The Language of Mathematics

5.1

***MathLinks 9*, pages 174–182**

Suggested Timing
80–100 minutes

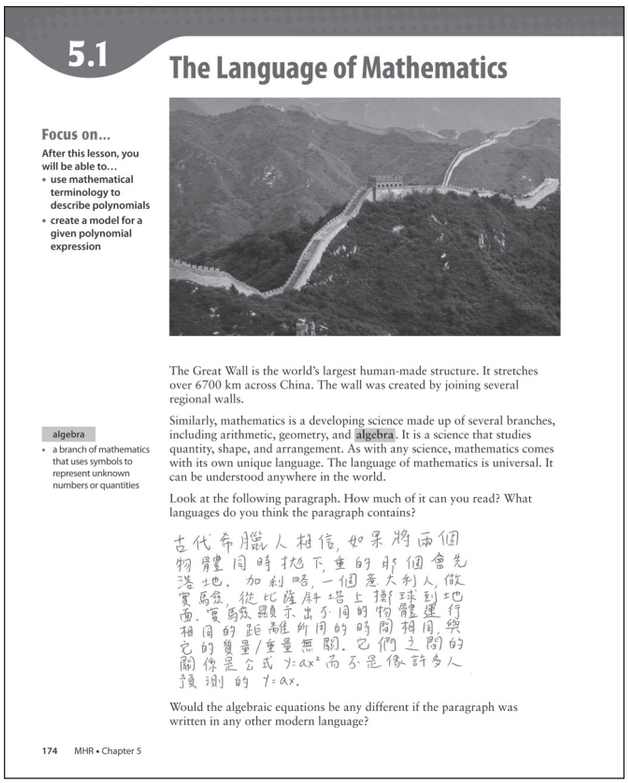
Materials
- concrete materials, such as algebra tiles

Blackline Masters
Master 2 Communication Peer Evaluation
Master 11 Algebra Tiles (Positive Tiles)
Master 12 Algebra Tiles (Negative Tiles)
BLM 5–3 Chapter 5 Warm-Up
BLM 5–5 Section 5.1 Extra Practice
BLM 5–6 Section 5.1 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
PR5 Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#2–4, 5, 7, 8, 12, 13, 15, 17, 20, 23, Math Link
Typical	#2–4, 7, 9, 10, 12, 13, 17–19, 21–26, Math Link
Extension/Enrichment	#2–4, 15, 21, 27–31



5.1 The Language of Mathematics

Focus on...
After this lesson, you will be able to...
- use mathematical terminology to describe polynomials
- create a model for a given polynomial expression

The Great Wall is the world's largest human-made structure. It stretches over 6700 km across China. The wall was created by joining several regional walls.

Similarly, mathematics is a developing science made up of several branches, including arithmetic, geometry, and algebra. It is a science that studies quantity, shape, and arrangement. As with any science, mathematics comes with its own unique language. The language of mathematics is universal. It can be understood anywhere in the world.

algebra
- a branch of mathematics that uses symbols to represent unknown numbers or quantities

Look at the following paragraph. How much of it can you read? What languages do you think the paragraph contains?

古代希臘人相信，如果將兩個物體同時拋下重的那個會先落地。加利略，一個意大利人，做實驗從比薩斜塔上擲球到地面，實驗顯示出不同的物體運行相同的距離所用的時間相同，與它的質量/重量無關。它們之間的關係是公式 $y=ax^2$ 而不是像許多人預測的 $y=ax$.

Would the algebraic equations be any different if the paragraph was written in any other modern language?

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Planning Notes

Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce material learned in previous sections.

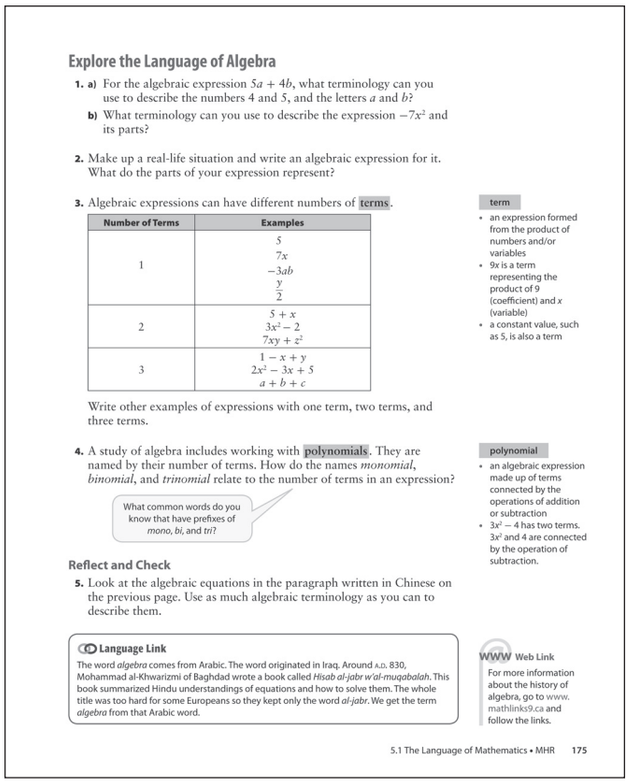
Explain to students that mathematics is often referred to as a universal language with many of the same symbols in every language.

The paragraph in the student resource is a Chinese translation of the following: The ancient Greeks believed that if two objects were dropped, the heavier one would strike the ground first. Galileo, an Italian, dropped cannonballs from the Leaning Tower of Pisa. He showed that different objects travelled the same distance in the same time regardless of mass/weight. The relationship was of the form $y = ax^2$, rather than $y = ax$ as many predicted.

Students with knowledge of a language other than English or Chinese should translate the paragraph about Galileo into their language. This translation could be supported by a parent or other community figure.

Use this exercise to confirm that the mathematical equations are the same in all languages when they are expressed using symbols. However, they would be different if expressed in words (i.e., y equals the product of a and x). This is intended to confirm mathematics as a universal language.

You may want to direct students' attention to the Language Link? on page 175. Perhaps initiate the discussion by asking students to indicate what aspects of algebra they have encountered in either grade 7 or 8.



Explore the Language of Algebra

1. a) For the algebraic expression $5a + 4b$, what terminology can you use to describe the numbers 4 and 5, and the letters a and b?
   b) What terminology can you use to describe the expression $-7x^2$ and its parts?

2. Make up a real-life situation and write an algebraic expression for it. What do the parts of your expression represent?

3. Algebraic expressions can have different numbers of terms.

Number of Terms	Examples
1	5 $7x$ $-3ab$ $\frac{y}{2}$
2	$5 + x$ $3x^2 - 2$ $7xy + z^2$
3	$1 - x + y$ $2x^2 - 3x + 5$ $a + b + c$

Write other examples of expressions with one term, two terms, and three terms.

term
- an expression formed from the product of numbers and/or variables
- $9x$ is a term representing the product of 9 (coefficient) and x (variable)
- a constant value, such as 5, is also a term

4. A study of algebra includes working with polynomials. They are named by their number of terms. How do the names *monomial*, *binomial*, and *trinomial* relate to the number of terms in an expression?

What common words do you know that have prefixes of *mono*, *bi*, and *tri*?

polynomial
- an algebraic expression made up of terms connected by the operations of addition or subtraction
- $3x^2 - 4$ has two terms. $3x^2$ and 4 are connected by the operation of subtraction.

Reflect and Check

5. Look at the algebraic equations in the paragraph written in Chinese on the previous page. Use as much algebraic terminology as you can to describe them.

Language Link

The word *algebra* comes from Arabic. The word originated in Iraq. Around A.D. 830, Mohammad al-Khwarizmi of Baghdad wrote a book called *Hisab al-jabr w'al-mugabalah*. This book summarized Hindu understandings of equations and how to solve them. The whole title was too hard for some Europeans so they kept only the word *al-jabr*. We get the term *algebra* from that Arabic word.

WWW Web Link

For more information about the history of algebra, go to www.mathlinks9.ca and follow the links.

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Explore the Language of Algebra

In this exploration, students examine algebraic expressions and their associated terminology. Encourage students to recall any times they have used variables (describing patterns, solving equations, graphing linear relations, etc.). This will provide a connection to prior learning of mathematical terminology.

Method 1 Have students work on the exploration in pairs. Once they have completed the steps, have each set of partners compare their results with those of another set of partners.

Method 2 Divide the class into two groups. Have each group work through the exploration together. Have each group create a word wall, showing any terminology they arrived at for #5. Conclude with a whole-class discussion. You may wish to ask questions, such as:

- What is the difference between an expression and an equation?
- How would your answers in #5 change if one of the equations was $y = x^2 + 1$?
- How many terms are on each side of the equation?

Refer to the thought bubble directed at #4. This is helpful in assisting students to remember the number of terms associated with a monomial, binomial, or trinomial. Have students identify words with the prefixes *mono*, *bi*, and *tri*. Examples could include:

- mono: monotone, monorail, monotony, monopoly, monogram, monochrome
- bi: bicycle, biceps, biannual, bicentennial, bifocal, bilateral, binary, bipartisan
- tri: tricycle, tricolour, trilingual, trimester, tripod, tribunal, trilogy

Meeting Student Needs

- You may decide that it would be better for the class to work through the Explore as a whole-class activity.
- As a way of introducing this topic, have students choose a language and, using the Internet, research the words used for counting to ten in that language. Then, have students find out how the numbers are written. Are the same symbols used as are used in English? Have a discussion speculating as to why math symbols are universal.
- Invite an Elder or a person from one of the surrounding areas who is fluent in Inuktitut or a First Nations language to speak to your class. Ask this guest to speak about how his or her ancestors taught math concepts or spoke about math many years ago in Aboriginal communities. In addition, discuss how math terminology is written or spoken today in the guest's native tongue.

ELL

- The introduction is a good opportunity for Chinese English language learners to demonstrate and explain what they know. Remind other students that this is what it is like for English language learners. Although the math is universal, the language presents a challenge.
- Allow English language learners to complete the Explore in their first language. Then, have them translate into English. The terminology they use in their first language will probably be more extensive, as they have greater access to prior knowledge in their first language.
- For #4 of the Explore, teach the meaning of *mono*, *bi*, and *tri*. Remind students of these meanings when they come across other words with these prefixes.

Answers

Explore the Language of Algebra

Answer may vary.

1. a) Example: 4 and 5 are coefficients and a and b are variables.

b) Example: The expression $-7x^2$ consists of one term with a coefficient of -7, a variable of x, and an exponent of 2.

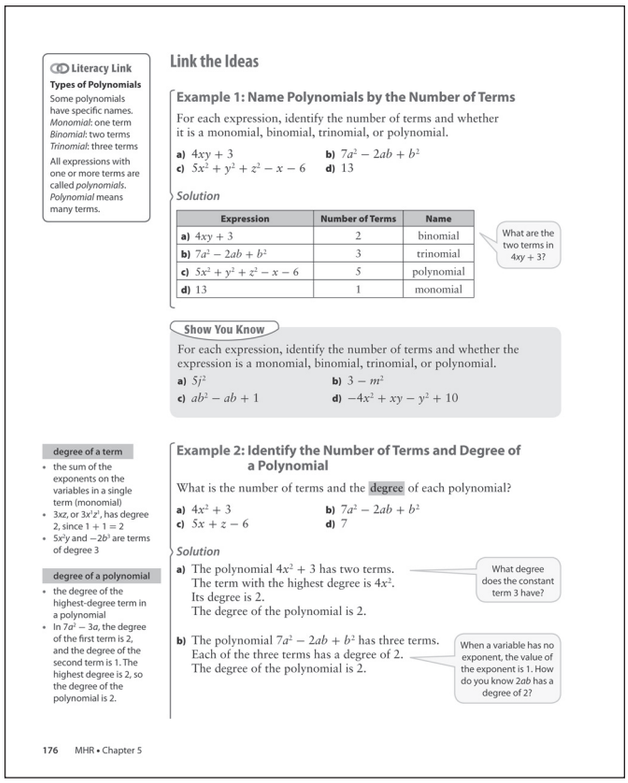
2. Example: The cost of 10 min of text messaging and 25 min of air time, where x is the cost per minute of text messaging, and y is the cost per minute of air time, can be expressed as $10x + 25y$.

3. Example: One term: $8y$; two terms: $a^2 - 4bc$; three terms: $y^2 - 5yz + 7$

4. Example: The prefix *mono* means one, and monomials have one term. The prefix *bi* means two, and binomials have two terms. The prefix *tri* means three, and trinomials have three terms.

5. Example: $y = ax^2$ and $y = ax$ are algebraic equations. y and ax^2 and ax are monomials. y, a, and x are called variables. The variable a acts as a coefficient to x. The number 2 is an exponent.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss the terminology associated with algebra. Try to have students use the correct words in the correct way. Encourage alternative wordings.	• Students should be comfortable with the mathematical vocabulary. Encourage each one to create a personal vocabulary list in their Foldable or notebook with terms and examples expressed in pictures, words, and symbols. • Encourage students to both speak and write the words. Where possible, encourage drawings and diagrams.



Literacy Link
Types of Polynomials
Some polynomials have specific names.
Monomial: one term
Binomial: two terms
Trinomial: three terms
All expressions with one or more terms are called *polynomials*. *Polynomial* means many terms.

Link the Ideas

Example 1: Name Polynomials by the Number of Terms

For each expression, identify the number of terms and whether it is a monomial, binomial, trinomial, or polynomial.

a) $4xy + 3$ **b)** $7a^2 - 2ab + b^2$
c) $5x^2 + y^2 + z^2 - x - 6$ **d)** 13

Solution

Expression	Number of Terms	Name
a) $4xy + 3$	2	binomial
b) $7a^2 - 2ab + b^2$	3	trinomial
c) $5x^2 + y^2 + z^2 - x - 6$	5	polynomial
d) 13	1	monomial

What are the two terms in $4xy + 3$?

Show You Know
For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.
a) $5j^2$ **b)** $3 - m^2$
c) $ab^2 - ab + 1$ **d)** $-4x^2 + xy - y^2 + 10$

degree of a term
- the sum of the exponents on the variables in a single term (monomial)
- $3xz$, or $3x^1z^1$, has degree 2, since $1 + 1 = 2$
- $5x^2y$ and $-2b^3$ are terms of degree 3

degree of a polynomial
- the degree of the highest-degree term in a polynomial
- In $7a^2 - 3a$, the degree of the first term is 2, and the degree of the second term is 1. The highest degree is 2, so the degree of the polynomial is 2.

Example 2: Identify the Number of Terms and Degree of a Polynomial

What is the number of terms and the degree of each polynomial?

a) $4x^2 + 3$ **b)** $7a^2 - 2ab + b^2$
c) $5x + z - 6$ **d)** 7

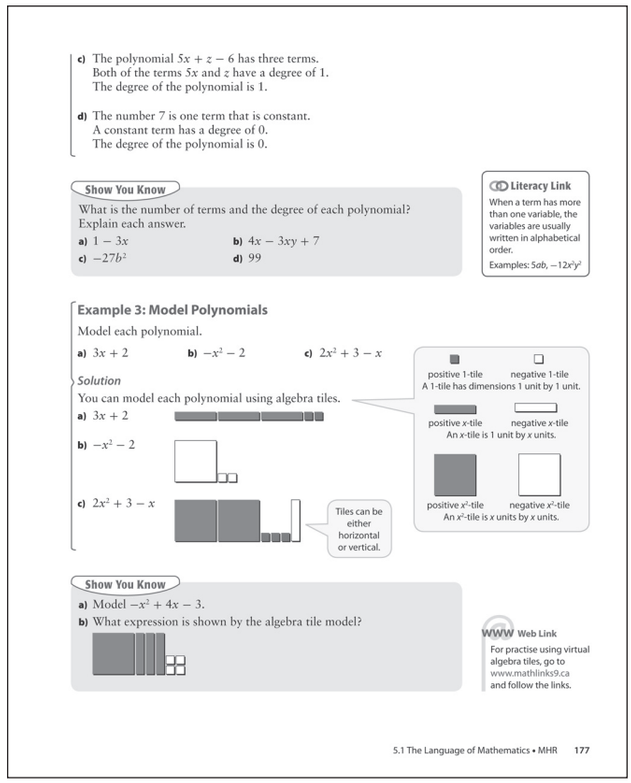
Solution

a) The polynomial $4x^2 + 3$ has two terms.
The term with the highest degree is $4x^2$.
Its degree is 2.
The degree of the polynomial is 2.

What degree does the constant term 3 have?

b) The polynomial $7a^2 - 2ab + b^2$ has three terms.
Each of the three terms has a degree of 2.
The degree of the polynomial is 2.

When a variable has no exponent, the value of the exponent is 1. How do you know $2ab$ has a degree of 2?



c) The polynomial $5x + z - 6$ has three terms.
Both of the terms $5x$ and z have a degree of 1.
The degree of the polynomial is 1.

d) The number 7 is one term that is constant.
A constant term has a degree of 0.
The degree of the polynomial is 0.

Show You Know
What is the number of terms and the degree of each polynomial? Explain each answer.
a) $1 - 3x$ **b)** $4x - 3xy + 7$
c) $-27b^2$ **d)** 99

Literacy Link
When a term has more than one variable, the variables are usually written in alphabetical order.
Examples: $5ab$, $-12x^2y^2$

Example 3: Model Polynomials

Model each polynomial.

a) $3x + 2$ **b)** $-x^2 - 2$ **c)** $2x^2 + 3 - x$

Solution

You can model each polynomial using algebra tiles.

a) $3x + 2$

b) $-x^2 - 2$

c) $2x^2 + 3 - x$

Tiles can be either horizontal or vertical.

positive 1-tile negative 1-tile
A 1-tile has dimensions 1 unit by 1 unit.
positive x-tile negative x-tile
An x-tile is 1 unit by x units.
positive x^2-tile negative x^2-tile
An x^2-tile is x units by x units.

Show You Know
a) Model $-x^2 + 4x - 3$.
b) What expression is shown by the algebra tile model?

Web Link
For practise using virtual algebra tiles, go to www.mathlinks9.ca and follow the links.

Link the Ideas

Example 1

Literacy Link Before students look at Example 1, relate the Literacy Link on page 176 to the words students identified with the prefixes *mono*, *bi*, and *tri* in the Explore. Remind students that any algebraic expression is a polynomial, but that some polynomials have special names according to their number of terms.

Encourage students to attempt Example 1 without looking at the solution. This may help some students overcome any fear they might have of algebra as a result of friends, parents, or siblings who say "I didn't get algebra!" or "I struggled with algebra."

For each part of Example 1, you might ask the following:
- What arithmetic operations create terms in algebra?
- What operations *do not* affect the number of terms in an expression?
- How many terms are in each of the expressions?
- What are the terms? (Have students circle them on the blackboard or on a flip chart.)

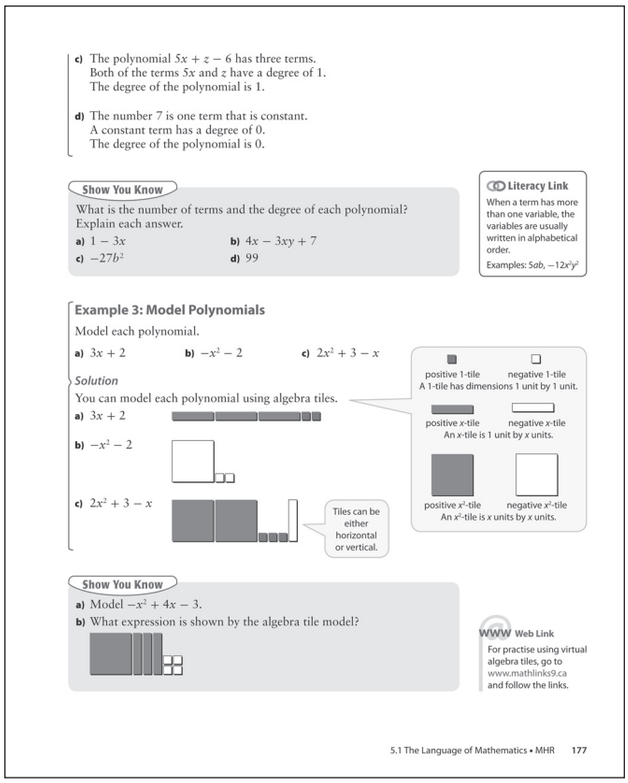
Ensure all students understand that
- all the expressions in Example 1 are polynomials, but that some of them have special names
- $4xy + 3$ has two terms ($4xy$ and 3)
- $4xy$ is one term with two variables: x and y

Have students work individually to do the questions in the Show You Know. Check to ensure that they all understand how to determine the number of terms in each expression.

Example 2

Have students use the same skills they used in Example 1 to find the number of terms in an algebraic expression. The second part of the exercise deals with the degree of the polynomial. Emphasize that the degree of a term is determined from the variables and their exponents. The degree of a polynomial is the same as the highest degree for any of its terms.

Rather than spending a lot of time going over each part of Example 2, it may be better to do one example in class. Ask students to work in pairs to attempt the other examples, checking the solution(s) given. They can then try the questions in the Show You Know. Remind students that the thought bubbles provide hints to help them in their thinking.



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A constant term has a degree of 0.
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Show You Know

What is the number of terms and the degree of each polynomial? Explain each answer.

a) $1 - 3x$ b) $4x - 3xy + 7$
c) $-27b^2$ d) 99

Literacy Link

When a term has more than one variable, the variables are usually written in alphabetical order.

Examples: $5ab$, $-12x^2y^2$

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Model each polynomial.

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Solution

You can model each polynomial using algebra tiles.

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Tiles can be either horizontal or vertical.

positive 1-tile negative 1-tile
A 1-tile has dimensions 1 unit by 1 unit.

positive x-tile negative x-tile
An x-tile is 1 unit by x units.

positive x^2-tile negative x^2-tile
An x^2-tile is x units by x units.

Show You Know

a) Model $-x^2 + 4x - 3$.
b) What expression is shown by the algebra tile model?

Web Link

For practise using virtual algebra tiles, go to www.mathlinks9.ca and follow the links.

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Encourage students by asking the following questions:

- How many terms are in the expression?
- Which term has the highest degree? What is its degree?
- What is the degree of the expression or polynomial?

Literacy Link Draw students' attention to the term $3xy$ in part b) of the Show You Know. Tell them to relate this term to the information in the Literacy Link on page 177, which says that when a term has more than one variable, the variables are usually placed in alphabetical order.

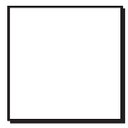
Example 3

This example serves as an introduction to algebra tiles and is intended to help students visualize and handle a physical model for an algebraic expression. As such, it is designed to increase students' understanding and to encourage them to avoid unnecessary memorization.

Provide students with algebra tiles. If students have prior experience with algebra tiles, you might begin the discussion with a series of questions:

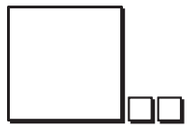
- Hold up a positive 1-tile. What are its dimensions?
- How is a negative 1-tile different?
- Hold up a positive x-tile. What are its dimensions?
- What colour is a negative x-tile?
- Hold up a positive x^2-tile. What are its dimensions?

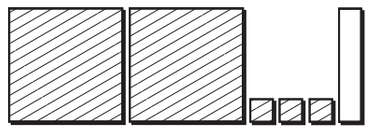
Have students work in pairs to show an algebra-tile model for each part of Example 3. Have students check their work against the solutions in the student resource. Encourage students to complete the Show You Know. When they are finished, ask them to explain the difference between part a) and part b).

You may also consider having students draw diagrams to represent the equations. This will introduce them to alternative ways of representing, and working with, equations. For example, students could use  to represent $+x^2$,  to represent $-x^2$,  to represent $+x$,  to represent $-x$,  to represent $+1$, and  to represent -1.

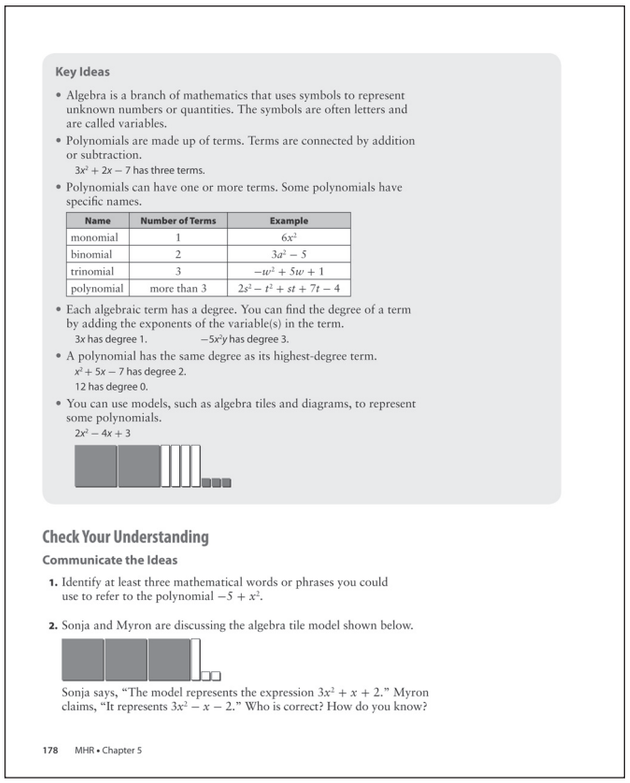
The parts in Example 3 can then be drawn as follows:

a) $3x + 2$ 

b) $-x^2 - 2$ 

c) $2x^2 + 3 - x$ 

For another example of how diagrams can be used, see question #20.



Key Ideas

- Algebra is a branch of mathematics that uses symbols to represent unknown numbers or quantities. The symbols are often letters and are called variables.
- Polynomials are made up of terms. Terms are connected by addition or subtraction.
  $3x^2 + 2x - 7$ has three terms.
- Polynomials can have one or more terms. Some polynomials have specific names.

Name	Number of Terms	Example
monomial	1	$6x^2$
binomial	2	$3x^2 - 5$
trinomial	3	$-w^2 + 5w + 1$
polynomial	more than 3	$2s^2 - t^2 + st + 7t - 4$

- Each algebraic term has a degree. You can find the degree of a term by adding the exponents of the variable(s) in the term.
  $3x$ has degree 1. $-5x^2y$ has degree 3.
- A polynomial has the same degree as its highest-degree term.
  $x^2 + 5x - 7$ has degree 2.
  12 has degree 0.
- You can use models, such as algebra tiles and diagrams, to represent some polynomials.
  $2x^2 - 4x + 3$

Check Your Understanding

Communicate the Ideas

1. Identify at least three mathematical words or phrases you could use to refer to the polynomial $-5 + x^2$.

2. Sonja and Myron are discussing the algebra tile model shown below.

Sonja says, "The model represents the expression $3x^2 + x + 2$." Myron claims, "It represents $3x^2 - x - 2$." Who is correct? How do you know?

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Key Ideas

The Key Ideas provide a detailed summary of what is most important in section 5.1. Have students prepare their own summary of the Key Ideas and record them in their Foldable. They should be discouraged from copying the Key Ideas and should use different examples from those found in the book. They should also limit their summaries to what is important to them, though they should also include information that they are uncertain about. Have students check each other's notes and examples for accuracy.

Meeting Student Needs

- You may decide that it would be better for the class to work through the examples as a whole-class activity. You could then assign the Show You Know as a small-group or pairs activity.
- Consider teaching this section over two periods if this strategy seems more appropriate for your class. Focus on the language and terminology in the first period. In the second, focus on using algebra tiles to represent expressions, and vice versa. Assign selected questions from Check Your Understanding to give students additional practice.
- If students have no experience with algebra tiles you will need to spend more time ensuring that all students understand them. You will need to allow some "play time" if students are not experienced in using concrete materials.
- Students who need supported learning may have to be reminded that a term is defined by multiplication.
- Point out that terms are separated by addition or subtraction signs.
- Have students who are struggling with determining the degree of a term place the exponent 1 on single variables. For example, x can be written x^1. This will help with determining the degree of a term.
- Some learners may benefit from orally completing some multiplication questions, explaining how to determine the exponent of the final answer.
- Have students struggling with multiplying the coefficients of terms with no apparent coefficient place the 1 in front of single variables. For example, have them write $3x^2 \times x$ as $3x^2 \times 1x$.
- You might suggest to students that they use diagrams to solve part a) of the Show You Know for Example 3.

Gifted and Enrichment

- Ask students to consider the limitations of using algebra tiles and then challenge them to construct solutions to the limitations. For example, they might say that algebra tiles can be used only with expressions that have constants with smaller values. A solution might be to assign larger values to some tiles. For example, students might attach a piece of masking tape labelled $10x$ to an x-tile to indicate that the tile represents 10 times its value.

Common Errors

- Some students who have little experience with manipulatives may want to play with the algebra tiles.

R_x Allow some time for students to become familiar with the tiles. Then, encourage them to concentrate on the mathematics under study.

- Some students may confuse x-tiles and x^2-tiles.

R_x Ask students for the dimensions of each tile. Ask which tile is square and help them associate the shape of the tile with the value that the tile represents (i.e., x^2).

- Some students may have difficulty determining the degree of a polynomial.

R_x Work through a progression of examples from monomials, to binomials, to trinomials, to polynomials. Working with monomials allows students to find the degree of a term. Once they are able to do that, they need only to check the degree of each term, and then identify the degree of the term with the highest degree.

Answers

Example 1: Show You Know

Expression	Number of Terms	Name
a) $5j^2$	1	monomial
b) $3 - m^2$	2	binomial
c) $ab^2 - ab + 1$	3	trinomial
d) $-4x^2 + xy - y^2 + 10$	4	polynomial

Example 2: Show You Know

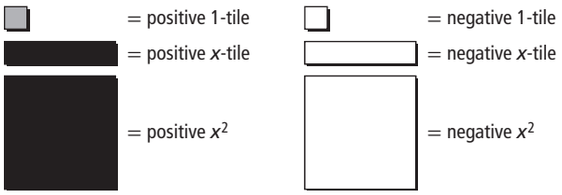
a) There are two terms, and the degree of the binomial is 1.

b) There are three terms, and the degree of the trinomial is 2.

c) There is one term, and the degree of the monomial is 2.

d) There is one term, and the degree of the monomial is 0.

Example 3: Show You Know



a)

b) $x^2 + 3x - 4$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • It is useful to change your questions continually by reversing the situation. For example, first, give an expression and ask for the number of terms. Then, give the number of terms and ask students to create an example. Some teachers think of this as reversing the questioning process. • Some students may require coaching to identify the type of polynomial being shown. Have them underline the prefixes for each word to help them focus on this link with the number of terms. • Encourage struggling learners to use their Foldable as a reference tool for terminology and possible examples.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to create polynomials with one or two characteristics. For example, write a polynomial with two terms that has degree 2. • Emphasize the importance of being able to quickly look at an expression and tell the number of terms. • Encourage struggling learners to identify the powers on each variable to assist in determining the total degree. • Remind students that the degree is linked to the variable and this is why a number or value that is constant has a degree of zero.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students create questions for each other. • Express questions in both the ways shown in the Show You Know (i.e., show an algebra-tile model and ask for its matching expression, or give an expression and have students model it). • Encourage students to model the questions using actual algebra tiles. If algebra tiles are not available, provide students with **Master 11 Algebra Tiles (Positive Tiles) and Master 12 Algebra Tiles (Negative Tiles)**. Ensure that all students understand the relationship between the algebra-tile model and the symbolic approach.

Key Ideas

- Algebra is a branch of mathematics that uses symbols to represent unknown numbers or quantities. The symbols are often letters and are called variables.
- Polynomials are made up of terms. Terms are connected by addition or subtraction.
  $3x^2 + 2x - 7$ has three terms.
- Polynomials can have one or more terms. Some polynomials have specific names.

Name	Number of Terms	Example
monomial	1	$6x^2$
binomial	2	$3a^2 - 5$
trinomial	3	$-w^2 + 5w + 1$
polynomial	more than 3	$2s^2 - t^2 + st + 7t - 4$

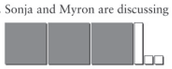
- Each algebraic term has a degree. You can find the degree of a term by adding the exponents of the variable(s) in the term.
  $3x$ has degree 1. $-5x^2y$ has degree 3.
- A polynomial has the same degree as its highest-degree term.
  $x^2 + 5x - 7$ has degree 2.
  12 has degree 0.
- You can use models, such as algebra tiles and diagrams, to represent some polynomials.
  $2x^2 - 4x + 3$

Check Your Understanding

Communicate the Ideas

1. Identify at least three mathematical words or phrases you could use to refer to the polynomial $-5 + x^2$.

2. Sonja and Myron are discussing the algebra tile model shown below.



Sonja says, "The model represents the expression $3x^2 + x + 2$." Myron claims, "It represents $3x^2 - x - 2$." Who is correct? How do you know?

3. Give two examples of a polynomial that satisfies all statements.
- consists of two terms
- contains two variables
- has degree 2
- one term is of degree 1 and has a coefficient of 1

4. When is it acceptable not to write the 1 in an algebraic expression? When must you write the 1? Give examples.

Practise

For help with #5 to #7, refer to Example 1 on page 176.

5. For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.
a) $3x^2 - 5x - 7$ **b)** $-11a$
c) $c^2 + cf + df - f^2$ **d)** 8

6. What is the number of terms and what is a name for each expression?
a) n **b)** $6 + 4x - x^2$
c) 0 **d)** $p^2 + 3pq$

7. Refer to the polynomials below to answer each question.

$6x$	-15
$3x - y$	$4c^2 - cd$
$7 + a + b$	$3m^2 - 4mn - 9n^2 + 1$

a) Which ones are monomials?
b) Which ones are trinomials?
c) Which ones have two terms?

For help with #8 to #10, refer to Example 2 on pages 176–177.

8. For each polynomial, what is the degree and number of terms?
a) $4 - b$ **b)** $fg + 2g$
c) $8x^2 - xy - y^2$

9. State the degree and number of terms for each polynomial.
a) $3xy + 1$ **b)** $11k^2 + 7k - 5$
c) 6

10. Refer to the polynomials below to answer each question.

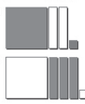
$3b^2$	$2 + p$
$4st + t - 1$	$2x^2 - y^2$

a) Which ones are binomials?
b) Which ones have degree 2?
c) What is the variable in the monomial?
d) Which polynomials have a constant term?

For help with #11 to #14, refer to Example 3 on page 177.

11. What expression is represented by each set of algebra tiles?

a)


b)


c)

12. Write the expression represented by each set of algebra tiles.

a)


b)


c)

Check Your Understanding

Communicate the Ideas

Have students work individually or in small groups to answer the questions.

For # 1, students use mathematical words and phrases for the polynomial $-5 + x^2$. Encourage students to use as many of the applicable terms as possible and justify why they chose them. Discuss these as a class for those students who may need some support.

For # 2, students are asked which student has correctly given the expression for the algebra tile model. This is a quick check to see if students know the symbols for a specific algebra tile combination.

In # 3, students are asked to create polynomials that satisfy multiple conditions. Have students share their answers with a partner or on the board so that a variety of possible responses can be seen.

In # 4, students are to give examples showing when it is necessary to show the 1 in an expression, and when it is not necessary. Students' responses could involve coefficients, exponents, and constants.

Practise

Students may work individually or in pairs on the Practise questions.

In the Practise, there are pairs of questions that are conceptually similar, though the wording of the questions is somewhat different. Students may be given a choice within each pair of questions: #5 or 6, #8 or 9, #11 or 12, #13 or 14.

Apply

Questions #15 to 20 ask students to combine their learning and the terminology or models from section 5.1 in slightly new ways. It is helpful to give students some choice in the questions to complete. For example, assign #19 and 20, and then let students select two additional questions from #15 to 20.

Questions #21 to 26 begin to relate the algebra to applications from the real world. Again, to provide choice for this series of questions, assign #21 and 23, and then let students select two or three additional questions.

13. Model each polynomial.
- **a)** $x^2 + x - 1$
- **b)** $3x + 2$
- **c)** $-2x$

14. Use a model to represent each polynomial.
- **a)** $-x^2 + 3$
- **b)** $2x^2 - 3x$
- **c)** 8

Apply

15. Represent each of the following with a diagram and an expression.
- **a)** binomial
- **b)** monomial of degree 1
- **c)** monomial of degree 2 with a coefficient of 9
- **d)** polynomial with four terms that is of degree 2

16. Use your knowledge of algebra tiles to answer the following questions.
- **a)** How are the dimensions of a 1-tile and an x-tile related?
- **b)** The rectangle shown was formed using an x^2-tile and three x-tiles. What is an expression for the length of the rectangle?

17. Write an algebraic expression for each of the following.
- **a)** the product of 6 and x
- **b)** the sum of $2x$ and 3
- **c)** the length of the rectangle below, which is made from algebra tiles

18. Make a model of an algebraic expression that includes at least one x^2-tile, at least two x-tiles, and two 1-tiles. Use materials or a diagram. Then, use symbols to show your expression. What type of polynomial is it?

19. For the polynomial $6x^2 - 5$, state the following.
- **a)** number of terms
- **b)** coefficient of the first term
- **c)** number of variables
- **d)** degree of polynomial
- **e)** constant term

20. Let  represent x^2,  represent x, and  represent 1. The same diagrams in yellow represent negative quantities.
- **a)** What is an expression for the polynomial shown?

- **b)** Make up a trinomial. Draw diagrams to represent your trinomial.

21. Write each statement as an algebraic expression. Include what your variables represent.
- **a)** Eight and a number are added together.
- **b)** Omar has some money in his wallet. How much money does he have after a friend gives him $5?
- **c)** A page is 4 cm longer than its width.
- **d)** The product of a number and 5 is increased by 2.
- **e)** The result of 3 times the number of people decreased by 21.

22. Describe a situation that could be modelled by each given polynomial.
- **a)** $3x + 5$
- **b)** $10 - x$

23. Marion gives French lessons in the evening. She charges $20 for adults and $15 for children. The expression $20a + 15c$ represents her earnings.
- **a)** What do the variables a and c represent?
- **b)** How much does Marion make if she gives lessons to four adults and nine children? Show your work.
- **c)** Write a new expression for Marion's earnings if she charges $3 more for adults and $2 more for children.

24. Tickets for a school concert are $10 for adults and $5 for students. Write an expression that shows the total income for the school concert. Tell what your variables represent.

25. A hockey league awards teams two points for a win, one point for a shoot-out loss, and no points for a loss in regulation time.

- **a)** Write an algebraic expression to represent the total points for a hockey team.
- **b)** What variable(s) did you use? Indicate what each variable represents.
- **c)** In the first 20 games of the season, Team A had 12 wins and 4 shoot-out losses. How many losses in regulation time did the team have?
- **d)** What were the total points for Team A?
- **e)** Team A was tied with Team B after 20 games. However, Team B had a different record than Team A. Show two possible records for Team B. Use your expression to show that the two hockey teams had the same number of total points.

26. A banquet hall can be rented for parties. An expression for the rental cost is $5n + 75$, where n is the number of people.
- **a)** What type of polynomial is $5n + 75$, and what is its degree?
- **b)** What could the numbers 5 and 75 represent?
- **c)** How much does it cost to rent the banquet hall for 150 people?

Extend

27. On a true/false test, there is a penalty for incorrect answers. Miranda's teacher advises the students not to guess at any of the 25 questions. The teacher awards 2 points for a correct answer, -1 point for a wrong answer, and 0 points if the question is not answered.
- **a)** Write a polynomial to represent a student's score on this test.
- **b)** What are the maximum and minimum scores possible on this test? Explain.
- **c)** What are all of the possible scores if Miranda got 20 questions correct? Explain.

28. What is the degree of $xy - abx + cdy - qr - prqz$ if x, y, and z are variables and a, b, c, d, p, q, and r are coefficients?

29. Ricardo draws the following rectangle with dimensions in metres.

- **a)** What is an expression for the perimeter of the rectangle?
- **b)** Write an equation showing how the length and width of the diagram would be related if the dimensions given were for a square.
- **c)** Solve your equation in part b) to find the value of x. Show your work.

Extend

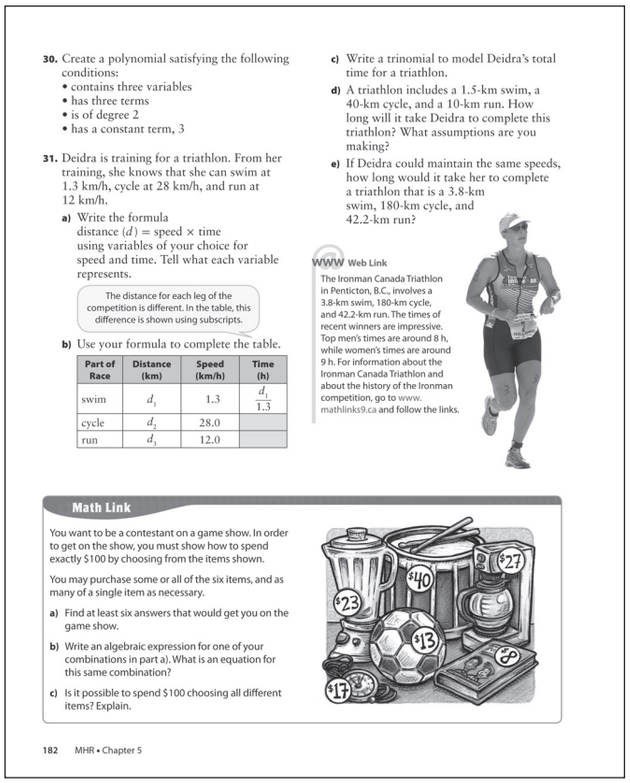
Each of the Extend questions is more challenging than those found earlier in the section. The challenge could come in the form of more difficult applications (#27 and 31) or a higher degree of abstraction (#28). The question may also be a prelude to upcoming work (#29).

Literacy Link Section 5.1 contains the most terms for students to learn. Have students provide a definition and an example for each of the terms in their concept map that is related to this chapter. Ask students to attach three ovals to the oval labelled *polynomial* and label them *monomial*, *binomial*, and *trinomial*. Then, have them provide examples of each of the three types of polynomials.

Math Link

In this Math Link, students are asked to determine how many items they can purchase for exactly $100. The prices are unusual in that half are prime numbers. This problem has at least nineteen unique solutions. The problem will be revisited in the section 5.2 Math Link where students will discover the usefulness of polynomial addition and subtraction in solving problems with multiple variables.

You may wish to work through this problem with students in this way:
- Introduce the problem in the last few minutes of class and send it home for students to find two or three possible combinations that total exactly $100.
- Have students show the possible answers on the board. It will likely be difficult to compare solutions because students will use different words/symbols for the sale items. Ask students how you might overcome this difficulty. Suggestions might include making an organized list, using specific names for each item, or using a specific letter to represent each item.
- Have students rewrite their answers using the techniques they brainstormed in the previous step.
- Assign the remainder of the Math Link.



30. Create a polynomial satisfying the following conditions:
- contains three variables
- has three terms
- is of degree 2
- has a constant term, 3

31. Deidra is training for a triathlon. From her training, she knows that she can swim at 1.3 km/h, cycle at 28 km/h, and run at 12 km/h.

a) Write the formula distance (d) = speed × time using variables of your choice for speed and time. Tell what each variable represents.

The distance for each leg of the competition is different. In the table, this difference is shown using subscripts.

b) Use your formula to complete the table.

Part of Race	Distance (km)	Speed (km/h)	Time (h)
swim	d_1	1.3	$\frac{d_1}{1.3}$
cycle	d_2	28.0	
run	d_3	12.0	

c) Write a trinomial to model Deidra's total time for a triathlon.

d) A triathlon includes a 1.5-km swim, a 40-km cycle, and a 10-km run. How long will it take Deidra to complete this triathlon? What assumptions are you making?

e) If Deidra could maintain the same speeds, how long would it take her to complete a triathlon that is a 3.8-km swim, 180-km cycle, and 42.2-km run?

WWW Web Link

The Ironman Canada Triathlon in Penticton, B.C., involves a 3.8-km swim, 180-km cycle, and 42.2-km run. The times of recent winners are impressive. Top men's times are around 8 h, while women's times are around 9 h. For information about the Ironman Canada Triathlon and about the history of the Ironman competition, go to www.mathlinks9.ca and follow the links.

Math Link

You want to be a contestant on a game show. In order to get on the show, you must show how to spend exactly $100 by choosing from the items shown.

You may purchase some or all of the six items, and as many of a single item as necessary.

a) Find at least six answers that would get you on the game show.

b) Write an algebraic expression for one of your combinations in part a). What is an equation for this same combination?

c) Is it possible to spend $100 choosing all different items? Explain.

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Meeting Student Needs

- Provide **BLM 5–5 Section 5.1 Extra Practice** to students who would benefit from more practice.
- Providing choices in what questions students will complete helps them take responsibility for their learning.
- The concepts covered in #4 of the Communicate the Ideas are important and could be challenging for some students. It might benefit students to have a class discussion about this question.
- It may be more effective for some students to complete the Practise questions for Example 1 immediately after working on that example. Follow the same approach for Examples 2 and 3 with these students. Then, have students attempt the Apply questions.
- For text-dense problems, such as #23 in the Apply section, guide students with prompts, such as the following:
  - What do you need to find out?
  - What information are you given?
  - How do you express "$3 more" in mathematical terms?
- Ask students if they follow any local league sports. Challenge them to research the statistics for a local team, and then revisit the question, using the statistics they find in their research.
- Some teachers may prefer to introduce the Math Links for sections 5.1 and 5.2 together after section 5.2 has been completed. This would allow for more concentrated time on the problem. It also provides more of an opportunity to show the benefits of using algebra for substituting equivalent values in expressions or equations.

ELL

- After students have completed the Communicate the Ideas, make a list of terms that describe different mathematical operations. For example, terms that describe addition include *plus*, *sum*, *increase by*, *and*. Post this list in the classroom.
- Complete one part of each Practise question on the board as an example for the English language learners so that they know what the question is asking.
- For #25, have a student with hockey experience explain what a shoot-out is and what regulation time is.
- For #27, teach the following words in context: *penalty*, *incorrect*, and *advises*.

Gifted and Enrichment

- As students become more familiar with formulaic polynomials, ask them to create methods to test their reliability or to experiment to produce their own formulas. For example, they might test the formula, distance = speed × time, by rolling a ball or a model car.

Common Errors

- Some students may mistakenly identify red unit tiles as negative rather than positive.

R_x Remind students that in this chapter all negative tiles are represented in white, and all coloured tiles are positive.

- Some students may consider a polynomial as having more than three terms.

R_x Explain that a polynomial has one or more terms, but that special names are given to polynomials with one, two, or three terms: *monomial*, *binomial*, and *trinomial*, respectively.

Answers

Communicate the Ideas

1. Example: The polynomial is a binomial because it has two terms. It is degree 2 because the term with the highest degree is x^2 and its degree is 2. It contains the variable x, an exponent of 2, and a constant of -5.

2. Myron is correct because the white tiles represent negative values.

3. Example: $xy + x$ and $5a^2 + b$

4. Example: When the coefficient of a term is 1, the 1 can be omitted from the term. For example, $1y$ can be written as y. When the exponent of a variable or number is 1, the exponent can be omitted. For example, x^1 can be written as x. The 1 must be written when it represents a constant. For example, $8y + 1$.

Math Link

a) Example:

Item Name	Number	Number	Number	Number	Number	Number
Blender ($23)				1	1	
Stop Watch ($17)	4	1	2	3	2	2
Book ($8)	4	2			2	
Ball ($13)		1	2	2		3
Drum ($40)			1			
Coffeemaker ($27)		2			1	1
Total	100	100	100	100	100	100

b) Example: $4w + 4b$, where w represents the number of watches and b represents the number of books. $4w + 4b = 100$

c) No, there is no combination of all different items that will give a total of $100.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #2–4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Check each student's answer to #2 to ensure that they understand the naming of different algebra tiles and can relate the tiles to the correct algebra symbols. • #3 is an important question to determine whether students can apply their understanding of the terminology to generate their own examples. This would be an excellent question to include in the students' Foldable as an assessment for learning, and as additional examples for the students' future reference. • Question # 4 is very important. If students struggle with this question, provide examples of polynomials with an exponent of 1, a coefficient of 1, and, finally, a constant term with a value of 1. • You may consider providing students with **Master 2 Communication Peer Evaluation** to help them provide feedback to one another on their responses to these questions.
Assessment *for* Learning	
Practise and Apply Have students do questions #5, 7, 8, 12, 13, 15, 18, 20, 23. Students who have no problems with these questions can go on to the remaining Apply questions.	• If students have difficulty with questions #5, 7, 8, or 10, it is likely a terminology problem. Review definitions and have students give examples for each term. Coach students through #5, 8 and 12, and then have them complete #6, 9, and 11 on their own. • Provide additional coaching with Example 3 for students who require help with #12 or 13. Ensure that students know the symbolic representation for each algebra tile used in this chapter. • Help students with #23 by getting them to explain what each coefficient and variable represents in the expression $20a + 15c$.
Math Link The Math Link on page 182 is intended to help students work toward the Wrap It Up! on page 203.	• Students who need help getting started could use **BLM 5-6 Section 5.1 Math Link**, which provides scaffolding. • Show students one combination of items whose cost is exactly $100. Encourage students to find other combinations and when someone does, have this student explain how the combination was found. This will help other students understand the thinking involved. • For part b), encourage students to identify what each variable represents.
Assessment *as* Learning	
Literacy Link (page 171) After completing section 5.1 of the unit, students should be able to fill in the first five ovals of the concept map: *algebra*, *polynomial*, *term*, *degree of a term*, and *degree of a polynomial*.	• You may want to model how the organizer works by discussing with the class how to complete the first oval (algebra). You may also want to help students complete the polynomial oval. Encourage students to attach three ovals to it: *monomial*, *binomial*, and *trinomial*.
Math Learning Log Have students respond to the following prompts: • Use examples to show similarities and differences between the degree of a term and the degree of a polynomial. • Create a polynomial and model it using algebra tiles or diagrams.	• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. • As a class, elicit examples of terms and polynomials. Ask what the degree is for each one. • Encourage concrete and kinesthetic learners to use manipulatives, and then to draw diagrams, to show a polynomial.