4.4 Similar Polygons

***MathLinks 9*, pages 154–159**

Suggested Timing
80–100 minutes

Materials
- tracing paper
- protractor
- ruler
- grid paper

Blackline Masters
Master 2 Communication Peer Evaluation
Master 7 Isometric Dot Paper
Master 8 Centimetre Grid Paper
Master 10 2 Centimetre Grid Paper
BLM 4–3 Chapter 4 Warm-Up
BLM 4–11 Section 4.4 Extra Practice
BLM 4–12 Section 4.4 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

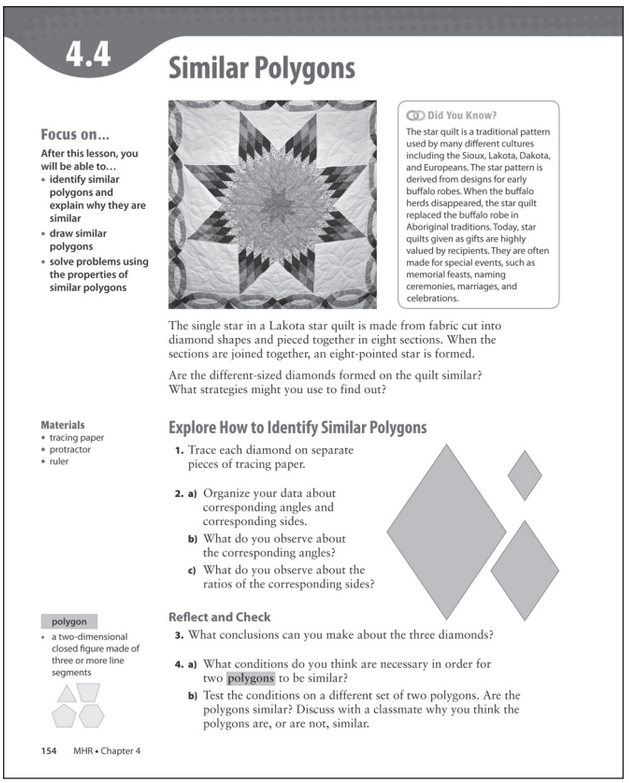
Specific Outcomes
SS3 Demonstrate an understanding of similarity of polygons.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2, 3a) *or* b), 5–7, Math Link
Typical	#1, 2, 3a) *or* b), 5–7, two of 8–12, Math Link
Extension/Enrichment	#1, 2, 7, 13–18, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce material learned in previous sections.

In this section, students explore similar polygons and solve problems involving similar polygons.



4.4 Similar Polygons

Focus on...
After this lesson, you will be able to...
- identify similar polygons and explain why they are similar
- draw similar polygons
- solve problems using the properties of similar polygons

Did You Know?
The star quilt is a traditional pattern used by many different cultures including the Sioux, Lakota, Dakota, and Europeans. The star pattern is derived from designs for early buffalo robes. When the buffalo herds disappeared, the star quilt replaced the buffalo robe in Aboriginal traditions. Today, star quilts given as gifts are highly valued by recipients. They are often made for special events, such as memorial feasts, naming ceremonies, marriages, and celebrations.

The single star in a Lakota star quilt is made from fabric cut into diamond shapes and pieced together in eight sections. When the sections are joined together, an eight-pointed star is formed.

Are the different-sized diamonds formed on the quilt similar? What strategies might you use to find out?

Materials
- tracing paper
- protractor
- ruler

Explore How to Identify Similar Polygons

1. Trace each diamond on separate pieces of tracing paper.
2. a) Organize your data about corresponding angles and corresponding sides.
   b) What do you observe about the corresponding angles?
   c) What do you observe about the ratios of the corresponding sides?

polygon
- a two-dimensional closed figure made of three or more line segments

Reflect and Check
3. What conclusions can you make about the three diamonds?
4. a) What conditions do you think are necessary in order for two polygons to be similar?
   b) Test the conditions on a different set of two polygons. Are the polygons similar? Discuss with a classmate why you think the polygons are, or are not, similar.

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As a class, review the opening text including the Did You Know? and the image of the star quilt. Have students discuss the opening questions about similar polygons and determine students' prior knowledge. As a class, discuss possible strategies for determining similar polygons.

Explore How to Identify Similar Polygons

In this Explore, students investigate corresponding angles and corresponding sides of polygons in order to determine if they are similar.

Method 1 Have students work in pairs to collect and organize data about the corresponding angles and sides of the three diamonds. Have them use a protractor and ruler to take measurements. As students work, circulate and prompt students to use math vocabulary when describing the properties of polygons. Consider having students discuss their findings with another pair of students before having a class discussion.

For #3 and 4 have students reflect on the class discussion and their own exploration before writing an individual response. Have students discuss #4b) with a classmate before discussing the findings as a class. You might ask:

- How are the conditions for similar polygons the same as the conditions for similar triangles? How are they different?
- Do you think the conditions for similar diamonds apply to all polygons? Explain.

Method 2 Have students work in small groups to explore the properties of polygons, and then discuss the findings as a class.

Meeting Student Needs

- Help students recall polygons by discussing the definition provided.
- Some students may benefit from seeing an enlarged copy of one diamond in the Explore drawn on the board and a smaller copy placed on an overhead. Lining up the two diamonds so they fit one over the other will help clarify the meaning of similarity. The images can then be enlarged or reduced.
- In #4b), you may wish to provide sets of polygons (one set that are similar and one set that are not) for students to use.
- Consider working through the Explore as a whole class.

ELL

- You may need to explain how the single star in a Lakota star quilt is made.
- Teach the following terms in context: *star, quilt, derived, buffalo robes*, *memorial feasts*, *naming ceremonies*, *marriages, diamond, sections,* and *closed figure*.

Gifted and Enrichment

- Have students research the symbolism of the different elements in a Lakota star quilt. Alternatively, invite someone who makes star quilts to display a quilt and talk about the star pattern.

Answers

Explore How to Identify Similar Polygons

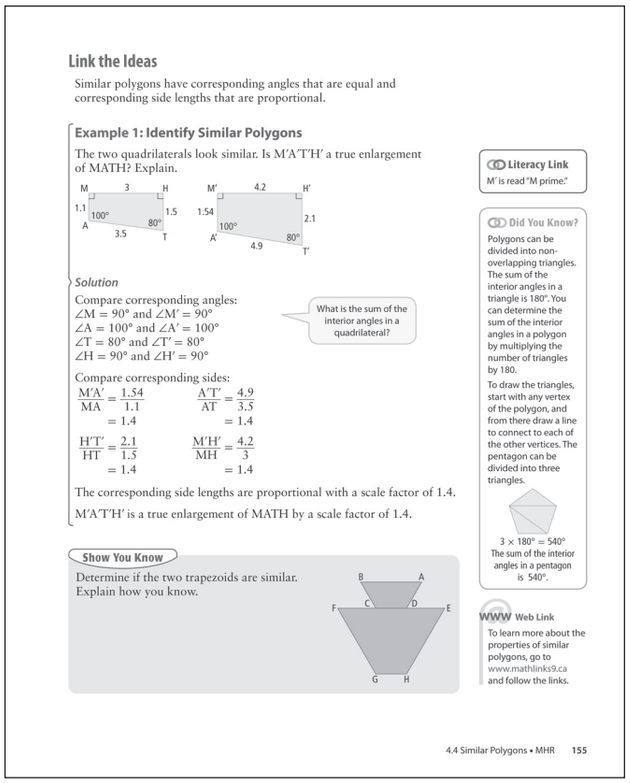
2. b) Example: All of the corresponding angles are equal in measure.

c) Example: The ratios for each set of corresponding sides are equal.

3. The three diamonds are similar.

4. a) Example: The two polygons must have corresponding angles equal in measure and corresponding sides that have the same scale factor.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss the conclusions and the conditions necessary for polygons to be similar. Check that students correctly interpret the term *similar* as it applies to polygons.	• Some students may benefit from using the class discussion as a springboard to develop their own response. • It is important that students understand that two conditions are necessary for polygons to be similar (angles equal in measure and side lengths that are proportional). Contrast that with triangles for which only one condition is necessary to prove similarity. • Some students may need prompting to compare the polygons. You may wish to have another set of polygons available for students to compare. • Encourage students to use the math vocabulary related to similar polygons.



Link the Ideas

Similar polygons have corresponding angles that are equal and corresponding side lengths that are proportional.

Example 1: Identify Similar Polygons

The two quadrilaterals look similar. Is M′A′T′H′ a true enlargement of MATH? Explain.

Solution

Compare corresponding angles:
∠M = 90° and ∠M′ = 90°
∠A = 100° and ∠A′ = 100°
∠T = 80° and ∠T′ = 80°
∠H = 90° and ∠H′ = 90°

Compare corresponding sides:

$\frac{M'A'}{MA} = \frac{1.54}{1.1} = 1.4$ $\quad \frac{A'T'}{AT} = \frac{4.9}{3.5} = 1.4$

$\frac{H'T'}{HT} = \frac{2.1}{1.5} = 1.4$ $\quad \frac{M'H'}{MH} = \frac{4.2}{3} = 1.4$

The corresponding side lengths are proportional with a scale factor of 1.4.

M′A′T′H′ is a true enlargement of MATH by a scale factor of 1.4.

Show You Know

Determine if the two trapezoids are similar. Explain how you know.

Literacy Link
M′ is read "M prime."

Did You Know?
Polygons can be divided into non-overlapping triangles. The sum of the interior angles in a triangle is 180°. You can determine the sum of the interior angles in a polygon by multiplying the number of triangles by 180.
To draw the triangles, start with any vertex of the polygon, and from there draw a line to connect to each of the other vertices. The pentagon can be divided into three triangles.
3 × 180° = 540°
The sum of the interior angles in a pentagon is 540°.

Web Link
To learn more about the properties of similar polygons, go to www.mathlinks9.ca and follow the links.

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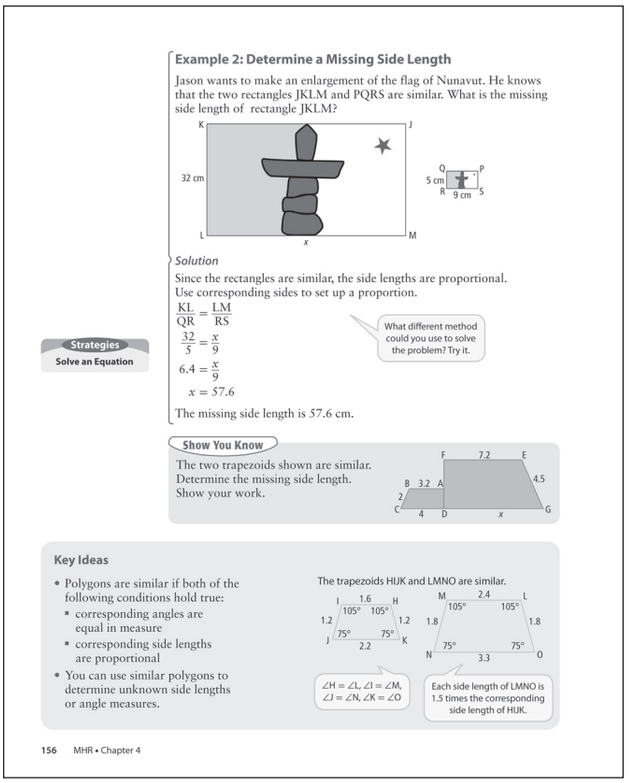

Example 2: Determine a Missing Side Length

Jason wants to make an enlargement of the flag of Nunavut. He knows that the two rectangles JKLM and PQRS are similar. What is the missing side length of rectangle JKLM?

Solution

Since the rectangles are similar, the side lengths are proportional. Use corresponding sides to set up a proportion.

$\frac{KL}{QR} = \frac{LM}{RS}$

$\frac{32}{5} = \frac{x}{9}$

$6.4 = \frac{x}{9}$

$x = 57.6$

The missing side length is 57.6 cm.

Strategies
Solve an Equation

What different method could you use to solve the problem? Try it.

Show You Know

The two trapezoids shown are similar. Determine the missing side length. Show your work.

Key Ideas

- Polygons are similar if both of the following conditions hold true:
  - corresponding angles are equal in measure
  - corresponding side lengths are proportional
- You can use similar polygons to determine unknown side lengths or angle measures.

The trapezoids HIJK and LMNO are similar.

∠H = ∠L, ∠I = ∠M, ∠J = ∠N, ∠K = ∠O

Each side length of LMNO is 1.5 times the corresponding side length of HIJK.

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Link the Ideas

Example 1

This example illustrates properties of similar polygons.

Read the introduction as a class. Ask students to reflect on what they discovered about the properties of similar polygons in the Explore. Once they know what they need to do, have students measure and compare the corresponding angles and corresponding sides of the two polygons. Ask:

- What are the corresponding angles? How does each set of corresponding angle measures compare?
- What are the lengths of the corresponding sides? Is each set of corresponding sides proportional? How do you know? What is the scale factor?

As a class, compare findings before walking through the solution in the student resource.

Literacy Link Direct students to the Literacy Link on page 155 that explains prime notation. Explain that prime notation is used for any image of a transformation from an original figure.

Direct students to the Did You Know? about determining the sum of the interior angles in a polygon. Have students try out the method using the quadrilateral in Example 1. Ask how they could use information about the sum of interior angles in a polygon to help solve problems involving polygons.

Have students work individually on the Show You Know. Expect students to use a protractor and a ruler to take measurements. Have students discuss their solution with a classmate and then the class.

Example 2

This example illustrates using similar polygons to determine a missing side length.

Read the introduction and have students work in pairs to determine the solution. Remind students to use what they learned about determining a missing side length for similar triangles. As they work, circulate and consider using the following prompts:

- What methods did you use to solve for a missing side length for similar triangles?
- How can you use the scale factor to determine the missing side?
- How can you determine the scale factor?
- How can you use a proportion to determine the missing side?
- How can you use corresponding sides to set up the proportion?

Have student pairs compare their solution with that of another pair who used a different method, and then discuss their solutions with the class. As a class, walk through the solution in the student resource.

Have students work individually on the Show You Know and then compare their solution with that of a classmate who used a different method.

Key Ideas

The Key Ideas highlight the properties of similar polygons and solving problems involving similar polygons. As a class, review the example and ask how students know that the trapezoids are similar.

Have students use their Foldable to make their own summary of the properties of similar polygons and provide an example of a set of polygons that are similar and a set of polygons that are not similar. Have students include a definition for *polygon*. Encourage students to provide their own example of solving for the missing side length of a polygon.

Meeting Student Needs

- Have students produce a reference chart for polygons that includes the number of triangles in each polygon and the sum of the interior angles.
- Some students may find it helpful to draw a diagram and colour code the corresponding sides of polygons. This is particularly effective for polygons that are attached to each other. For polygons whose corresponding angles and sides are not in the same location, encourage students to redraw the polygons as two separate polygons and position them so that the angles and sides are in the same location.
- For Example 2, some students may need assistance to set up the proportion.
- Consider working through Examples 1 and 2 as a whole class and having students work in pairs to complete the Show You Knows.

ELL

- Teach the terms *non-overlapping* and *vertex* in context.
- Explain that a *trapezoid* is a quadrilateral with at least one pair of parallel sides.

Gifted and Enrichment

- Invite students to explore the Web Link related to properties of similar polygons described on page 155 in the student resource.

Answers

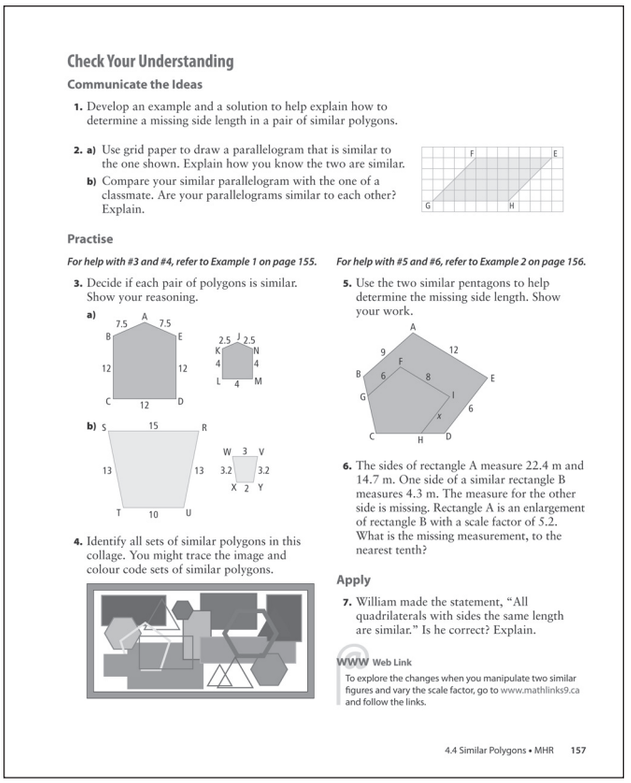
Example 1: Show You Know

Example: Yes, the trapezoids are similar. The corresponding angles have identical measures. Each pair of corresponding sides has the same scale factor.

Example 2: Show You Know

$x = 9$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from tracing the polygons to compare angles and sides. • Some students may benefit from verbalizing how to determine if the polygons are similar. • Check that students know that both conditions for similarity must be met. Ask: – What do you notice about the measures of the angles in both trapezoids? – Write ratios comparing the side lengths of the larger trapezoid to the side lengths of the smaller trapezoid. What do you notice about the numerical value of the ratios? – How do you know if these shapes are similar? • Some students may benefit from completing an additional problem similar to the Show You Know.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Help students recall the methods used to solve for missing side lengths of similar triangles. • Encourage students to check that their proportion is the same as that of a classmate. • Ensure students understand that the order in which they write their proportions must be consistent. For example, small to large = small to large. • Some students may benefit from completing an additional problem similar to the Show You Know.



Check Your Understanding

Communicate the Ideas

1. Develop an example and a solution to help explain how to determine a missing side length in a pair of similar polygons.

2. a) Use grid paper to draw a parallelogram that is similar to the one shown. Explain how you know the two are similar.
   b) Compare your similar parallelogram with the one of a classmate. Are your parallelograms similar to each other? Explain.

Practise

For help with #3 and #4, refer to Example 1 on page 155.

3. Decide if each pair of polygons is similar. Show your reasoning.

a)

b)

4. Identify all sets of similar polygons in this collage. You might trace the image and colour code sets of similar polygons.

For help with #5 and #6, refer to Example 2 on page 156.

5. Use the two similar pentagons to help determine the missing side length. Show your work.

6. The sides of rectangle A measure 22.4 m and 14.7 m. One side of a similar rectangle B measures 4.3 m. The measure for the other side is missing. Rectangle A is an enlargement of rectangle B with a scale factor of 5.2. What is the missing measurement, to the nearest tenth?

Apply

7. William made the statement, "All quadrilaterals with sides the same length are similar." Is he correct? Explain.

WWW Web Link

To explore the changes when you manipulate two similar figures and vary the scale factor, go to www.mathlinks9.ca and follow the links.

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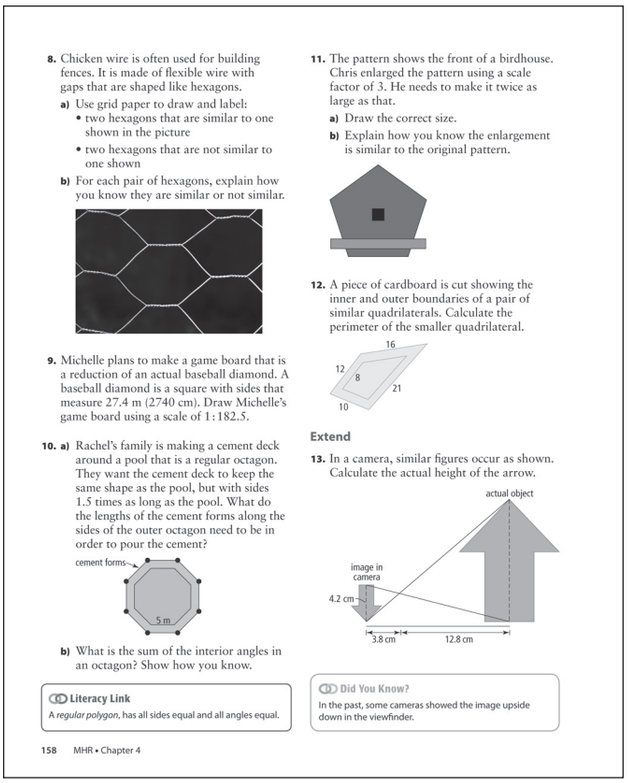

8. Chicken wire is often used for building fences. It is made of flexible wire with gaps that are shaped like hexagons.
   a) Use grid paper to draw and label:
   - two hexagons that are similar to one shown in the picture
   - two hexagons that are not similar to one shown

   b) For each pair of hexagons, explain how you know they are similar or not similar.

9. Michelle plans to make a game board that is a reduction of an actual baseball diamond. A baseball diamond is a square with sides that measure 27.4 m (2740 cm). Draw Michelle's game board using a scale of 1:182.5.

10. a) Rachel's family is making a cement deck around a pool that is a regular octagon. They want the cement deck to keep the same shape as the pool, but with sides 1.5 times as long as the pool. What do the lengths of the cement forms along the sides of the outer octagon need to be in order to pour the cement?

   b) What is the sum of the interior angles in an octagon? Show how you know.

Literacy Link

A *regular polygon*, has all sides equal and all angles equal.

11. The pattern shows the front of a birdhouse. Chris enlarged the pattern using a scale factor of 3. He needs to make it twice as large as that.
   a) Draw the correct size.
   b) Explain how you know the enlargement is similar to the original pattern.

12. A piece of cardboard is cut showing the inner and outer boundaries of a pair of similar quadrilaterals. Calculate the perimeter of the smaller quadrilateral.

Extend

13. In a camera, similar figures occur as shown. Calculate the actual height of the arrow.

Did You Know?

In the past, some cameras showed the image upside down in the viewfinder.

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Check Your Understanding

Communicate the Ideas

These questions allow students to explain their understanding of similar polygons.

For #1, students develop a problem and a solution involving similar polygons. Some students may benefit from verbalizing how to find a missing side length before they record their solution.

For #2, students draw a parallelogram similar to one shown and then compare their parallelogram with that of a classmate who used a different size of grid paper. Make 1-cm and 2-cm grid paper available to students. Students should observe a variety of parallelograms that are all similar.

Practise

Consider giving students a choice of doing one of #3 and 4 and one of #5 and 6 initially to demonstrate their understanding.

For #3, consider having students work with a partner. Have each partner complete either part a) or b), exchange their solution, and check each other's work and provide feedback.

For #6, encourage students to draw a diagram.

Apply

These questions provide a range of contexts for students to solve problems involving similar polygons. Give students some choice in the questions they answer based on their interest and/or familiarity with the contexts. Encourage students to use a method of their choice to solve the problems.

For #7, encourage students to draw a diagram.

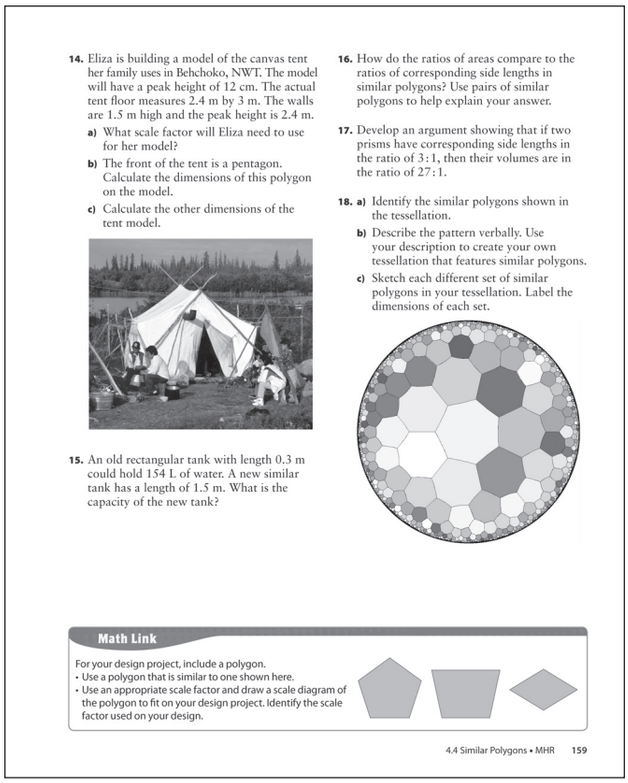
For #8, make **Master 8 Centimetre Grid Paper**, **Master 9 0.5 Centimetre Grid Paper**, and **Master 10 2 Centimetre Grid Paper** available to students.

For #9, consider showing students a picture of a baseball diamond.

Literacy Link For #10, direct students to the Literacy Link on page 158 that defines a regular polygon, such as an octagon.

Extend

Some of these questions involve area, volume, and tessellation. Consider allowing students to select one or two questions based on their interest. Encourage students to draw diagrams.



14. Eliza is building a model of the canvas tent her family uses in Behchoko, NWT. The model will have a peak height of 12 cm. The actual tent floor measures 2.4 m by 3 m. The walls are 1.5 m high and the peak height is 2.4 m.

a) What scale factor will Eliza need to use for her model?

b) The front of the tent is a pentagon. Calculate the dimensions of this polygon on the model.

c) Calculate the other dimensions of the tent model.

15. An old rectangular tank with length 0.3 m could hold 154 L of water. A new similar tank has a length of 1.5 m. What is the capacity of the new tank?

16. How do the ratios of areas compare to the ratios of corresponding side lengths in similar polygons? Use pairs of similar polygons to help explain your answer.

17. Develop an argument showing that if two prisms have corresponding side lengths in the ratio of 3 : 1, then their volumes are in the ratio of 27 : 1.

18. a) Identify the similar polygons shown in the tessellation.

b) Describe the pattern verbally. Use your description to create your own tessellation that features similar polygons.

c) Sketch each different set of similar polygons in your tessellation. Label the dimensions of each set.

Math Link

For your design project, include a polygon.
- Use a polygon that is similar to one shown here.
- Use an appropriate scale factor and draw a scale diagram of the polygon to fit on your design project. Identify the scale factor used on your design.

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For #13, direct students to the Did You Know? that explains why the image in the camera is upside down in comparison to the actual object.

Literacy Link Direct students to the leg in their spider map entitled Similar Polygons and have them complete a definition for *polygon*, using words, diagrams, and mathematical expressions. Have students include an example of a polygon and calculate the sum of its angles.

Math Link

The Math Link provides an opportunity for students to apply their understanding of similar polygons.

In this Math Link, students add a polygon to fit on their design project. They will need to use a protractor and a ruler to take measurements of one of the three polygons provided.

Make **Master 8 Centimetre Grid Paper**, **Master 9 0.5 Centimetre Grid Paper**, and **Master 10 2 Centimetre Grid Paper** available to students for their scale diagram.

Meeting Student Needs

- Some students may benefit from working with a partner to answer some of the questions.
- For #4, you may wish to substitute a collage that has fewer polygons in it.
- For #5, some students may find it helpful to trace the polygons and colour code the corresponding angles and sides, or draw them as separate polygons.
- For #8, some students may prefer to use isometric dot paper to draw the hexagons. You may wish to provide **Master 7 Isometric Dot Paper**.
- Provide **BLM 4–11 Section 4.4 Extra Practice** to students who would benefit from more practice.

ELL

- Allow English language learners to discuss #1 in their first language, and then express their thinking in English.
- For #2, explain that a *parallelogram* is a quadrilateral with two pairs of parallel lines.
- Teach the following terms in context: *chicken wire*, *hexagons, game board*, *baseball*, *baseball diamond*, *cement deck*, *birdhouse*, *boundaries*, *camera*, *viewfinder, canvas tent, tank*, *prisms*, and *tessellation*.
- Clarify the meaning of any questions that students are unclear about.

Gifted and Enrichment

- Invite students to draw several versions of similar polygons that fit into each other and create an interesting design. Ask them to create another design using dissimilar polygons that fit into each other. Challenge them to analyse the role of similarity in creating visual impressions. Ask which design they prefer and why.
- Have students explore fractals. They may find the Web Link on this TR page interesting.

Web Link

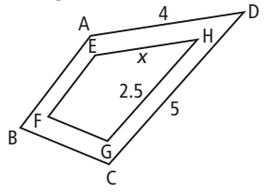
To explore properties of similar polygons, go to www.mathlinks9.ca and follow the links.

To explore fractals and self-similarity, go to www.mathlinks9.ca and follow the links.

Answers

Communicate the Ideas

1. Example:



The two quadrilaterals ABCD and EFGH are similar. What is the missing side length, x?
Solution: Since the quadrilaterals are similar, the side lengths are proportional.
$\frac{CD}{GH} = \frac{AD}{EH}$; $\frac{5}{2.5} = \frac{4}{x}$; $x = 2$.

2. Look for a similar parallelogram drawn on grid paper. Example:
For a parallelogram that is enlarged using a scale factor of 1.5:
$\angle E = \angle E' = 45°$; $\angle F = \angle F' = 45°$; $\angle G = \angle G' = 45°$; $\angle H = H' = 45°$.
$\frac{E'F'}{EF} = 1.5$; $\frac{F'G'}{FG} = 1.5$; $\frac{G'H'}{GH} = 1.5$; $\frac{H'E'}{HE} = 1.5$.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • For #1, allow students to verbalize their explanation before recording it. • For #2, let students know that there are a variety of similar parallelograms possible. Some students may find it easier to draw a similar parallelogram directly inside the original one. • You may wish to have students use **Master 2 Communication Peer Evaluation** to assess each other's responses to #1.
Assessment *for* Learning	
Practise and Apply Have students do #3a) *or* b), and 5 to 7. Students who have no problems with these questions can go on to the remaining Apply questions.	• Students who need assistance with #3a) may benefit from additional coaching with Example 1. They may benefit from verbalizing the corresponding angles and sides before beginning to solve. Coach students through the solution, and then have them try part b) on their own to check for understanding. • Students who need assistance with #5 and 6 may benefit from additional coaching with Example 2. Coach students through the solution for #5, and then have students try #6. Encourage them to draw a diagram for #6. • For #5, students may find it easier to redraw separate pentagons and label the sides before setting up the proportion. • For #6, help students visualize the problem by having them draw and label the rectangles first. • Consider using #7 as an assessment *as* learning piece. The responses could be collected and reviewed before having students keep them in their Foldable.
Math Link The Math Link on page 159 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 163.	• It is recommended that all students complete the Math Link. • You may decide to limit the choice of polygon for some students and provide some possible measurements. • Some students may wish to use a different polygon than what is shown. Tell them to get your approval before using it. • Students who need help getting started could use **BLM 4–12 Section 4.4 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Literacy Link By the end of section 4.4, have students complete a definition for *polygon*.	• Some students may benefit from using their Foldable to record definitions for terms as they are introduced. Then, at the end of the section or the chapter, have them summarize the definitions in their own words and organize them on the spider map. • Some students may find it helpful to draw a diagram and then verbalize what it shows before writing a definition.
Math Learning Log Have students respond to the following questions: • How can you determine if two polygons are similar? • How can you use similar polygons to determine an unknown angle measure? Use a diagram to help you explain.	• Allow students to reference visuals of similar polygons from the student resource to help with their explanations.