) $7-2(3^2)$
   b) $(-4-3)^2+(-3)^2$
   c) $(-2)^6\div4^3$
   d) $24-2^2+(7^2-5^2)$

Apply

10. For each pair of expressions, which one has a greater value? How much greater is it?
    a) $3(2)^3$ $2(3)^2$
    b) $(3\times4)^2$ $3^2\times4^2$
    c) 6^3+6^3 $(6+6)^3$

11. Find the step where Justin made an error. Show the correct answer.

$(-3+6)^2-4\times3^2$
$=3^2-4\times3^2$ Step 1
$=9-4\times9$ Step 2
$=5\times9$ Step 3
$=45$ Step 4

12. Find the step where Katarina made an error. What is the correct answer?

$32+(-2)^3+5(4)^2$
$=32+(-8)+5\times8$ Step 1
$=-4+5\times8$ Step 2
$=-4+40$ Step 3
$=36$ Step 4

13. Write an expression with powers to determine the difference between the volume of the small cube and the volume of the large cube. What is the difference?

14. Read the following riddle and then answer the questions below.

In downtown Victoria, there are seven pink houses. Every pink house has seven pink rooms, every pink room has seven cats, and every cat has seven kittens.

   a) How many pink rooms are there?
   b) How many kittens are there?
   c) Write an expression using powers of 7 to determine the total number of houses, rooms, cats, and kittens. Evaluate your expression.

15. Write an expression with powers to determine the difference between the area of the large square and the area of the small square. What is the difference?

16. A red square with a side length of 8 cm is placed on a yellow square with a side length of 10 cm. Write an expression with powers to determine the visible yellow area. What is the visible yellow area?

Check Your Understanding

Communicate the Ideas

For #1, 3, and 4, students are provided with an opportunity to directly address several common mistakes or misconceptions often made when working with exponential expressions.

Practise

Provide students with sufficient opportunities to practise simplifying exponential expressions. Note that #5, 6, and 7 provide three different ways of practising these skills.

For #8 and 9, students are provided with an opportunity to apply order of operations in multi-step problems.

Apply

In #10, students apply order of operations to compare the relative values of two expressions.

For #11 and 12, students perform error analysis. Encourage students to read carefully through each step of the incorrect solutions presented.

For #14, some students may need to sketch a diagram to get a sense of how many kittens are present in the houses.

For #15 and 16, students are introduced to visual modelling for factoring differences in squares.

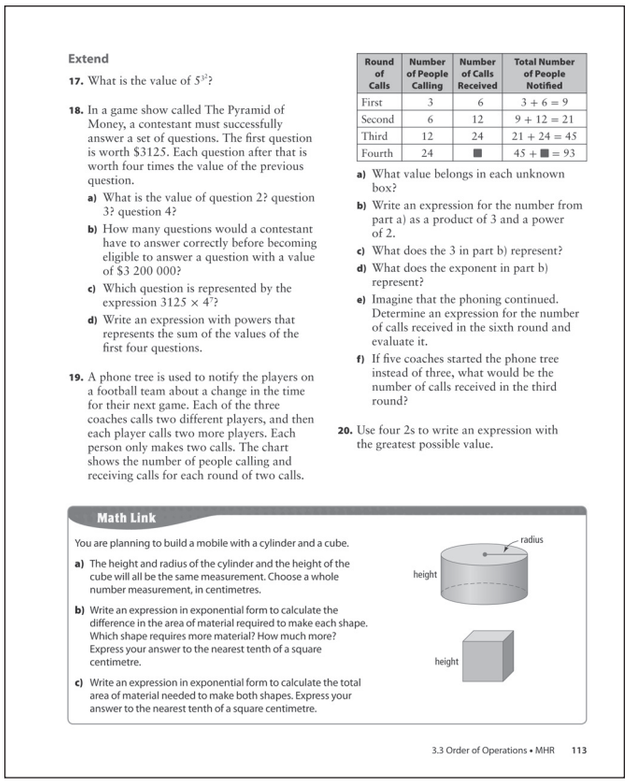
Extend

Some students may benefit from using a calculator to solve #18, though it is not necessary.

For #19, you may need to explain what a phone tree is.

Students will get an idea for maximizing the value in #20 if they complete #17 first.

Literacy Link After section 3.3, work with students to start the lower left leg of the spider map, entitled Order of Operations. Brainstorm and discuss as a class the information needed to begin this leg.



Extend

17. What is the value of 5^{3^2}?

18. In a game show called The Pyramid of Money, a contestant must successfully answer a set of questions. The first question is worth $3125. Each question after that is worth four times the value of the previous question.

a) What is the value of question 2? question 3? question 4?

b) How many questions would a contestant have to answer correctly before becoming eligible to answer a question with a value of $3 200 000?

c) Which question is represented by the expression 3125×4^7?

d) Write an expression with powers that represents the sum of the values of the first four questions.

19. A phone tree is used to notify the players on a football team about a change in the time for their next game. Each of the three coaches calls two different players, and then each player calls two more players. Each person only makes two calls. The chart shows the number of people calling and receiving calls for each round of two calls.

Round of Calls	Number of People Calling	Number of Calls Received	Total Number of People Notified
First	3	6	3 + 6 = 9
Second	6	12	9 + 12 = 21
Third	12	24	21 + 24 = 45
Fourth	24	■	45 + ■ = 93

a) What value belongs in each unknown box?

b) Write an expression for the number from part a) as a product of 3 and a power of 2.

c) What does the 3 in part b) represent?

d) What does the exponent in part b) represent?

e) Imagine that the phoning continued. Determine an expression for the number of calls received in the sixth round and evaluate it.

f) If five coaches started the phone tree instead of three, what would be the number of calls received in the third round?

20. Use four 2s to write an expression with the greatest possible value.

Math Link

You are planning to build a mobile with a cylinder and a cube.

a) The height and radius of the cylinder and the height of the cube will all be the same measurement. Choose a whole number measurement, in centimetres.

b) Write an expression in exponential form to calculate the difference in the area of material required to make each shape. Which shape requires more material? How much more? Express your answer to the nearest tenth of a square centimetre.

c) Write an expression in exponential form to calculate the total area of material needed to make both shapes. Express your answer to the nearest tenth of a square centimetre.

Math Link

This Math Link allows students to create and simplify several exponential expressions. You may need to ensure students recall methods for calculating the surface area of cubes and cylinders.

Meeting Student Needs

- Provide **BLM 3–8 Section 3.3 Extra Practise** to students who would benefit from more practice.

ELL

- Ensure that students understand what is meant by area (of a square). Though the calculation may be familiar to them, the term may not.
- Demonstrate one part of each Practise question on the board so that students understand what they are being asked to do.
- Have students draw a picture of the scenario in #14 to help them understand the problem.
- Model #19 for students as they may not understand what a phone tree is.

Common Errors

- In the Math Link, some students may not choose appropriate dimensions for their shapes.

R$_x$ Have students refer to a ruler to determine a reasonable value for the height and radius of each shape.

Answers

Communicate the Ideas

1. In the expression -2^2, the coefficient is -1. The answer is obtained by multiplying -1 by the value of 2^2 or 4. So, $-2^2 = -4$. In the expression $(-2)^2$, the coefficient is 1. The answer is obtained by determining the value of $(-2)^2$. So, $(-2)^2 = 4$.

2. Example: Follow the order of operations.

$$\begin{aligned}(5 - 2)^2 + (-4)^3 &= 3^2 + (-4)^3 \text{ (Brackets)}\\ &= 9 + (-64) \text{ (Exponents)}\\ &= -55 \text{ (Add)}\end{aligned}$$

3. Step 3 is incorrect. Exponents need to be evaluated first, followed by multiplication. The correct answer is 28.

4. Maria should have cubed 5 before multiplying by 128.

Math Link

a) Example: 5 cm

b) $2\pi 5^2 + 2\pi(5)(5) - 6(5^2)$.
The cylinder requires more material by 164.2 cm^2.

c) $2\pi 5^2 + 2\pi(5)(5) + 6(5^2) \approx 464.2$ cm^2

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Consider having students use **Master 2 Communication Peer Evaluation** to comment on each other's responses to these questions. • Note that #1 provides students an opportunity to demonstrate their understanding of the importance of parentheses. It is also important for students to know how their calculators handle these questions. • Students might use #2 as a model example in their Foldable. • For #3 and 4, students use error analysis to discover the mistakes in the solutions. Encourage students to check with a partner that they are in agreement about the incorrect step.
Assessment *for* Learning	
Practise Have students do #5–8, 10, 11, and 15. Students who have no problems with these questions can go on to the Apply questions.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students can do #5 to 7 to review the basic concepts covered in Example 1. In #5, students focus on the importance of parentheses. Struggling learners should be encouraged to write the question in expanded form before evaluating to help them visualize where the negative signs are placed and the effects the parentheses have on the question. In #6 and 7, students visualize the expanded form and write it in exponential form before evaluating. • Encourage student who are having difficulty with #8 to refer to their mnemonic for order of operations. Having them verbalize the process first may also assist some students. • For students having difficulty with #10, have them write out the question in expanded form first. This may assist them in seeing the values that are being repeatedly multiplied. • Note that #11 serves as a useful formative assessment question. Listen for students who are unable to identify the error as this means that they are making the same mistake as presented in the problem. Provide additional coaching and assign #12 to check for understanding. • The visual in #15 will assist students in solving the question; however, ensure they are able to write an appropriate expression that models their calculation. Ask them to explain how the order of operations is important in their expression.
Math Link The Math Link on page 113 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 123.	• Some students may require some extra coaching regarding the area formulas of shapes. • Ensure that students note the exponents in the surface area formulas. They may get confused by the fact that the formulas they have become familiar with are being referred to as exponential expressions. • Students who need help getting started could use **BLM 3–9 Section 3.3 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Literacy Link At the end of section 3.3, have students work in pairs to complete the lower left leg of the spider map, entitled Order of Operations.	• Have students list all the rules and terms they learned that are associated with order of operations involving powers. • The rules that students record might include the mnemonic BEDMAS or Please Excuse My Dear Aunt Sally. The terms might include *coefficient*.
Math Learning Log Have students respond to the following prompts: • Create a subtraction of two exponential expressions using the numbers 0, 1, 2, 3, 4, and 5. Then, solve. • If I were to evaluate $(-4) \times (2 + 1)^2 3 + 10$, the steps I would follow are …	• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with.