Using Exponents to Solve Problems

3.4

***MathLinks 9*, pages 114–119**

Suggested Timing
50–60 minutes

Materials
- ruler (optional)
- scissors (optional)

Blackline Masters
Master 2 Communication Peer Evaluation
BLM 3–3 Chapter 3 Warm-Up
BLM 3–10 Section 3.4 Extra Practice
BLM 3–11 Section 3.4 Math Link

Mathematical Processes
- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

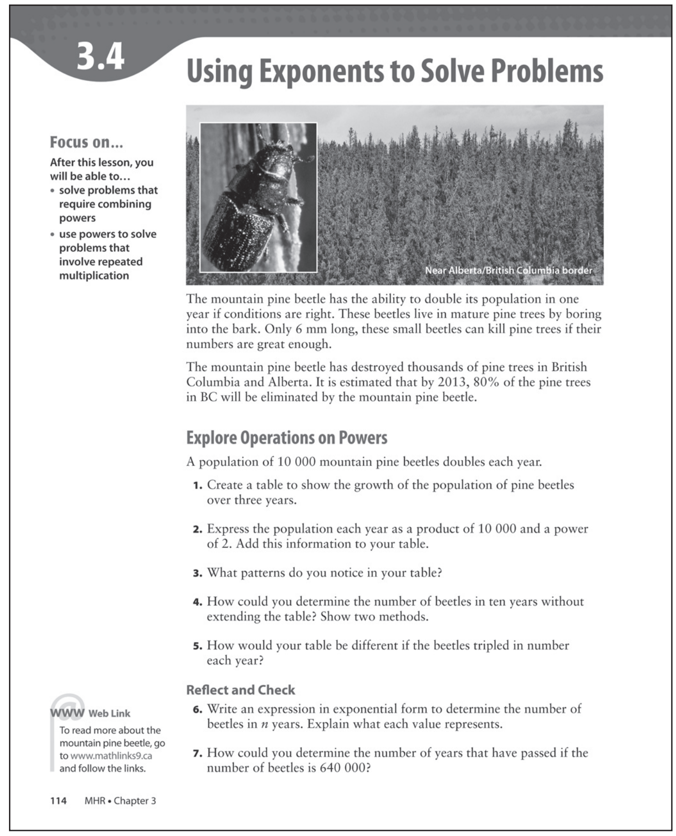
Specific Outcomes
N1 Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:
- representing repeated multiplication using powers
- using patterns to show that a power with an exponent of zero is equal to one
- solving problems involving powers.

N2 Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.
N4 Explain and apply the order of operations, including exponents, with and without technology.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 7, 9, Math Link
Typical	#1–3, 5, 7, 9, 11, Math Link
Extension/Enrichment	#1, 2, 6, 9, 11

Planning Notes

Discuss the information about pine beetles with students and mention how the mountain pine beetle continues to destroy thousands of pine trees in Western Canada. Some questions that may be explored during the discussion are:
- Why is the pine beetle suddenly such a concern?
- Why are a large number of dead trees a problem?
- Where is the beetle infestation found?


3.4 Using Exponents to Solve Problems

Focus on...
After this lesson, you will be able to...
- solve problems that require combining powers
- use powers to solve problems that involve repeated multiplication

Near Alberta/British Columbia border

The mountain pine beetle has the ability to double its population in one year if conditions are right. These beetles live in mature pine trees by boring into the bark. Only 6 mm long, these small beetles can kill pine trees if their numbers are great enough.

The mountain pine beetle has destroyed thousands of pine trees in British Columbia and Alberta. It is estimated that by 2013, 80% of the pine trees in BC will be eliminated by the mountain pine beetle.

Explore Operations on Powers

A population of 10 000 mountain pine beetles doubles each year.

1. Create a table to show the growth of the population of pine beetles over three years.
2. Express the population each year as a product of 10 000 and a power of 2. Add this information to your table.
3. What patterns do you notice in your table?
4. How could you determine the number of beetles in ten years without extending the table? Show two methods.
5. How would your table be different if the beetles tripled in number each year?

Reflect and Check

6. Write an expression in exponential form to determine the number of beetles in n years. Explain what each value represents.
7. How could you determine the number of years that have passed if the number of beetles is 640 000?

Web Link
To read more about the mountain pine beetle, go to www.mathlinks9.ca and follow the links.

114 MHR • Chapter 3

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce material learned in previous sections.

Explore Operations on Powers

In this Explore, students create and simplify an exponential expression in context. Students discover that some growth is exponential and that a population doubling or tripling corresponds to the base of the exponential term.

Method 1 Have students work in pairs to construct the table of data in #1 and 2. After students complete all five questions, discuss the last three questions as a class.

Method 2 Have a student construct the table on the overhead or whiteboard at the front of the class. As a class, complete the table on the overhead and determine the answers for #3, 4, and 5. Students should complete #6 and 7, in pairs or individually.

As students work on the Explore, you may wish to ask questions such as the following:

- How might you organize your data?
- What headings might you use?
- What strategies can you use to deal with the large numbers associated with the beetle population?

Meeting Student Needs

- You may wish to have students research how the pine beetle affects a particular community, perhaps their own community. For example, students living in the North may seek answers to the following questions:
  – Why is the pine beetle moving north?
  – Will it pose a problem for the Arctic?
- You may wish students to research other examples of exponential growth in order to help them understand this type of growth. For example, they might research exponential growth in animal populations, such as fruit flies.
- If you introduced the beaded medallion in section 3.1, you may wish to revisit it. If the medallion were doubled in size, ask students how they would write the number of each colour exponentially.
- It might be better for students to work through the Explore as a whole-class activity.

ELL

- Teach the following terms in context: *double its population*, *conditions*, *mature*, *boring into the bark*, *eliminated*, and *tripled*.

Common Errors

- Some students may find it a challenge to deal with the large numbers in the Explore.

R$_x$ Encourage students to use mental math to help verify that their answers are reasonable.

Answers

Explore Operations on Powers

1., 2.

Year	Population	Exponential Form
Start	10 000	$10\ 000(2^0)$
1	20 000	$10\ 000(2^1)$
2	40 000	$10\ 000(2^2)$
3	80 000	$10\ 000(2^3)$

3. Example: The population doubles each year. When written in exponential form, the exponent for the base of 2 is the same as the number of years.

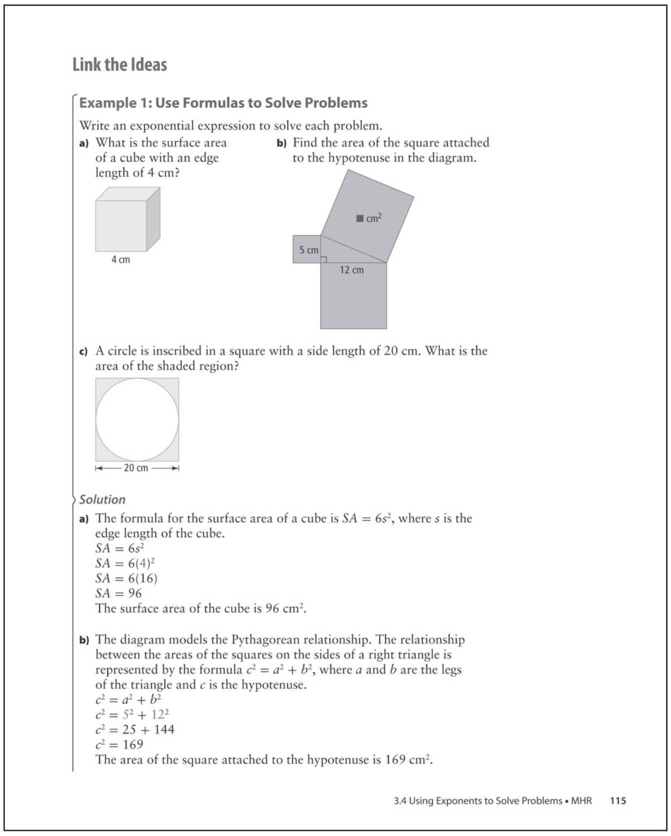
4. Example: Multiply 10 000 by 2^{10} or take the population in year 2 and multiply it by 2^8.

5. Example: The population would be 3 times the previous year. For year n, the population would be $10\ 000(3^n)$.

6. $10\ 000(2^n)$. 10 000 represents the starting population, 2 represents the doubling each year, and n represents the number of years.

7. Example: Divide by 10 000. The quotient is 64. Then, find the power of 2 that equals 64: $2^6 = 64$. The number of years is 6.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. When students determine the base and exponent of the power, check that they use sound reasoning.	• It may be best to complete the Reflect and Check as a class. Some students may have a difficult time with a variable as an exponent. Develop several concrete examples and values that pattern the beetle growth before introducing the variable exponent. • Provide several values that represent n years for students to try on their calculators to ensure that they understand the meaning of n. • As a class, develop a formula for when the population would triple each year. This will provide some scaffolding to assist students in questions that come up in the Practise and Apply. • If students are completing the Reflect and Check individually, for #7, encourage students to share their method with a classmate so that students have an opportunity to see more than one method for solving this problem.



Link the Ideas

Example 1: Use Formulas to Solve Problems

Write an exponential expression to solve each problem.

a) What is the surface area of a cube with an edge length of 4 cm?

4 cm

b) Find the area of the square attached to the hypotenuse in the diagram.

■ cm^2 5 cm 12 cm

c) A circle is inscribed in a square with a side length of 20 cm. What is the area of the shaded region?

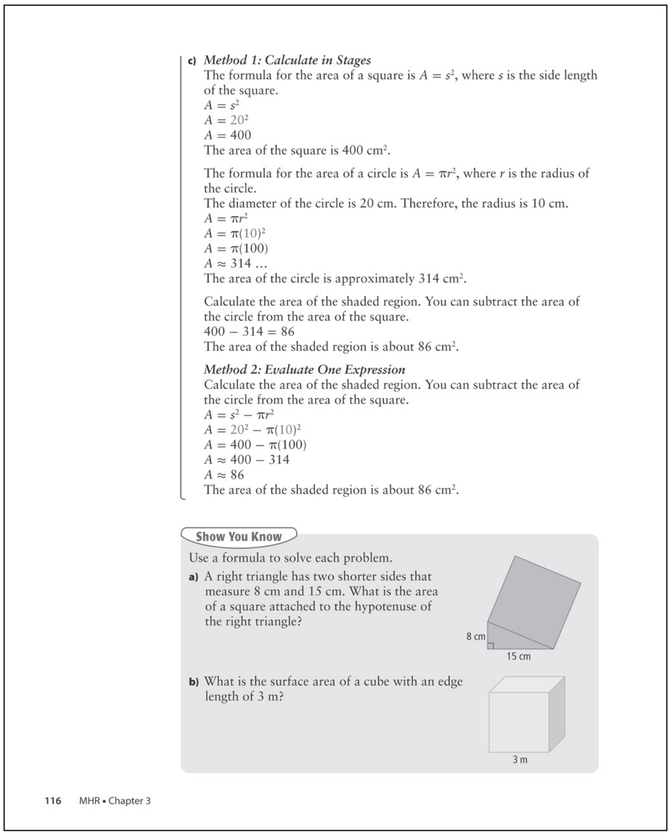
20 cm

Solution

a) The formula for the surface area of a cube is $SA = 6s^2$, where s is the edge length of the cube.
$SA = 6s^2$
$SA = 6(4)^2$
$SA = 6(16)$
$SA = 96$
The surface area of the cube is 96 cm^2.

b) The diagram models the Pythagorean relationship. The relationship between the areas of the squares on the sides of a right triangle is represented by the formula $c^2 = a^2 + b^2$, where a and b are the legs of the triangle and c is the hypotenuse.
$c^2 = a^2 + b^2$
$c^2 = 5^2 + 12^2$
$c^2 = 25 + 144$
$c^2 = 169$
The area of the square attached to the hypotenuse is 169 cm^2.

3.4 Using Exponents to Solve Problems • MHR 115



c) *Method 1: Calculate in Stages*
The formula for the area of a square is $A = s^2$, where s is the side length of the square.
$A = s^2$
$A = 20^2$
$A = 400$
The area of the square is 400 cm^2.

The formula for the area of a circle is $A = \pi r^2$, where r is the radius of the circle.
The diameter of the circle is 20 cm. Therefore, the radius is 10 cm.
$A = \pi r^2$
$A = \pi(10)^2$
$A = \pi(100)$
$A \approx 314 \ldots$
The area of the circle is approximately 314 cm^2.

Calculate the area of the shaded region. You can subtract the area of the circle from the area of the square.
$400 - 314 = 86$
The area of the shaded region is about 86 cm^2.

Method 2: Evaluate One Expression
Calculate the area of the shaded region. You can subtract the area of the circle from the area of the square.
$A = s^2 - \pi r^2$
$A = 20^2 - \pi(10)^2$
$A = 400 - \pi(100)$
$A \approx 400 - 314$
$A \approx 86$
The area of the shaded region is about 86 cm^2.

Show You Know

Use a formula to solve each problem.

a) A right triangle has two shorter sides that measure 8 cm and 15 cm. What is the area of a square attached to the hypotenuse of the right triangle?

8 cm 15 cm

b) What is the surface area of a cube with an edge length of 3 m?

3 m

116 MHR • Chapter 3

Link the Ideas

Example 1

In this example, students use formulas that contain exponents to simplify exponential expressions in context. Students must determine the appropriate formula, substitute the correct values, and then simplify using proper order of operations.

Coach students by asking:

- What is the formula for the surface area of a cube?
- What is the relationship between the areas of the squares attached to the sides of a right triangle?
- How does the diameter of the circle relate to the side length of the square?

Example 2

In this example, students develop an appropriate exponential expression to solve problems involving exponential growth of bacteria. Through patterning, students will see that repeated multiplication is required to determine the number of bacteria after a specific time. From this realization, students formulate an exponential expression that determines the number of bacteria after n hours.

As students complete the questions, check whether they are solving by extending the pattern. Check that they are organizing the data clearly, and that they are generalizing successfully in part c).

Key Ideas

The Key Ideas summarize the concept that formulas and patterns that involve repeated multiplication can be represented using exponents.

Brainstorm formulas that contain powers. Students can practise simplifying exponential expressions by evaluating various formulas.

Have students record the Key Ideas in their Foldable, using their own words and examples.

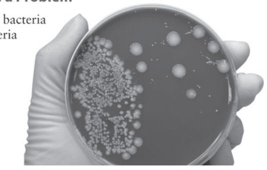
Meeting Student Needs

- Students may benefit from doing the examples as a whole-class activity. They might then complete the first part of the Show You Know as a small-group or paired activity and the second part as individual student work.

Example 2: Develop a Formula to Solve a Problem

A dish holds 100 bacteria. It is known that the bacteria double in number every hour. How many bacteria will be present after each number of hours?

a) 1 **b)** 5 **c)** n



Solution

a) After 1 h, the bacteria population doubles.
$100 \times 2 = 200$
After 1 h, there will be 200 bacteria.

b) In a period of 5 h, the bacteria population doubles five times.
$100 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 100(2^5)$
$= 100(32)$
$= 3200$
After 5 h, there will be 3200 bacteria.

c) After n hours, the bacteria population doubles n times.
Number of bacteria $= 100(2^n)$
After n hours, there will be $100(2^n)$ bacteria.

Strategies
Use a Variable

Show You Know

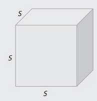
A type of bacterium is known to triple every hour. There are 50 bacteria to start with. How many will there be after each number of hours?

a) 3 **b)** 5 **c)** t

Key Ideas

- Powers are found in many formulas. When repeated multiplication is present in a formula, it is represented as a power. The use of powers keeps the formula as short as possible.
- Many patterns that involve repeated multiplication can be modelled with expressions that contain powers.

Volume of a cube $= s^3$



3.4 Using Exponents to Solve Problems • MHR 117

Common Errors

- Some students may struggle with recalling the correct formula for Example 1.

R_x Assist students in recalling formulas for the surface area of cubes, the Pythagorean relationship, and the area of circles.

- In Example 2, some students may not know how to generalize to n hours in part c).

R_x Have students continue the pattern for several more hours. Encourage students to record the number of bacteria for the first 10 h in a table before generalizing.

Answers

Example 1: Show You Know

a) 289 cm^2

b) 54 m^2

Example 2: Show You Know

a) 1350

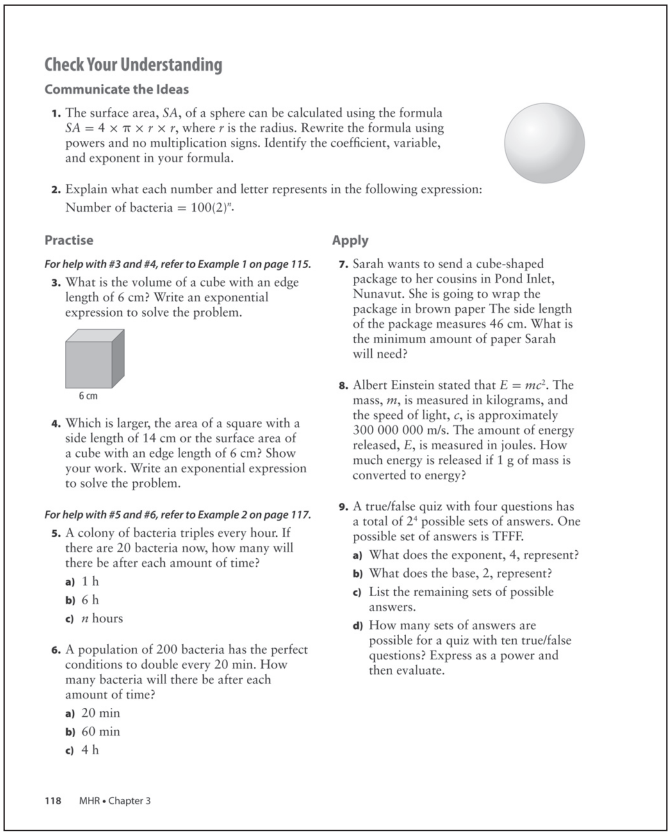
b) 12 150

c) $50(3^t)$

ELL

- Teach the following terms in context: *hypotenuse*, *inscribed*, *shaded*, *Pythagorean*, *formula*, *right triangle*, *surface area*, and *cube*.
- Although students may not know the term *Pythagorean relationship*, they may have learned the concept. Knowing whether or not any challenges students face are language-based will help you to assess how to assist them with their learning.
- For Example 2, clarify that the question is asking how much bacteria there will be after each time period.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Check if students are finding the length of the hypotenuse in part a) and then squaring the value, or if they stop after they apply the Pythagorean relationship and find the square of the hypotenuse. • Encourage them to refer to part b) of the solution for Example 1.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to use repeated multiplication and extend the pattern before writing the expression as an exponential expression. • Suggest that students refer back to the class work associated with the Explore, where the tripling of populations was investigated.



Check Your Understanding

Communicate the Ideas

1. The surface area, SA, of a sphere can be calculated using the formula $SA = 4 \times \pi \times r \times r$, where r is the radius. Rewrite the formula using powers and no multiplication signs. Identify the coefficient, variable, and exponent in your formula.

2. Explain what each number and letter represents in the following expression: Number of bacteria $= 100(2)^n$.

Practise

For help with #3 and #4, refer to Example 1 on page 115.

3. What is the volume of a cube with an edge length of 6 cm? Write an exponential expression to solve the problem.

6 cm

4. Which is larger, the area of a square with a side length of 14 cm or the surface area of a cube with an edge length of 6 cm? Show your work. Write an exponential expression to solve the problem.

For help with #5 and #6, refer to Example 2 on page 117.

5. A colony of bacteria triples every hour. If there are 20 bacteria now, how many will there be after each amount of time?
   a) 1 h
   b) 6 h
   c) n hours

6. A population of 200 bacteria has the perfect conditions to double every 20 min. How many bacteria will there be after each amount of time?
   a) 20 min
   b) 60 min
   c) 4 h

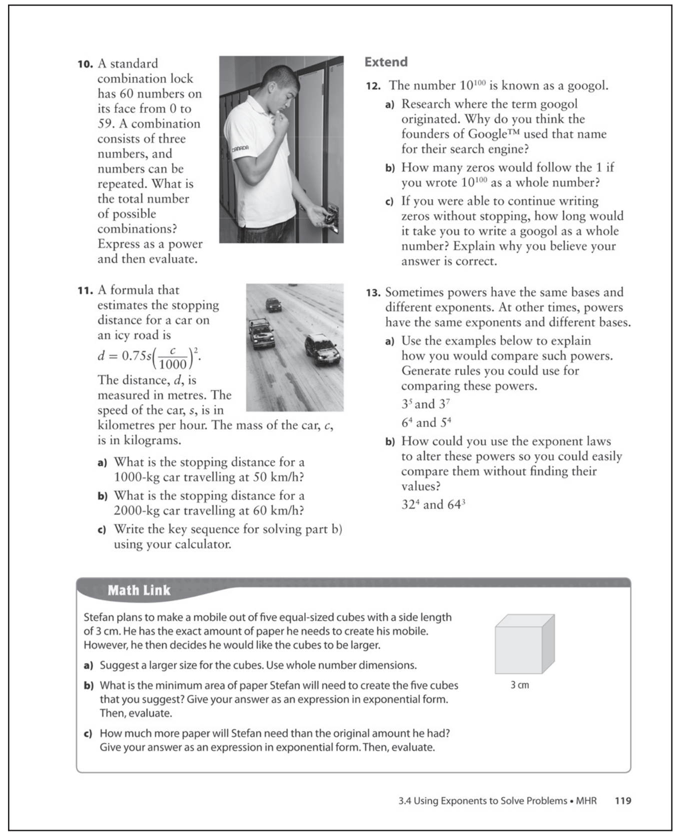
Apply

7. Sarah wants to send a cube-shaped package to her cousins in Pond Inlet, Nunavut. She is going to wrap the package in brown paper The side length of the package measures 46 cm. What is the minimum amount of paper Sarah will need?

8. Albert Einstein stated that $E = mc^2$. The mass, m, is measured in kilograms, and the speed of light, c, is approximately 300 000 000 m/s. The amount of energy released, E, is measured in joules. How much energy is released if 1 g of mass is converted to energy?

9. A true/false quiz with four questions has a total of 2^4 possible sets of answers. One possible set of answers is TFFF.
   a) What does the exponent, 4, represent?
   b) What does the base, 2, represent?
   c) List the remaining sets of possible answers.
   d) How many sets of answers are possible for a quiz with ten true/false questions? Express as a power and then evaluate.

118 MHR • Chapter 3



10. A standard combination lock has 60 numbers on its face from 0 to 59. A combination consists of three numbers, and numbers can be repeated. What is the total number of possible combinations? Express as a power and then evaluate.

11. A formula that estimates the stopping distance for a car on an icy road is $d = 0.75s\left(\frac{c}{1000}\right)^2$. The distance, d, is measured in metres. The speed of the car, s, is in kilometres per hour. The mass of the car, c, is in kilograms.
   a) What is the stopping distance for a 1000-kg car travelling at 50 km/h?
   b) What is the stopping distance for a 2000-kg car travelling at 60 km/h?
   c) Write the key sequence for solving part b) using your calculator.

Extend

12. The number 10^{100} is known as a googol.
   a) Research where the term googol originated. Why do you think the founders of Google™ used that name for their search engine?
   b) How many zeros would follow the 1 if you wrote 10^{100} as a whole number?
   c) If you were able to continue writing zeros without stopping, how long would it take you to write a googol as a whole number? Explain why you believe your answer is correct.

13. Sometimes powers have the same bases and different exponents. At other times, powers have the same exponents and different bases.
   a) Use the examples below to explain how you would compare such powers. Generate rules you could use for comparing these powers.
   3^5 and 3^7
   6^4 and 5^4
   b) How could you use the exponent laws to alter these powers so you could easily compare them without finding their values?
   32^4 and 64^3

Math Link

Stefan plans to make a mobile out of five equal-sized cubes with a side length of 3 cm. He has the exact amount of paper he needs to create his mobile. However, he then decides he would like the cubes to be larger.

a) Suggest a larger size for the cubes. Use whole number dimensions.
b) What is the minimum area of paper Stefan will need to create the five cubes that you suggest? Give your answer as an expression in exponential form. Then, evaluate.
c) How much more paper will Stefan need than the original amount he had? Give your answer as an expression in exponential form. Then, evaluate.

3 cm

3.4 Using Exponents to Solve Problems • MHR 119

Check Your Understanding

Communicate the Ideas

In #1, students take a formula in expanded form and write it in exponential form. Remind students that π is a numerical coefficient. Coach students by asking:

- What is the coefficient of the power?
- What is the base of the power?

Question #2 provides a good review of the Explore and Example 2. Students should realize that the numerical coefficient is the starting number and the base of the power indicates that the bacteria are doubling in number every n units of time. Coach students by asking:

- What does $n = 0$ represent?
- What does the expression simplify to when $n = 0$?
- Why can you not multiply 100 and 2 before raising the base to the exponent n?

Practise

Encourage students to construct exponential expressions before solving #3 and 4. For #5 and 6, some students will need to create a table and find the answers through repeated multiplication.

Apply

Some of these questions are fairly text dense. Help students extract and apply the math from the contexts.

For #7, students may benefit from sketching the net in order to help understand the question.

For #8, you may wish to have students research Albert Einstein and his accomplishments.

For #9, students may be reminded that in grades 7 and 8 they determined the possible outcomes for probability experiments using the multiplication method.

For #10, students may be surprised at the size of the answer. Some students may be confused about which number is the base and which number is the exponent.

For #11c), students' calculators will vary and the sequencing may be different. This is a good question to solve with technology due to the size of the numbers. However, students should be encouraged to estimate the answer before computing.

Extend

In #12, students explore the concept of the googol. This question gives students an opportunity to see the benefits of using exponents. Students are required to research the term and to estimate how long it would take to write a googol as a whole number.

In #13, students have an opportunity to expand their understanding of powers by creating rules for comparing powers with the same bases and different exponents, and powers with the same exponents and different bases. They are also required to see the relationship between 32^4 and 64^3 and determine how to write them as exponents so that they can be more easily compared.

Literacy Link After section 3.4, work with students to start the lower right leg of the spider map, entitled Solving Problems. Brainstorm and discuss as a class the information needed to begin this leg.

Math Link

This Math Link allows students to create and simplify exponential expressions. The dimensions for the cubes may be kept for the Wrap It Up! at the end of the chapter.

Meeting Student Needs

- Provide **BLM 3–10 Section 3.4 Extra Practice** to students who would benefit from more practice.

ELL

- English language learners may find the language in #1 and 2 challenging. Allow them to work with a partner, or do not have them answer these questions.
- Teach the following terms in context: *colony of bacteria*, *perfect conditions*, *combination lock*, and *stopping distance*.
- Read #7 with students, as you draw a picture. Label and point out the side that measures 46 cm.
- Consider reducing the number of word problems given to English language learners. You may wish to have them skip #8 as the language may pose a challenge.

Gifted and Enrichment

- For #8, have students research how the formula for stopping distance might change for vehicles other than cars.

Common Errors

- Some students may have difficulty visualizing the problem in the Math Link.

R_x Encourage students to draw net diagrams for each cube.

Answers

Communicate the Ideas

1. $SA = 4\pi r^2$. The coefficient is 4π, the variable is r, and the exponent is 2.

2. The coefficient, 100, represents the number of bacteria to start with. The base, 2, represents that the number of bacteria doubles every time period. The exponent, n, represents the number of times the bacteria doubles.

Math Link

a) Example: 4 cm

b) $5(6)(4^2) = 480$ cm^2

c) $5(6)(4^2) - 5(6)(3^2) = 480 - 270 = 210$ cm^2

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from evaluating the expression in #2 for the first four or five whole numbers in order to answer the question. • You may wish to ask students leading questions to break down #2 into more manageable units such as: – What does the coefficient of 100 represent? – What does 2^n represent? – When the two parts are multiplied together, what does the answer represent?
Assessment *for* Learning	
Practise Have students do #3, 5, 7, and 9. Students who have no problems with these questions can go on to the Apply questions.	• Encourage students to create exponential expressions prior to solving #3. • Students should have a good understanding from Communicate the Ideas #2 how to answer #5. If not, refer them back to #2 and Example 2. You may wish to ask these leading questions to help start students: – What does the coefficient 20 represent? – What does "triples every hour" look like as an expression? – When the two parts are multiplied together, what does the answer represent? • Some students may benefit from having a metre stick to view when answering #7. • Students struggling with #9 may wish to draw a tree diagram first or sketch a mock outline of four true/false questions to visualize the possibilities.
Math Link The Math Link on page 119 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 123.	• For part c) students might verify their solution by cutting the required nets for the cubes from paper. • Some students may benefit from having a metre stick to view when answering part c). You might consider having them include a reasonable amount of overlap. • Students who need help getting started could use **BLM 3–11 Section 3.4 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Literacy Link At the end of section 3.4, have students work in pairs to complete the lower right leg of the spider map, entitled Solving Problems.	• Have students list all the rules and terms they learned that are associated with solving problems involving powers. • The terms that students record might include *formulas*.
Math Learning Log Have students respond to the following prompts: • A population of flies doubles in number every day. How many flies will there be in four days? ten days? Create a formula that will determine the number of flies in *d* days. Explain what each number represents. • The part I find most confusing about exponential questions is … The part that is the easiest is …	• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. • Either of the questions would serve as formative assessment questions for students' Foldables.