What Have You Got to Hide?

Prime numbers have fascinated mathematicians for thousands of years. The earliest known reference to prime numbers dates to 500 B.C.E. At that time Pythagoras' school of Greek mathematicians discovered prime numbers and realized that these are the building blocks of whole numbers.

Prime numbers may be used to create encrypted data such as that used to hide personal information being sent on the internet. You be the cryptographer!

The following polynomials can be used to create prime numbers:
- $n^2 - n + 41$
- $n^2 + n - 1$
- $n^2 - n + 17$

Your challenge is to investigate whether the addition and/or subtraction of these polynomials can create prime numbers.

1. Test each polynomial using at least three possible n values. Show your testing process and the results of your test.
2. If decimals or fractions are substituted for n, are your results prime numbers? Show your testing process and results. What is your conclusion?
3. Add and/or subtract several combinations of the given polynomials. Test the resulting polynomials to see if they generate prime numbers when values are substituted in. Show your work.

Challenges • MHR 205

***MathLinks 9*, page 205**

Suggested Timing

40–50 minutes

Materials

- calculator
- computer with a spreadsheet program (optional)

Blackline Masters

Master 1 Project Rubric

Mathematical Processes

- ✓ Communication
- ✓ Connections
- ✓ Mental Mathematics and Estimation
- ✓ Problem Solving
- ✓ Reasoning
- ✓ Technology
- ✓ Visualization

Specific Outcomes

PR5 Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).

PR6 Model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2).

Planning Notes: What Have You Got to Hide?

You may wish to use the following steps to introduce and complete this Challenge:

1. Have a class discussion where students share what they know about encryption, decryption, cryptography, and prime number (make sure they know that 1 is not a prime number). With the class, discuss:
   - Why has encryption become increasingly important and commonplace in our digital world?
   - Why is it important to protect our information on the Internet?
   - What types of passwords and encryption can you think of?
2. Discuss with the class what it means to have a restriction. Have them share examples of situations that involve a restriction (for example, everyone has the right to vote, but you have to be over 18 years of age). What does it mean for a mathematical expression to have a restriction? Have students previously seen restrictions in their math studies? For example, what restriction do square roots have?
3. Emphasize that, for this exercise, all justifications and explanations must be included. Clarify that the task is to:
   - substitute at least three different values for n in each of the three polynomials given to see if a prime number results.
   - try to find values that will not produce a prime number for the three polynomials given. If any are found, use these values to place restrictions on n.
   - form new polynomials by adding and subtracting combinations of the three polynomials given. Check to see if these new polynomials can be used to find prime numbers.
4. Review the **Master 1 Project Rubric** with students so that they will know what is expected.

Meeting Student Needs

- Some students may have questions on restrictions of values for n when using the given polynomials to find prime numbers. Discuss why restrictions may need to be placed on the values for n. For example, replacing n with negative integers may not result in a prime number. By definition, a prime number must be a natural number, so replacing n with a fraction or decimal may not result in a prime number.
- Some students may have difficulty recognizing whether a large number is a prime number. Suggest that these students restrict the values of n to single-digit numbers.
- Discuss with students how computers make repetitive tasks easier. How could a simple program or spreadsheet assist with the tasks in this challenge? (Note: When developing a spreadsheet program, the testing process involves dividing the generated number by prime numbers less than the square root of the generated number. For example: If testing 983, the square root of 983 is approximately 31.35. So prime numbers less than 31 would be tested to see if they divide evenly into 983.) Have students set up a spreadsheet program to evaluate the polynomials and enter various values of n.

ELL

- Teach the following terms in context: *prime numbers*, *fascinated*, and *generate*.

Gifted and Enrichment

- Have students research the role of polynomials in the finding of new prime numbers by using the web links provided below or doing a web search using a phrase such as *polynomials and prime numbers*.
- Have students research the role of prime numbers in encryption by using the web links provided below or doing a web search using the phrase *encryption of data and prime numbers*.
- Have students explain why encryption of personal data is necessary. They might wish to use the web links provided below or do a web search using the terms *Internet security*, *Internet privacy*, *encryption*, *cryptography*, or *decryption*.

WWW Web Link

Students can learn more about the following by going to www.mathlinks9.ca and following the links:

- the role of polynomials in finding new prime numbers
- the role of prime numbers in encryption
- the role of security and privacy issues on the Internet

Answers

What Have You Got to Hide?

1. Example: $n^2 - n + 41$: $1^2 - 1 + 41 = 41$, $2^2 - 2 + 41 = 43$, $(-3)^2 - (-3) + 41 = 53$; $n^2 + n - 1$: $12 + 1 - 1 = 1$, $32 + 3 - 1 = 11$, $(-5)^2 + (-5) - 1 = 19$; $n^2 - n + 17$: $2^2 - 2 + 17 = 19$, $4^2 - 4 + 17 = 29$, $(-3)^2 - 3 + 17 = 23$
2. Example: $n^2 - n + 41$: $\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 41 = 40\frac{3}{4}$. Decimals and fractions substituted into the expressions will not give prime numbers because they are not integers.
3. $n^2 - n + 41 + n^2 + n - 1 = 2n^2 + 40$: $2(1)^2 + 40 = 42$. The resulting expressions do not give prime numbers.

This Challenge can be used for either Assessment *for* Learning or Assessment *of* Learning.

Assessment	Supporting Learning
Assessment *for* Learning	
What Have You Got to Hide? Discuss the Challenge with students. Have students complete the activity in pairs.	• As a class, brainstorm strategies for approaching this Challenge.
Assessment *of* Learning	
What Have You Got to Hide? Introduce the Challenge to students. Have students complete the activity in pairs.	• **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on this Challenge. Page 288 provides notes on how to use this rubric for the Challenge. • To view student exemplars, go to www.mathlinks9.ca, access the Teacher Centre on the Online Learning Centre, go to Assessment, and then follow the links.

The chart below shows the **Master 1 Project Rubric** for tasks such as this Challenge, What Have You Got to Hide?, and provides notes that specify how to identify the level of specific answer for this project.

Score/Level	Holistic Descriptor	Specific Question Notes
5 (Standard of Excellence)	❑ Applies/develops **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution ❑ Procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding ❑ Uses **significant** mathematical language to explain their understanding and provides **in-depth** support for their conclusion	• provides a complete and correct solution with most justification present
4 (Above Acceptable)	❑ Applies/develops **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding ❑ Procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution ❑ Uses **appropriate** mathematical language to explain their understanding and provides **clear** support for their conclusion	• provides a complete response to all parts of the question, with a weak conclusion
3 (Meets Acceptable)	❑ Applies/develops **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding ❑ Procedures are **basic** and may contain a **major error or omission** ❑ Uses **common** language to explain their understanding and provides **minimal** support for their conclusion	Demonstrates one of the following: • provides a correct and complete response to #1 and 2 • provides a correct and complete response to #2 and 3 • provides correct partial solutions to all parts of the question • provides a correct and complete response to #1 and 3
2 (Below Acceptable)	❑ Applies/develops **some relevant** mathematical processes making **minimal** comparisons/connections that lead to a **partial solution** ❑ Procedures are **basic** and may contain **several major mathematical errors** ❑ Communication is **weak**	• provides a correct and complete response to any part and a significant start to another part
1 (Beginning)	❑ Applies/develops an **initial start** that may be **partially correct** or could have led to a correct solution ❑ Communication is **weak or absent**	• provides a correct completion to any part

For student exemplars, go to www.mathlinks9.ca and follow the links.