Scale Diagrams

Suggested Timing		Scale Diagrams
80–100 minutes	Focus on	
Materials	After this lesson, you will be able to • identify scale	
ruler	diagrams and interpret the scale	
N= - -!* M=-+	factor • determine the scale factor for scale	
Blackline Masters Master 2 Communication Peer Evaluation	diagrams • determine if a given	
3LM 4–3 Chapter 4 Warm-Up	diagram is proportional to the original shape	Car manufacturers create scale drawings that show what a new car will look like.
BLM 4–7 Section 4.2 Extra Practice	scale	An actual car measures 339.2 cm in length and 163.2 cm in height. It is drawn to a scale of 1:32. Is the drawing an accurate representation of
LM 4–8 Section 4.2 Math Link	 a comparison between the actual size of an 	the actual model? What different strategies can you develop to find out?
	object and the size of its diagram • can be expressed as a	Explore the Accuracy of a Diagram
lathematical Processes	ratio, as a fraction, as a percent, in words, or in a diagram	 What measurements would help you compare the diagram of the can to the actual car? Take the measurements.
Communication (C)	 the scale 1:32 means that 1 cm on the 	2. Compare the measurements. What conclusions can you make?
Connections (CN)	diagram represents 32 cm on the actual car	Reflect and Check
Mental Math and Estimation (ME)	Materials • ruler	 a) How did you set up your calculations to determine if the diagram accurately represents the actual car?
Problem Solving (PS)		b) What information did you need to determine whether the diagrar is an accurate representation of the actual car?
Reasoning (R)		4. a) Choose an object and draw one view of it. Estimate the scale
Technology (T)		between your drawing and the actual object. b) Use the method you developed to determine how accurately the
Visualization (V)		drawing represents the actual object. 5. Compare vour method with the one used by a classmate. How are t
		5. Compare your method with the one used by a classmate. Flow are the methods similar? How are they different? Which method seems more efficient? Explain.
pecific Outcomes		L
S4 Draw and interpret scale diagrams of 2-D shapes.		4.2 Scale Diagrams • MHR

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 4 <i>or</i> 5, 6 <i>or</i> 7, 8 <i>or</i> 9, one of 10–12, Math Link
Typical	#1–3, 4 <i>or</i> 5, 6 <i>or</i> 7, 8 <i>or</i> 9, one of 10–12, three of 13–18, 19, Math Link
Extension/Enrichment	#1–3, one of 10–12, 19–22, Math Link

Planning Notes

Have students complete the warm-up questions on BLM 4–3 Chapter 4 Warm-Up to reinforce material learned in previous sections.

In this section, students build on what they learned in the previous section by identifying scale diagrams, determining the scale factor for scale diagrams, and determining if a given diagram is proportional to the original shape.

As a class, read about and discuss the car model. Discuss the definition for *scale* in the margin. Have students clarify the difference between scale (a comparison between the actual size of an object and the size of its

diagram) and scale factor (a constant amount by which all dimensions are multiplied). Invite students to discuss the scale of any models they are familiar with.

4.2 Scale Diagrams • MHR 139

You might have students brainstorm strategies to determine whether the drawing accurately represents the actual model.

Explore the Accuracy of a Diagram

In the Explore, students determine the relationship between the measurements of the diagram of the car and the actual car.

Method 1 Have students work individually to complete #1 and 2, and then discuss their results with a partner or a small group. Note that some discrepancies may be a result of inaccurate measurements. You might circulate as students work and use the following prompts:

- How do you know your measurements are correct?
- How can you verify your measurements?
- · How could someone have different measurements than you do?

You may wish to discuss the importance of taking accurate measurements prior to beginning the Explore.

Have students complete #3 and 4 on their own, and then discuss #5 with a classmate, before discussing #3 to 5 as a class. Summarize the methods and the similarities and differences on the board, and then have the class decide which method seems more efficient.

Method 2 Have students work in pairs or small groups to answer #1 and 2. Then, have students discuss the results as a class.

Meeting Student Needs

• Consider working through the Explore as a whole class.

ELL

• English language learners may not be familiar with the terms *car manufacturers, representation,* and *efficient*. Teach any unfamiliar terms in context.

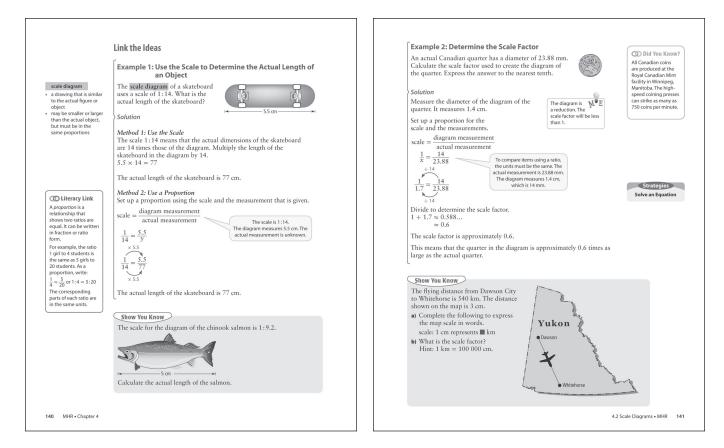
• Read through the questions with students, and rephrase questions as needed to clarify understanding.

Answers

Explore the Accuracy of a Diagram

- **1.** Example: length and height; Length = 10.6 cm, height = 5.1 cm
- **2.** Example: The ratio for each set of measurements is the same. 1 cm on the diagram represents 32 cm on the actual car.
- **3.** a) Example: I set up a ratio for length: $\frac{\text{diagram measurement}}{\text{actual measurement}}$, and then divided both the numerator and denominator by the length of the diagram. I did the same calculation for the height of the car. Then, I compared both ratios. Since both ratios are the same, I concluded that the diagram accurately represents the actual car.
 - b) Measurements of the length and the height of the car in the drawing

Assessment	Supporting Learning	
Assessment <i>as</i> Learning		
Reflect and Check Listen as students discuss what they discovered during the Explore. For #4b), check that students have developed a method that works.	 Some students may benefit from using the class discussion as a springboard to develop their own response. For #4, allow students to work with a partner. 	



Link the Ideas

Example 1

Example 1 illustrates two methods of using the scale on a scale diagram to determine the actual length of an object, namely, using the scale and using a proportion.

As a class, discuss the definition for *scale diagram* before walking through the example as a class. When drawing a scale diagram, emphasize the importance of including information about the scale used in the diagram.

Students may likely be familiar with Method 1 from the Explore. Use the following prompts:

- What does each value in the statement $5.5 \times 14 = 77$ represent?
- Explain why using the scale works to find the actual length.

Method 2 uses a proportion. Ask:

- What does each value in the proportion $\frac{1}{14} = \frac{5.5}{x}$ represent?
- What different ways can you determine the missing value? (Example: Multiply 5.5×14 .)
- Explain why the proportion is set up in this way.
- Can the proportion be set up any other way? If so, does it provide a correct answer?

• Is there other information you can find using a proportion like this? (Example: Determine the scale if the diagram length and the actual length are known, *or* determine the diagram length if the scale and the actual length are known.)

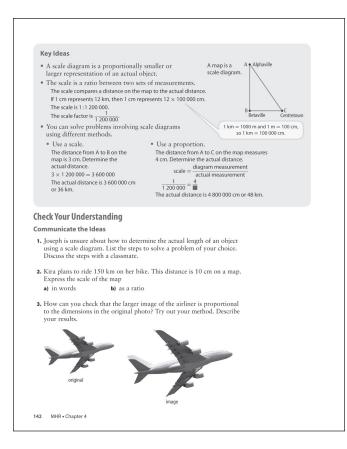
Have students use a method of their choice to complete the Show You Know individually, and then compare their answer to that of a classmate who used a different method. You might have students verify their answer using a different method.

Literacy Link Use the Literacy Link on page 140 of the student resource to clarify the term *proportion*. Emphasize the importance of using the same units for the corresponding parts of each ratio.

Example 2

Example 2 illustrates using a proportion to determine the scale factor.

As a class, read the problem. Direct students to the Did You Know?, which explains that Canadian coins are produced at the Royal Canadian Mint in Winnipeg, Manitoba. You might ask students how they could use what they learned in Example 1 to



solve the problem. (Set up a proportion for the scale and the measurements.)

Walk through the solution as a class. Ask:

- Do you expect the scale factor to be less than 1 or greater than 1? Why?
- What does each value in the proportion $\frac{1}{y} = \frac{14}{23.88}$ represent?
- How else might you have converted units so that they are the same?
- What steps do you need to follow to calculate the scale factor?
- Generally, how do you calculate the scale factor? (Measure a pair of corresponding sides in a diagram and an actual object and then calculate the ratio of the diagram measurement to the actual measurement.)
- How could you check that your calculated scale factor is correct? (Choose a different pair of corresponding sides and determine their ratio. It should be the same.)

For the Show You Know, discuss that map scales normally use different units of measurement in their scale ratios. Students need to be careful to record the measurement units correctly. Have students complete the Show You Know individually, and then compare their answers with those of a classmate.

Key Ideas

The Key Ideas summarize scale diagrams and scales as well as how to solve problems involving scale diagrams using a scale or using a proportion. As a class, walk through the example that illustrates solving a problem in which the units are not the same.

Explain that it is possible to set up a proportion with different units. A proportion can involve ratio or rate. For instance, have students consider a proportion such as $\frac{1 \text{ cm}}{500 \text{ m}} = \frac{x \text{ cm}}{3000 \text{ m}}$. Point out that the rates have the same combination of units. You might model setting up the example in the Key Ideas $\frac{1 \text{ cm}}{12 \text{ km}} = \frac{4 \text{ cm}}{x \text{ km}}$, and then solve for *x*.

Have students use their Foldable to make their own summary of the Key Ideas and provide their own example of using a scale and a proportion to solve a problem involving a scale diagram. As part of their summary, have students include definitions and an example for *scale*, *scale diagram*, and *proportion*. Have them use an example to explain how a scale factor is different than a scale.

Meeting Student Needs

- Some students may benefit from reactivating their skills with ratios and proportions.
- Remind students that when determining the scale of a diagram, the scale always has a 1 either in the numerator or the denominator. For example, a scale given as 5 can be written as $\frac{5}{1}$. Prompt students to realize that each unit in the original is 5 units in the diagram.
- For Example 1, help students verbalize their understanding of the meaning of the scale factor for a reduction. Since the diagram of the quarter is a reduction, the scale factor will be less than 1. In solving the proportion, it is necessary to divide $(\frac{1}{1.7} = 0.6)$ to determine the scale factor used for the diagram.
- For Example 2, some students may benefit from practice with setting up proportions using a different scenario. Consider providing measurements for another coin, such as a loonie or a nickel.
- Have students research the actual dimensions of a structure such as a tipi. Have them use this information to determine the scale factor used for a diagram of a tipi that they find.

- Consider working through Examples 1 and 2 as a whole class and having students work in pairs to complete the Show You Knows.
- Some students may benefit from completing an additional Show You Know related to each example. For Example 1, have students determine the actual length of a snow machine from a diagram that shows its length as 5 cm. The scale for the diagram is 1:28. For Example 2, have students determine the scale factor for a diagram that shows the flying distance from Cambridge Bay to Taloyoak as 5 cm when the actual distance is 460 km.

ELL

• English language learners may not be familiar with terms such as *skateboard*, *Chinook salmon*, *Canadian quarter*, *diameter*, *flying distance*, and *map scale*. Teach any unfamiliar terms in context.

Common Errors

- Some students may confuse scale and scale factor.
- R_x Clarify that scale is a ratio between two sets of measurements of an actual object and a diagram of the object. The scale factor is the constant factor by which all dimensions of an object are enlarged or reduced in a scale diagram.

- Some students may forget to convert units when setting up a ratio.
- R_x Emphasize checking that the units are the same before setting up a ratio.
- In setting up a proportion, some students may confuse the order of the diagram measurement and the actual measurement.
- R_x Explain that the numerator of the scale corresponds to the diagram measurement and the denominator of the scale corresponds to the actual measurement.

Answers

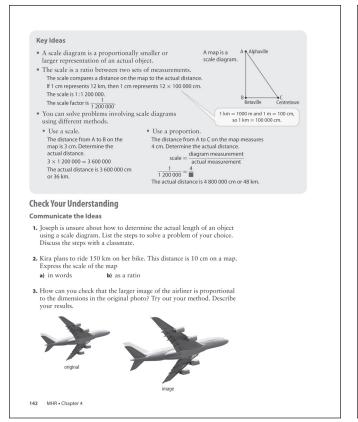
Example 1: Show You Know 46 cm

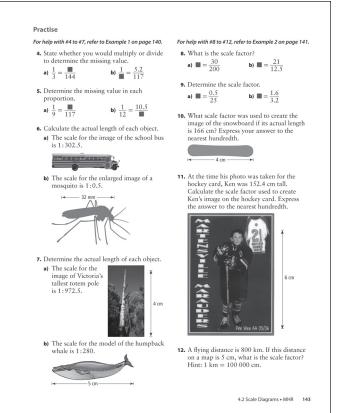
Example 2: Show You Know

a) 1 cm represents 180 km

b) 1 cm : 18 000 000 cm or 1 cm : 180 km

Assessment	Supporting Learning
Assessment for Learning	
Example 1 Have students do the Show You Know related to Example 1.	Encourage students to verbalize their thinking.You may wish to have students work with a partner and use both methods.
Example 2 Have students do the Show You Know related to Example 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Have students refer to Example 2 to help set up the proportion. Encourage students to check that their proportion is the same as that of a classmate.





Check Your Understanding Communicate the Ideas

These questions provide an opportunity for students to explain their understanding of solving problems involving scale, scale diagrams, and scale factors. Have students work individually to complete the questions.

For #1, students list steps for solving a problem they develop. Have students discuss with a classmate their problem and the steps for solving it. You might have students troubleshoot for errors in setting up a proportion by checking that the units are the same and that the order of the diagram measurement and the actual measurement in the proportion corresponds to the scale.

For #2 and 3, you might have students work in pairs to discuss their ideas before providing an individual response.

For #2, students express a map scale in words and as a ratio.

For #3, note that the plane is an Airbus 380. The actual wingspan is 79.8 m and the length is 73 m. Have students exchange their response with that of a classmate who used a different method. Have students try their partner's method and provide feedback.

Practise

Note that #6 and 7, #8 and 9, and #10 and 11 are sets of similar questions. Consider giving students a choice to do one question from each set initially to demonstrate their understanding.

For #12, you may need to prompt some students to convert to the same units.

Some students may not need to set up proportions for some of the questions. Encourage them to use a method of their choice, when applicable.

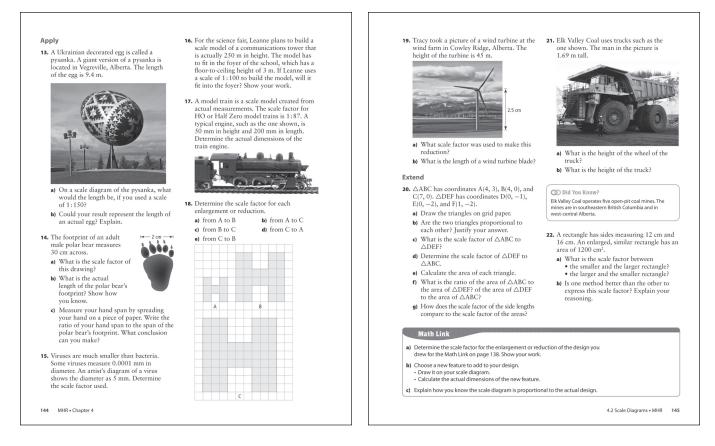
Apply

The Apply questions provide a range of contexts that involve scale, scale diagrams, and scale factors. You might assign questions based on student interest and/ or familiarity with the contexts.

For #17, explain that HO, or Half Zero, is a type of scale. It is pronounced "Aitch-Oh." It is the most popular scale of railway model worldwide. A typical HO engine is around 50 mm tall and 100 mm to 300 mm in length.

Extend

Encourage students to draw diagrams to help them answer the Extend questions.



Consider asking students to select one or two questions based on their interest.

For #21, direct students to the Did You Know? that follows the question.

Literacy Link Direct students to the leg on their spider map entitled Scale Diagrams and have them complete definitions for *scale*, *scale diagram*, and *proportion*, using words, diagrams, and mathematical expressions. Encourage them to include an example for each term.

Math Link

The Math Link allows students to apply their understanding of scale factors and proportional diagrams.

In this Math Link, students determine the scale factor for the design they drew for the Math Link on page 138. They also add a new feature to their design, calculate its dimensions, and explain how they know the scale diagram is proportional to the actual design. Note that students may need to conduct research to determine the actual dimensions of the new feature they choose.

Meeting Student Needs

- Some students may benefit from coaching to set up proportions for #6, 7, 10, and 11.
- Provide BLM 4–7 Section 4.2 Extra Practice to students who would benefit from more practice.

ELL

- Allow students to discuss #1 to 3 in their first language, and then express their thinking in English.
- Teach the following terms in context: *mosquito*, *totem pole*, *humpback whale*, *hockey card*, *Ukrainian decorated egg*, *pysanka*, *polar bear*, *footprint*, *hand span*, *viruses*, *bacteria*, *science fair*, *communications tower*, *foyer*, *floor-to-ceiling height*, *model train*, and *wind turbine*.

Gifted and Enrichment

- Have students search for a scale drawing. Have them research the actual size of the object and state the scale factor used to create the image.
- Have students solve the following problem: Rectangle ABCD measures 8 cm by 10 cm.
 Rectangle EFGH measures 4 cm by 6 cm. Are the two rectangles proportional to each other? Explain.
- Have students research the creation of scale models for movies. They might research the scaling up of small objects (e.g., insects) or the scaling down of large objects (e.g., ocean liners).



For a lesson plan designed for students to explore the relationship between scale factor and surface area and volume, go to www.mathlinks9.ca and follow the links. Students can measure the dimensions of a common object, multiply each dimension by a scale factor, and examine a model using the multiplied dimensions. Then, they compare the surface area and volume of the original object and the enlarged one.

For lessons designed for students to explore ratio, proportion, scale factor, and similarity using perimeter, area, volume, and surface area of various rectangular shapes, go to www.mathlinks9.ca and follow the links. An overview is provided for different lessons and links to student applets.

Answers

Communicate the Ideas

- 1. Example: The scale diagram of a fish uses a scale of 1:150. The image of the fish is 5 cm long. What is the actual length of the fish? To solve, multiply the length of the image by the scale factor to determine the length of the actual object. $(5 \times 150 = 750$. The actual fish is 7.5 m in length.)
- **2.** a) 1 cm represents 1 500 000 cm or 15 km.
 b) 1:1 500 000 or 1 cm:15 km
- **3.** Example: Measure the length of the original and the larger image. Determine the scale factor. Then, measure the height of both images and determine the scale factor. For each set of measurements, the scale factor is 1.5. Therefore, the dimensions of the larger image are proportional to the original photo.

Assessment	Supporting Learning		
Assessment <i>as</i> Learning			
Communicate the Ideas Have all students complete #1 to 3.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. For #1, encourage students to choose an object that they are familiar with. Remind them to include the scale and label the diagram. Alternatively, some students may benefit from being provided with a scale factor as a starting point for their explanation. For #2, remind students to convert to the same units. Some students may need a reminder about the difference in expressing a ratio and a proportion. For #3, encourage students to fully explain their method and how they determined the scale factor. Have them compare their answer with a classmate's. You may wish to have students use Master 2 Communication Peer Evaluation to assess each other's responses to #1 and 3. 		
Assessment for Learning			
Practise Have students do #4 <i>or</i> 5, 6 <i>or</i> 7, 8 <i>or</i> 9, and one of 10 to 12. Students who have no problems with these questions can go on to the Apply questions.	 Students who need help with #4, 5, 8, or 9 may need coaching to review whether to divide or multiply to find the missing value. They may benefit from revisiting Examples 1 and 2. Coach students through #4 and 8, and then have students complete #5 and 9. For #6, coach students through setting up the proportions, and then have students try #7. Students should have successfully completed either #6 or 7 before doing #10 to 12 as these later questions require the ability to express and calculate scale factors. For #10, coach students through setting up and solving the proportion. Then, have students try #11 on their own to check for understanding. 		
Math Link The Math Link on page 145 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 163.	 It is recommended that all students complete the Math Link. It is important for students to continue to use the sheet on which they started their design. Students who need help getting started could use BLM 4–8 Section 4.2 Math Link, which provides scaffolding. 		
Assessment as Learning	Assessment <i>as</i> Learning		
Literacy Link By the end of section 4.2, have students complete definitions for <i>scale</i> , <i>scale</i> <i>diagram</i> , and <i>proportion</i> .	 Some students may benefit from using their Foldable to record definitions for terms as they are introduced. Then, at the end of the section or the chapter, have them summarize the definitions in their own words and organize them on the spider map. Show students who struggle with the concept of scale an actual set of objects, in which one is double the size of the other. Have students verbalize what <i>double the size</i> means. 		
 Math Learning Log Have students respond to the following questions: How are the terms <i>scale factor</i>, <i>scale</i>, and <i>scale diagram</i> the same? How are they different? What information would help you determine the scale on a diagram? 	 Have students use a chart to help organize the similarities and differences. The completed chart may be a useful formative assessment piece to collect and review. Then, return it to students to keep in their Foldable. Encourage students to refer to the worked examples and notes in their Foldable and spider map. Depending on students' learning styles, have them provide oral or written answers. Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. 		