Similar Triangles

<i>MathLinks 9</i> , pages 146–153	43	
Suggested Timing 80–100 minutes	Focus on	Similar Triangles
Materials • tracing paper • ruler • protractor • grid paper	After this lesson, will be able to • determine sim triangles • determine if diagrams are proportional • solve problem the properties similar triangle	you S S S Working Success Together of S Students and Staff
Blackline Masters Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper BLM 4–3 Chapter 4 Warm-Up BLM 4–9 Section 4.3 Extra Practice BLM 4–10 Section 4.3 Math Link	Materials • tracing paper • ruler • protractor corresponding corresponding • have the same r position in geom figures A	Bonnie and Justin created these logos for the Student Council. Their advisor tells them that the triangles are similar. How can she tell? What do you know about similar figures? What strategies can you develop to determine if triangles are similar? Explore How to Identify Similar Triangles 1. Trace each logo on separate pieces of tracing paper. 2. a) Measure the angles in each logo. What do you notice about the sorresponding angles? b) Measure the side lengths in each logo. What do you notice about the
Mathematical Processes Communication (C) Connections (CN) Mental Math and Estimation (ME) Problem Solving (PS) Reasoning (R) Technology (T) Visualization (V)	B Corresponding a 2 dand 2 d Can ad 2 f Corresponding d A and DE B dand DE A Cand EF A Cand DF	ratios of the corresponding sides of the triangles? Reflect and Check a. a) What conclusions can you make about the corresponding angles of the two triangles? b) What conclusions can you make about the corresponding sides of the two triangles? c) What conditions do you think are necessary in order for two triangles to be similar? b) Test the conditions on a different set of two triangles. Are the triangles similar? Discuss with a classmate why you think the triangles are, or are not, similar.

Specific Outcomes

SS4 Draw and interpret scale diagrams of 2-D shapes.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2, 4 <i>or</i> 5, 6 <i>or</i> 7, 8, 9 <i>or</i> 10, 12, Math Link
Typical	#1, 2, 4 <i>or</i> 5, 6 <i>or</i> 7, 8, 9 <i>or</i> 10, two of 12–15, Math Link
Extension/Enrichment	#1, 2, 15, 16, 18, three of 19–23, Math Link

Planning Notes

Have students complete the warm-up questions on BLM 4–3 Chapter 4 Warm-Up to reinforce material learned in previous sections.

In this section, students explore similar triangles and solve problems involving similar triangles.

As a class, review the opening text and visual. Have students discuss the opening questions about similar triangles and determine students' prior knowledge. You might have students discuss the questions in

groups, and then have the groups report to the class in order for students to challenge each other and think through their ideas.

Explore How to Identify Similar Triangles

In this Explore, students investigate corresponding angles and corresponding sides of two triangles in order to determine if they are similar.

Method 1 Before directing students to the definitions for corresponding angles and corresponding sides, consider having them trace the logos on separate pieces of tracing paper and identify the matching (corresponding) angles and matching (corresponding) sides in the triangles. Have students work individually and use a ruler and protractor to explore the two triangles. Once students have had an opportunity to explore, have them discuss their findings with the class. Record the findings on the board. At this time, introduce the vocabulary corresponding angles and corresponding sides. Have students use these terms as they collect more information about the two triangles.

Consider having students discuss their findings in small groups, before asking a spokesperson from each group to report to the class.

For #3 and 4, have students reflect on the class discussion and their own exploration before writing an individual response. Have students discuss #4b) with a classmate before discussing the findings in a class discussion. As they work, circulate and prompt students to use math vocabulary when describing the properties of the two triangles.

Method 2 Have students work in pairs to explore the properties of the two triangles, and then discuss the findings as a class.

Meeting Student Needs

- Some students may find it helpful if you provide one of the logos copied on an overhead transparency. They can then overlay the other logo on it and check the measures of angles.
- In #4b), you may wish to provide sets of triangles (one set that are similar and one set that are not) for students to use.
- Consider working through the Explore as a whole class.

ELL

- You may need to explain the terms *logo, similar* and *relative position* in context.
- For #4, explain that students are to state what needs to be true in order for two triangles to be similar.

Common Errors

- Some students may not correctly identify the corresponding angles and corresponding sides.
- $\mathbf{R}_{\mathbf{x}}$ Point out that the logos are positioned so that the corresponding angles and sides are in the same location in each triangle. If the triangles were not positioned in this way, tell students to match the corresponding sides by looking across from the angles that are equal in each triangle.

Answers

Explore How to Identify Similar Triangles

- **2.** a) Example: The corresponding angles are equal in measure.b) Example: The ratios for each set of corresponding sides are equal.
- **3.** a) Example: The corresponding angles are equal in size.b) Example: The corresponding sides are related by the same scale factor.
- **4.** a) Similar triangles have corresponding angles that are equal in measure and corresponding sides that are proportional in length.

Assessment	Supporting Learning	
Assessment <i>as</i> Learning		
Reflect and Check Listen as students discuss what they discovered during the Explore.	 Some students may benefit from using the class discussion as a springboard to revise their own response. Some students may need support with the concept of similarity. Coach students who are having difficulty with #4 by asking: If you were to measure the angles of similar triangles, what would you find? How does each set of corresponding angles compare? If you were to measure the side lengths of similar triangles, what would you find? What are the lengths of the proportional sides? Is each set of corresponding sides proportional? How do you know? If you set up a ratio of the side lengths comparing the smallest to the largest triangle, what would you expect to observe about the ratios? Encourage students to verbalize their responses to #4 and to draw and label diagrams. 	



Link the Ideas

Example 1

This example illustrates properties of similar triangles.

Read the introduction as a class. Ask students to reflect on what they discovered in the Explore about the properties of similar triangles. Prompt students, if necessary, to state that similar triangles have corresponding angles that are equal in measure and corresponding sides that are proportional in length. Once they know what they need to find, have students measure and compare

the corresponding angles and corresponding sides of the two triangles. Ask:

- What are the corresponding angles? How does each set of corresponding angle measures compare?
- What are the lengths of the corresponding sides? Is each set of corresponding sides proportional? How do you know? What is the scale factor?

As a class, compare findings and have students conclude whether the triangles are similar. Then, walk through the solution together. **Literacy Link** Direct students to the Literacy Link on page 147, which explains the symbol ~. Have students note how to name similar triangles in the order of their corresponding angles. Provide an example to reinforce the point: If $\triangle ABC$ is similar to $\triangle DEF$, then, without a diagram, it is known that $\angle A$ corresponds to $\angle D$, $\angle B$ corresponds to $\angle E$, and $\angle C$ corresponds to $\angle F$.

Direct students to the second Literacy Link on page 147 that explains how to name angles.

Have students work individually on the Show You Know. Expect students to use a protractor and a ruler to take measurements. You may wish to have students choose one question, and then discuss their solution with a classmate who did a different question. As a class, discuss the solutions to both questions.

Example 2

This example illustrates determining if two triangles are similar and using similar triangles to determine a missing side length.

Read the introduction as a class. Then, have students work in pairs to determine the solution.



As they work, circulate and consider using the following prompts to help students with part a):

- How might you check that \triangle KLM is similar to \triangle TUV?
- Which angles can you compare?
- How do the measures of the corresponding angles compare?
- Which sides can you compare?
- Are the corresponding sides proportional? How do you know?
- How much larger is △KLM? What is the scale factor?

For part b), ask:

- How can you use the scale factor to determine the missing side?
- How can you determine the scale factor?
- How can you use a proportion to determine the missing side?
- How else might you determine the missing length?

Have student pairs or groups compare their solutions with those of another pair or group, and then discuss their solutions with the class. Note the methods used to solve part b). If students have used only one method, you might demonstrate an alternative method. As a class, walk through the solution in the student resource. For part a), explain that proving one condition for similarity is sufficient. Reinforce that if one condition is met, the other condition is also met. For part b), walk through both methods.

Have students work individually on the Show You Know and then compare their solutions with those of a classmate who used a different method.

Key Ideas

The Key Ideas highlight the properties of similar triangles and solving problems involving similar triangles by using a scale factor or using a proportion. As a class, have students review the example provided. Ask how students know that \triangle PQR is not similar to \triangle ABC or \triangle DEF.

Have students use their Foldable to make their own summary of the properties of similar triangles and to provide an example of a set of triangles that are similar and a set of triangles that are not similar. As part of their summary, have students include a definition for *similar*. Encourage students to provide their own example of solving for the missing side length of a triangle.

Meeting Student Needs

- Some students may benefit from recalling equivalent fractions to solve the unknown in a proportion.
- Some students may need coaching to understand the connection between the corresponding proportional sides and the scale factor. For Example 1, prompt them to realize that the scale factor is the constant amount that each side of △ABC is divided by to produce the corresponding side of △EFG.
- Consider working through Examples 1 and 2 as a whole class and having students work in pairs to complete the Show You Knows.
- Some students may benefit from completing an additional Show You Know related to each example.

ELL

• English language learners may not be familiar with *math puzzles*. Teach this term in context.

Gifted and Enrichment

• Invite students to explore the Web Link related to properties of similar triangles described on page 148 in the student resource.

Common Errors

- Students may compare both corresponding angles and corresponding sides when determining similar triangles.
- $\mathbf{R}_{\mathbf{x}}$ Remind students that it is necessary to compare only one set of measurements.

- Some students may compare angles or sides that are not the corresponding angles or sides.
- $\mathbf{R}_{\mathbf{x}}$ If a set of triangles is not positioned so that the corresponding angles and sides are in the same location, encourage students to redraw the triangles so that they are in the same position. Alternatively, have students match the corresponding sides by looking across from the angles that are equal in each triangle.

Answers

Example 1: Show You Know

- **a)** Example: Yes, the triangles are similar. The corresponding angles are equal in measure and the corresponding sides are proportional with a scale factor of 1.5.
- **b)** Example: No, the triangles are not similar. The three pairs of corresponding sides are not proportional in length.

Example 2: Show You Know

a) x = 2.5 **b)** x = 9.9

Assessment	Supporting Learning	
Assessment <i>for</i> Learning		
Example 1 Have students do the Show You Know related to Example 1.	 Encourage students to verbalize their thinking about how to determine similarity. You may wish to have students work with a partner. Encourage students to refer to the Explore for a method. Some students may benefit from tracing the triangles to compare angles and sides or to use tick marks or dashes to identify the corresponding angles and sides. This may help in setting up the ratios. 	
Example 2 Have students do the Show You Know related to Example 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Some students may find it helpful to redraw the triangles as two separate triangles. Help students recall the different methods for solving the problem. Refer students to Example 2 for help with setting up the proportion. Alternatively, provide coaching to help set up the proportion. Encourage students to check that their proportion is the same as that of a classmate. 	





Check Your Understanding

Communicate the Ideas

These questions provide an opportunity for students to explain their understanding of the properties of similar triangles. Have students work individually to complete the questions.

For #1 and 3, students explain the properties of similar triangles. Encourage students to draw diagrams to illustrate their responses.

For #2, students identify the error made in stating that two given triangles are similar. Consider having students compare their response with a classmate's.

Practise

These questions give students practice with identifying corresponding angles and corresponding sides, determining similar triangles, and determining the missing side length of a triangle using similar triangles. Encourage students to use the method of their choice to solve problems.

Note that #4 and 5, #6 and 7, and #9 and 10 are sets of similar questions. Consider giving students a choice to do one question from each set initially to demonstrate their understanding.

For #8, students may find it helpful to draw the triangles.

Apply

These questions provide a range of contexts for students to solve problems involving similar triangles. You might assign questions based on student interest and/or familiarity with the contexts. Encourage students to use a method of their choice to solve the problems.

For #12 to 15, encourage students to redraw the diagram as two separate triangles to help solve the problems.

Extend

Consider allowing students to select problems based on their interest. Encourage them to use a method of their choice to solve the problems.

For #19, 20, and 21, encourage students to draw diagrams to help them solve the problems.

Literacy Link Direct students to the leg on their spider map entitled Similar Triangles and have them complete definitions for *corresponding angles, corresponding sides,* and *similar* using words, diagrams, and mathematical expressions. They should include an example of similar triangles.



Math Link

The Math Link provides an opportunity for students to apply their understanding of similar triangles.

In this Math Link, students design a signature logo (that includes a triangle) to fit on their design project. You may need to clarify that the signature logo, which combines a personal signature and a logo, should provide a strong and recognizable graphic image of the design project. They will need to use a protractor and a ruler to take measurements. Encourage students to be creative but remind them to keep the logo design fairly simple as they will need to represent their logo on a scale diagram as well. Make **Master 8 Centimetre Grid Paper** and **Master 9 0.5 Centimetre Grid Paper** available for students to create their scale diagram. Emphasize that the logo must be sized to fit on their design project.

Meeting Student Needs

- Some students may benefit from coaching to set up proportions.
- For #8 and 16, some students may benefit from coaching to help create diagrams.

• Provide **BLM 4–9 Section 4.3 Extra Practice** to students who would benefit from more practice.

ELL

- Allow students to discuss #2 in their first language, and then express their thinking in English.
- Teach the following terms in context: *ramp*, *loading dock*, *vertical support*, *base*, *extension ladder*, *vertical wall*, *casts*, *shadow*, *slide*, *extra support beam*, *tower*, *mirror*, *mirror reflection*, *maximum altitude*, *model rocket*, *storeys*, and *launch pad*.
- Encourage students to draw diagrams. For #21, work with students to draw a diagram.

Gifted and Enrichment

- Challenge students to solve the following problem:
- How might a laser beam, a telescope, and a 100-m rope marked off in 1-m segments be used to determine the distance across a canyon without crossing over to the other side of the canyon? Use a diagram involving similar triangles to show your thinking. (The laser beam could be used to determine and mark reference points on the other side of the canyon. The wide end of the similar triangle would be on the other side and its length could be determined by measuring its parallel equivalent on the same side as the observer.)

• Have students explore different conditions for similarity of triangles (SSS, ASA, SAS, AAS, Hyp-S). They may find the related Web Link on this TR page helpful. Have them present their findings in a format of their choice.

Web Link

You may wish to have students use technology for their Math Link design project. To access open-source freeware paint programs, go to www.mathlinks9.ca and follow the links.

To explore similar and congruent triangles, go to www.mathlinks9.ca and follow the links.

Answers

Communicate the Ideas

- **1.** Example: The corresponding angles are equal in measure and the corresponding sides are proportional in length.
- **2.** Example: No, the corresponding sides are not proportional in length.
- **3.** Example: Two triangles with equal sides and equal angles are similar. Since the corresponding sides are equal, the ratio of each pair of corresponding sides will be 1:1.

Assessment	Supporting Learning			
Assessment as Learning				
Communicate the Ideas Have all students complete #1 and 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. For #1, students may benefit from drawing a diagram to help explain their answer. For #2, encourage students to discuss their response before recording it. 			
Assessment <i>for</i> Learning				
Practise and Apply Have students do #4 <i>or</i> 5, 6 <i>or</i> 7, 8, 9 <i>or</i> 10, and 12. Students who have no problems with these questions can go on to the remaining Apply questions.	 Students who need assistance with #4 to 7 may benefit from additional coaching with Example 1. Have them verbalize the meaning of <i>corresponding</i>. Coach them through the solution for #6, and then have students try #7 on their own. For #4, students may find it helpful to use tracing paper to trace the triangles, and compare the corresponding angles and sides. For #5, have them trace the triangles so that the corresponding angles and sides are in the same position on both triangles. For #8, have students draw diagrams to help visualize corresponding angles and sides. Students who need assistance with #9 or 10 may benefit from additional coaching with Example 2. They may find it helpful to redraw the triangles so that the orientations are the same. For #9, coach students through the solution, and then have students try #10 on their own. For #12, encourage students to draw and label the diagrams with those of a classmate. 			
Math Link The Math Link on page 153 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 163.	 It is recommended that all students complete the Math Link. Reinforce that a signature logo includes a name. You might provide students with a triangle superimposed on grid paper to help them draw the scale diagram for the project. Alternatively, you may wish students to use the triangle provided on BLM 4–10 Section 4.3 Math Link so that students begin with the same size. Students who need help getting started could use BLM 4–10 Section 4.3 Math Link, which provides scaffolding. 			
Assessment <i>as</i> Learning				
Literacy Link By the end of section 4.3, have students complete definitions for <i>corresponding</i> <i>angles, corresponding sides,</i> and <i>similar</i> .	 Some students may benefit from using their Foldable to record definitions for terms as they are introduced. Then, at the end of the section or the chapter, have them summarize the definitions in their own words and organize them on the spider map. Some students may find it helpful to draw a diagram and then verbalize what it shows before writing a definition. 			
 Math Learning Log Have students respond to the following questions: How can you determine if two triangles are similar? Develop a word problem for a missing length in a pair of similar triangles. Then, solve it. Show your work. 	 Some students may benefit from verbalizing the solution to their problem first before recording it. Allow students to modify a problem from the student resource. 			