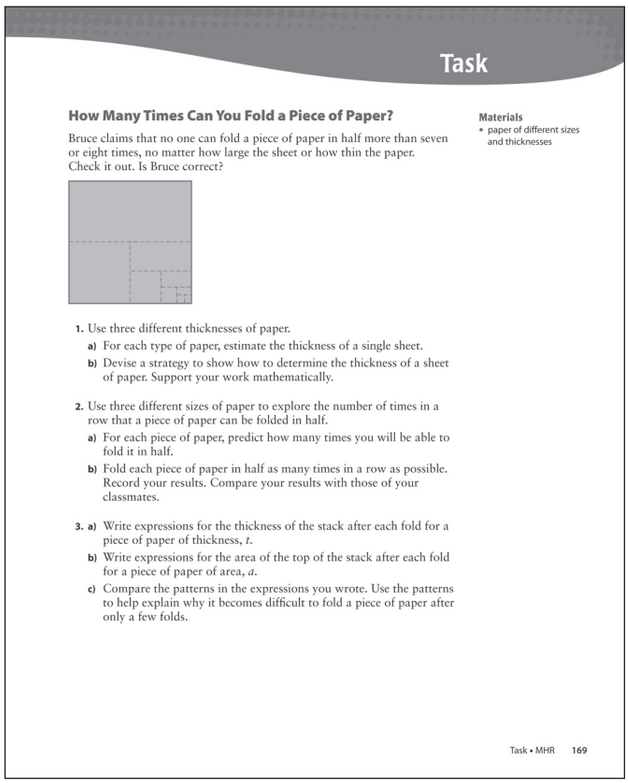
Task



Task

How Many Times Can You Fold a Piece of Paper?

Bruce claims that no one can fold a piece of paper in half more than seven or eight times, no matter how large the sheet or how thin the paper. Check it out. Is Bruce correct?

Materials
- paper of different sizes and thicknesses

1. Use three different thicknesses of paper.
   a) For each type of paper, estimate the thickness of a single sheet.
   b) Devise a strategy to show how to determine the thickness of a sheet of paper. Support your work mathematically.
2. Use three different sizes of paper to explore the number of times in a row that a piece of paper can be folded in half.
   a) For each piece of paper, predict how many times you will be able to fold it in half.
   b) Fold each piece of paper in half as many times in a row as possible. Record your results. Compare your results with those of your classmates.
3. a) Write expressions for the thickness of the stack after each fold for a piece of paper of thickness, t.
   b) Write expressions for the area of the top of the stack after each fold for a piece of paper of area, a.
   c) Compare the patterns in the expressions you wrote. Use the patterns to help explain why it becomes difficult to fold a piece of paper after only a few folds.

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***MathLinks 9*, page 169**

Suggested Timing
40–50 minutes

Materials
- paper of different sizes and thicknesses

Blackline Masters
Master 1 Project Rubric

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
SS3 Demonstrate an understanding of similarity of polygons.
N1 Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:
- solving problems involving powers.

N3 Demonstrate an understanding of rational numbers by:
- solving problems that involve arithmetic operations on rational numbers.

Planning Notes

You may wish to use the following steps to introduce and complete this task:

1. In advance, assign students to bring in examples of three different types of paper for a class challenge. Tell them that you are going to have a contest to see who can fold a piece of paper the most times. Appropriate types of paper might be photocopy, printer, tissue, rice, toilet, notebook, newsprint, origami, foil, and gift wrap.
2. Have students display the different types of paper they brought. Ask:
   - How do the characteristics of these types of paper differ?
   - How is each type suited to its use?
   - Which type of paper is the thinnest?
   - Which is the thickest?
   - How might you determine the thickness of a single piece?
   - Predict how many times you might be able to fold your paper samples in half.
3. Read through the task as a class and ensure that students understand what they are to do. As students work, circulate and consider using the following prompts:
   - How will you organize the data for your investigation?
   - How can you use your data to write an expression for the thickness of the folded paper after each fold?
   - How can you use your data to write an expression for the area of the top of the folded paper after each fold?
   - What patterns do you see in your set of expressions?
   - How might you use these patterns to help explain why it is difficult to continue folding the paper?
4. Clarify that the task is to
   - devise a strategy and show how to determine the thickness of a piece of paper
   - predict how many times each piece of paper can be folded in half
   - fold pieces of paper in half as many times as possible without mechanical means

- measure and record the number of folds and the area of the top of the stack after each fold
- write expressions for the thickness and the area of the stack after each fold

5. Review the **Master 1 Project Rubric** with students so that they will know what is expected.

Meeting Student Needs

- Some students may need coaching to calculate the thickness of paper.
  - Use papers that are neatly stacked (e.g., photocopy paper comes in 500 sheet packages).
  - Divide the height of the stack by the number of sheets to find the thickness of one sheet.
  - Repeat the procedure with two, then three, and finally four stacks. Recalculate the thickness of a single sheet.
  - Calculate the average thickness from the results of four stacks to get a more accurate measurement.
- Some students may need coaching to calculate the area of the top of a stack.
  - Start with one sheet of notebook paper. Record the actual dimensions and area before any folds.
  - Fold the sheet once and record the new dimensions and area.
  - Record the dimensions and area after each subsequent fold.

  Ask:
  - How do the dimensions of the stack change with each fold?
  - How does the area of the top change with each fold?
  - How can you write an expression showing this?

Gifted and Enrichment

- Have students research how other people have dealt with this paper-folding challenge. Ask how they might duplicate or beat the record. Students may find the related Web Link on this TR page helpful.
- Give students the following challenge: Given a folded piece of paper, how might you determine the size of the paper when it is unfolded? Write a formula that would enable you to determine the unfolded size of a piece of paper based on the top area of the stack.

Web Link

For details about folding a piece of paper more than seven or eight times, go to www.mathlinks9.ca and follow the links. The site includes a formula that can be used to calculate the number of times that a piece of paper can be folded, given its size and thickness.

Answers

How Many Times Can You Fold a Piece of Paper?

1. b) Example: Stack 100 sheets of paper, measure the stack, and then divide by 100 sheets. Alternatively, stack enough sheets of paper to make a stack exactly 1 cm high, and then divide 1 cm by the number of sheets of paper.

3. a), b) Sample results are displayed in the table, where t is the thickness of one piece of paper and a is the area of one piece of paper.

Number of Folds	Thickness of Stack	Area of Stack
0	t	a
1	$2t$	$\left(\frac{1}{2}\right)a$
2	$4t$	$\left(\frac{1}{4}\right)a$
3	$8t$	$\left(\frac{1}{8}\right)a$
4	$16t$	$\left(\frac{1}{16}\right)a$
5	$32t$	$\left(\frac{1}{32}\right)a$
6	$64t$	$\left(\frac{1}{64}\right)a$
n	$(2^n)(t)$	$\left(\frac{1}{2^n}\right)a$

c) Example: On the sixth fold, the thickness is 64 times the original, making the paper too thick to fold again. Similarly, the area is $\frac{1}{64}$ the original size, making it too small to fold by hand.

Assessment	Supporting Learning
Assessment *of* Learning	
How Many Times Can You Fold a Piece of Paper? Introduce the task. Have students provide individual responses.	• **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this task. Page 234 provides notes on how to use this rubric for the task. • To view student exemplars, go to www.mathlinks9.ca, access the Teacher Centre on the Online Learning Centre, go to Assessment, and then follow the links. • For a second task, complete with teaching notes and student exemplars, go to www.mathlinks9.ca, access the Teacher Centre on the Online Learning Centre, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as this Task and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 **(Standard of Excellence)**	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution
4 **(Above Acceptable)**	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	Demonstrates one of the following: • provides a complete response to all parts of the problem, with weak communication in explanations or in recording results • provides a complete response with an incorrect answer for either #3a) or b) • provides a complete response to all parts of the problem, based on less than three types of paper
3 **(Meets Acceptable)**	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	Demonstrates one of the following: • provides a correct and complete response to #1 and 2; response may be based on two types of paper • provides a complete and correct response to #3 • provides correct partial solutions to all parts of the problem
2 **(Below Acceptable)**	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	Demonstrates one of the following: • provides a correct response to #1 with a significant start to #2 or 3; makes a start to justify thinking but it is weak • provides a correct and complete response to #2
1 **(Beginning)**	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	Demonstrates one of the following: • provides a correct and complete #2 • makes a correct start to #1 or 3