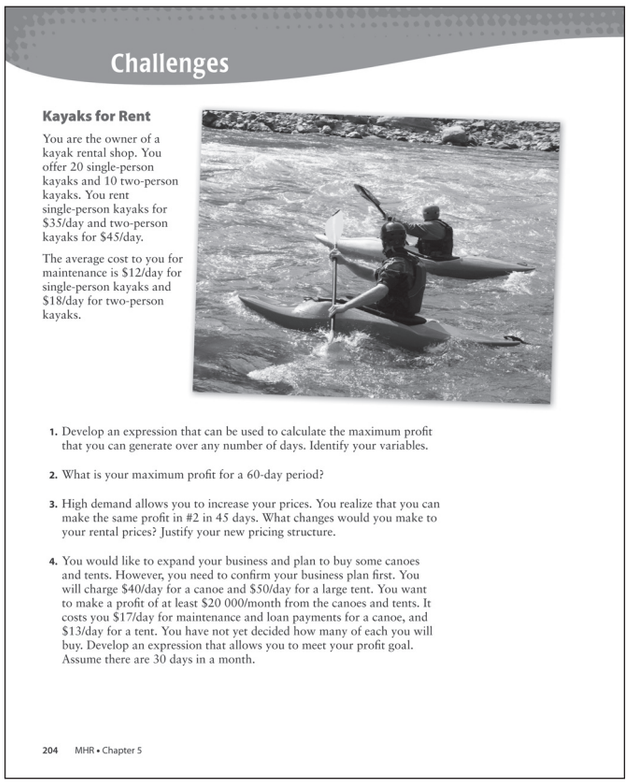
Challenges



Challenges

Kayaks for Rent

You are the owner of a kayak rental shop. You offer 20 single-person kayaks and 10 two-person kayaks. You rent single-person kayaks for $35/day and two-person kayaks for $45/day.

The average cost to you for maintenance is $12/day for single-person kayaks and $18/day for two-person kayaks.

1. Develop an expression that can be used to calculate the maximum profit that you can generate over any number of days. Identify your variables.
2. What is your maximum profit for a 60-day period?
3. High demand allows you to increase your prices. You realize that you can make the same profit in #2 in 45 days. What changes would you make to your rental prices? Justify your new pricing structure.
4. You would like to expand your business and plan to buy some canoes and tents. However, you need to confirm your business plan first. You will charge $40/day for a canoe and $50/day for a large tent. You want to make a profit of at least $20 000/month from the canoes and tents. It costs you $17/day for maintenance and loan payments for a canoe, and $13/day for a tent. You have not yet decided how many of each you will buy. Develop an expression that allows you to meet your profit goal. Assume there are 30 days in a month.

204 MHR • Chapter 5

***MathLinks 9*, pages 204–205**

Suggested Timing
40–60 minutes

Blackline Masters
Master 1 Project Rubric

Mathematical Processes
- ☑ Communication
- ☑ Connections
- ☑ Mental Mathematics and Estimation
- ☑ Problem Solving
- ☑ Reasoning
- ☑ Technology
- ☐ Visualization

Specific Outcomes
PR5 Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).
PR6 Model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2).

Planning Notes: Kayaks for Rent

You may wish to use the following steps to introduce and complete this challenge:

1. Have a class discussion about the concepts of income, expenses, and profit. It is not necessary to get into detailed discussions on the types of profit as this is not the focus of the question. However, a brief distinction between *gross* and *net* may enhance the lesson.
2. Assist students in recalling the use of variables to represent an unknown quantity. Ask:
   - What variables do you need for #1?
   - What will each variable represent?
   - How can you show profit using these variables?
   - What processes do you need to use?
3. Ask:
   - How will the answer to #2 differ from the answer to #1? Why?
   - How will the answer to #3 differ from the answer to #2? Why?
   - What strategies can you use to answer #3?
   - What variables will you use to answer #4?
4. Encourage students to share their ideas for strategies to answer #3. Some possible strategies include Solve an Equation, Work Backwards, or Estimate and Check. Have students report on how they might use each strategy and why.
5. Clarify that the task is to
   - define variables to represent unknown quantities
   - write polynomial expressions based on the variables
   - simplify the expressions by collecting like terms
   - solve an expression
6. Review the **Master 1 Project Rubric** with students so that they will know what is expected.

Meeting Student Needs

- You may need to guide some students to understand that profit is calculated by subtracting expenses from income.
- Have students share with a partner the strategy they used for #3. Encourage students to try alternative strategies and think about which strategy they prefer.
- Because there are several possible answers to the questions, set up one possibility on the board and discuss how it works. Challenge students to develop one or more other possible solutions.

- Wrap-up this challenge by having a discussion with students about what other factors might have to be considered when starting up a business.

ELL

- Teach the following terms in context: *kayak rental shop*, *single-person*, *two-person*, *average cost*, *maintenance*, *develop*, *generate*, *high demand*, *increase*, *pricing structure*, *canoes*, *tents*, and *assume*.

Gifted and Enrichment

- You may wish to encourage discussion with students about what other factors you might need to consider, such as weather, interest level, tourism fluctuations, etc.
- Encourage students to make the profit in 45 days as close as possible to the profit in #2.
- For #3, have students suggest other ways to increase the profit and show how it would affect the total profit. For example:
  – change the maintenance to after every second use
  – increase the prices by 25%
- Challenge students to find three possible solutions to #4. Have them justify why they feel they may need more canoes than tents, or vice versa.
- Have students answer the following question:
  Given that there are only ten one-person kayaks and ten two-person kayaks available, determine if the following profit goals are possible. If yes, determine the minimum number of kayaks rented per day that would still result in that profit goal.
  a) $15 000 profit per month
  b) $12 000 profit per month

  Answer: Example:
  a) To reach the profit goal of $15 000 per month:
  $\frac{15\,000}{30} = 500$
  The profits would have to be at least $500 per day.
  Assuming all 20 kayaks are rented out every day, the maximum profit is $22(10) + 27(10) = 490$.
  The maximum profit is $490 per day, so it is not possible to achieve $15 000/month in profits.

  b) To reach the profit goal of $12 000 per month:
  $\frac{12\,000}{30} = 400$
  The maximum profit is $490 per day, so it is possible to achieve $12 000/month in profits.
  $22n + 27w \geq 400$, where n is the number of one-person kayaks and w is the number of two-person kayaks.
  $22 \times 9 + 27 \times 8 = 198 + 216 = 414$
  One possible solution is nine one-person kayaks and eight two-person kayaks to be rented out each day, which would result in a profit of $12 420/month.
- Have students answer the following question:
  Given that there are only ten one-person kayaks and ten two-person kayaks available, determine if the following profit goals are possible. If yes, determine the minimum number of kayaks rented per day that would still result in that profit goal.
  a) $15 000 profit per month
  b) $12 000 profit per month

  a) To reach the profit goal in a 30-day month, the business would have to make at least $500 per day. If all 20 kayaks were rented out every day, the maximum profit would be $490 per day, which would not amount to $15 000/month in profits in a 30-day month, but would in a 31-day month. So, they could get that profit in July and August, but not in June and September.
  b) With a maximum profit of $490 per day, it is possible to achieve $12 000/month in profits. Example: Nine one-person kayaks and eight two-person kayaks rented out each day would result in a profit of $12 420/month in a 30-day month.

Answers

Kayaks for Rent

1. Example: $20s(35 - 12) + 10t(45 - 18) = 460s + 270t$, where s represents the number of days that all 20 single-person kayaks are rented and t represents the number of days that all 10 two-person kayaks are rented.

2. Profit $= 730d$
$= 730 \times 60$
$= 43\ 800$

3. Example: Increasing the price for a single-person kayak to $42 per day, and two-person kayaks to $55 per day, and calculating the maximum profit over a 45-day period with the same maintenance costs gives a profit of $43 650.

4. Example: $30c(40 - 17) + 30t(50 - 13) = 20\ 000$, where c represents the number of canoes purchased and t represents the number of tents purchased.

This Challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment	Supporting Learning
Assessment *for* Learning	
Kayaks for Rent Discuss the Challenge with students. Have students complete the activity in pairs.	• As a class, brainstorm strategies for approaching this Challenge. • Modelling a possible solution to #3 on the board may provide a springboard to assist students in attempting their own strategy.
Assessment *of* Learning	
Kayaks for Rent Introduce the Challenge to students. Have students complete the activity in pairs.	• **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Challenge. Page 285 provides notes on how to use this rubric for the Challenge. • To view student exemplars, go to www.mathlinks9.ca, access the Teacher Centre on the Online Learning Centre, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as this Challenge, Kayaks for Rent, and provides notes that specify how to identify the level of specific answers for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 (Standard of Excellence)	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution with most justification present
4 (Above Acceptable)	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	Demonstrates one of the following: • provides a complete response to all parts of the question, with a weak justification • provides a complete response to all parts of the question, with a minor calculation error that may affect the overall answers but not the understanding of the problem
3 (Meets Acceptable)	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	Demonstrates one of the following: • provides a correct and complete response to #1, 2, and 3; the profit may not be as close to the actual profit, but the student demonstrates a basic understanding of the problem • provides a correct and complete response to #2 and 3 • provides correct partial solutions to all parts of the question
2 (Below Acceptable)	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	Demonstrates one of the following: • provides a correct and complete response to #1 and 2 • provides correct and complete response to #2 and a significant start to #3, based on an incorrect #1 • provides a correct #4
1 (Beginning)	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	Demonstrates one of the following: • completes or correctly starts #1 • provides a correct #2 based on an incorrect #1 • provides a correct start to #4

For student exemplars, go to www.mathlinks9.ca and follow the links.