Equivalent Expressions

5.2

***MathLinks 9*, pages 183–189**

Suggested Timing
100–120 minutes

Materials
- concrete materials, such as algebra tiles

Blackline Masters
Master 6 Square Dot Paper
Master 7 Isometric Dot Paper
Master 8 Centimetre Grid Paper
Master 9 0.5 Centimetre Grid Paper
Master 11 Algebra Tiles (Positive Tiles)
Master 12 Algebra Tiles (Negative Tiles)
BLM 5–3 Chapter 5 Warm-Up
BLM 5–7 Section 5.2 Extra Practice
BLM 5–8 Section 5.2 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

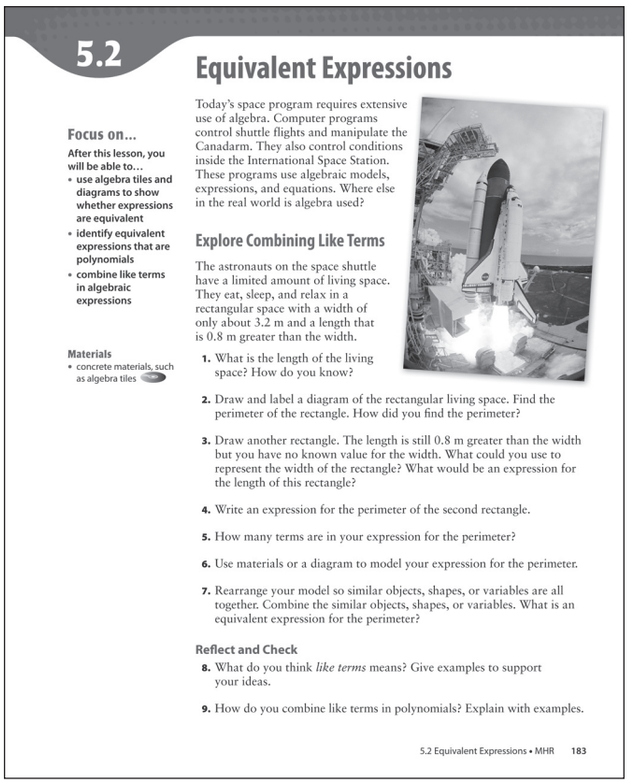
Specific Outcomes
PR6 Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2).

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, either 2 or 3, 4, 6, 8, 10, 12, 16, 17, 21, Math Link
Typical	#1, 2, 4, 6–8, 11–13, 15–17, 19–22, Math Link
Extension/Enrichment	#1–4, 18, 23–25

Planning Notes

Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce material learned in previous sections.

After reading the opening paragraph with the class, discuss how algebraic expressions might be used in the space program. Then, have the class brainstorm where else in the real world algebraic expressions are used.



5.2

Equivalent Expressions

Focus on...

After this lesson, you will be able to...
- use algebra tiles and diagrams to show whether expressions are equivalent
- identify equivalent expressions that are polynomials
- combine like terms in algebraic expressions

Materials
- concrete materials, such as algebra tiles

Today's space program requires extensive use of algebra. Computer programs control shuttle flights and manipulate the Canadarm. They also control conditions inside the International Space Station. These programs use algebraic models, expressions, and equations. Where else in the real world is algebra used?

Explore Combining Like Terms

The astronauts on the space shuttle have a limited amount of living space. They eat, sleep, and relax in a rectangular space with a width of only about 3.2 m and a length that is 0.8 m greater than the width.

1. What is the length of the living space? How do you know?
2. Draw and label a diagram of the rectangular living space. Find the perimeter of the rectangle. How did you find the perimeter?
3. Draw another rectangle. The length is still 0.8 m greater than the width but you have no known value for the width. What could you use to represent the width of the rectangle? What would be an expression for the length of this rectangle?
4. Write an expression for the perimeter of the second rectangle.
5. How many terms are in your expression for the perimeter?
6. Use materials or a diagram to model your expression for the perimeter.
7. Rearrange your model so similar objects, shapes, or variables are all together. Combine the similar objects, shapes, or variables. What is an equivalent expression for the perimeter?

Reflect and Check
8. What do you think *like terms* means? Give examples to support your ideas.
9. How do you combine like terms in polynomials? Explain with examples.

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The Canadarm is a piece of technology developed in Canada for NASA. It is used to manipulate objects outside the space shuttle, and to move objects into or out of the cargo bay. Despite the fact that it only weighs about 450 kg and cannot support itself in normal gravity, in space it can move objects that are nearly 30 000 kg, to any position with an accuracy of about 5 cm.

Explore Combining Like Terms

In this exploration, students examine the perimeter of the living space in the space shuttle, using arithmetic and algebra techniques. Students should be able to work their way through this Explore either individually or in small groups. In this activity, students express perimeter using:
- arithmetic operations
- algebraic expressions
- algebra-tile models

Method 1 Complete #1 to 3 as a class, eliciting all responses from students. Students should then complete #4 to 7 individually, since these questions are related to the concepts learned in section 5.1. Have students discuss #8 and 9 with a partner.

Method 2 Have students work in pairs to complete #1 to 7 of the Explore. Then, have each group contribute to a class discussion of questions #8 and 9. Expand the discussion to include related mathematical terminology from section 5.1: *term*, *coefficient*, *variable*, *exponent*, *monomial*, and *polynomial*. You may wish to ask questions such as:

- What is an algebraic expression? Give an example.
- How many terms are in your expression?
- In the expression $3x + 4$, how many terms are there? What algebra tiles may you use to model the expression?

Meeting Student Needs

- It may be better for your class to work through the Explore as a whole-class activity.
- Have students research the fur trade, either on the Internet or by speaking with an Elder in the Aboriginal community, if possible. Discuss with students how items such as beaver, muskrat, rabbit, wolf, and other pelts were equivalent in value to other trading items.

ELL

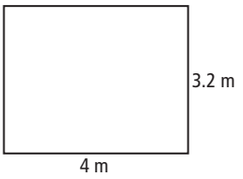
- Teach the following terms in context: *extensive*, *shuttle flights*, *greater*, *width*, *equivalent*, and *rearrange*.
- Work through the steps of the Explore with English language learners, ensuring they understand each one before they move on.

Answers

Explore Combining Like Terms

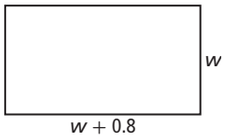
1. 4 m, because $3.2 + 0.8 = 4.0$

2.



Example: To find the perimeter, multiply 4 by 2, multiply 3.2 by 2, and add the two products: $2 \times 4 + 2 \times 3.2 = 14.4$ m.

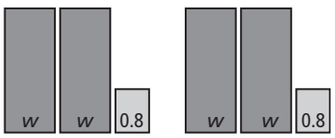
3. Example: The width could be represented by the variable w. The expression representing the length would be $w + 0.8$.



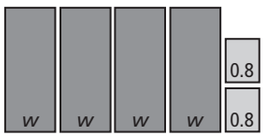
4. Example: $w + w + 0.8 + w + w + 0.8$

5. Answer is dependent on the answer to #4. Example: Six terms

6. Example:



7. Example:

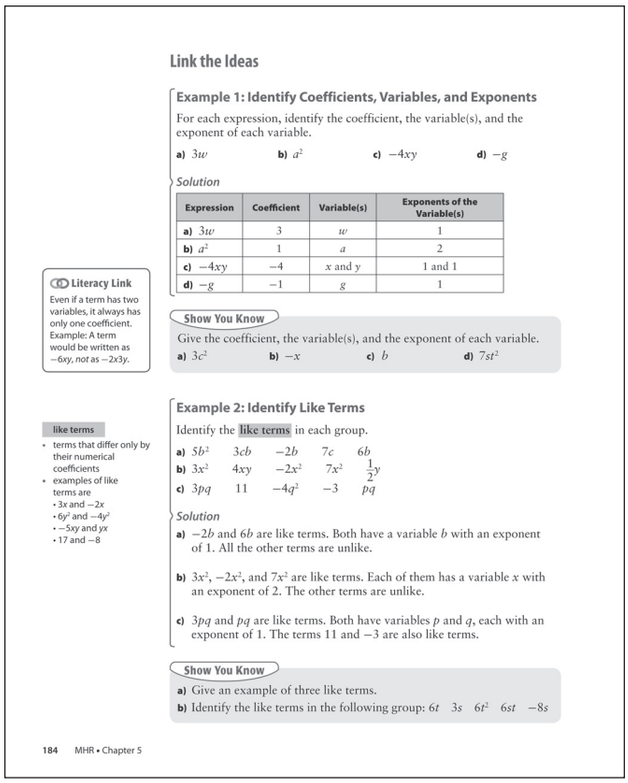


An equivalent expression for the perimeter is $4w + 1.6$.

8. Example: Like terms are terms that can be modelled by tiles of the same shape and size. For example, $2x$ and $3x$ are like terms.

9. Example: To combine like terms, you add the coefficients. For example, $2x + 3x = 5x$.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Have students generalize their findings. Check their responses for understanding.	• Encourage students to write symbolic expressions for each concrete model they build. Always match symbols to models. • Clarify different ways to find the perimeter of a rectangle. Ask which method might work best in this situation. • Emphasize that there are several ways to approach most problems. Have students compare what they have done with what other students did. • You may wish to provide students with **Master 6 Square Dot Paper**, **Master 7 Isometric Dot Paper**, **Master 8 Centimetre Grid Paper**, or **Master 9 0.5 Centimetre Grid Paper** for drawing their diagrams in the Explore.



Link the Ideas

Example 1: Identify Coefficients, Variables, and Exponents

For each expression, identify the coefficient, the variable(s), and the exponent of each variable.

a) $3w$ b) a^2 c) $-4xy$ d) $-g$

Solution

Expression	Coefficient	Variable(s)	Exponents of the Variable(s)
a) $3w$	3	w	1
b) a^2	1	a	2
c) $-4xy$	-4	x and y	1 and 1
d) $-g$	-1	g	1

Literacy Link
Even if a term has two variables, it always has only one coefficient. Example: A term would be written as $-6xy$, *not* as $-2x3y$.

Show You Know
Give the coefficient, the variable(s), and the exponent of each variable.
a) $3c^2$ b) $-x$ c) b d) $7st^2$

Example 2: Identify Like Terms

Identify the like terms in each group.

a) $5b^2$ $3cb$ $-2b$ $7c$ $6b$
b) $3x^2$ $4xy$ $-2x^2$ $7x^2$ $\frac{1}{2}y$
c) $3pq$ 11 $-4q^2$ -3 pq

like terms
- terms that differ only by their numerical coefficients
- examples of like terms are
  - $3x$ and $-2x$
  - $6y^2$ and $-4y^2$
  - $-5xy$ and yx
  - 17 and -8

Solution

a) $-2b$ and $6b$ are like terms. Both have a variable b with an exponent of 1. All the other terms are unlike.

b) $3x^2$, $-2x^2$, and $7x^2$ are like terms. Each of them has a variable x with an exponent of 2. The other terms are unlike.

c) $3pq$ and pq are like terms. Both have variables p and q, each with an exponent of 1. The terms 11 and -3 are also like terms.

Show You Know
a) Give an example of three like terms.
b) Identify the like terms in the following group: $6t$ $3s$ $6t^2$ $6st$ $-8s$

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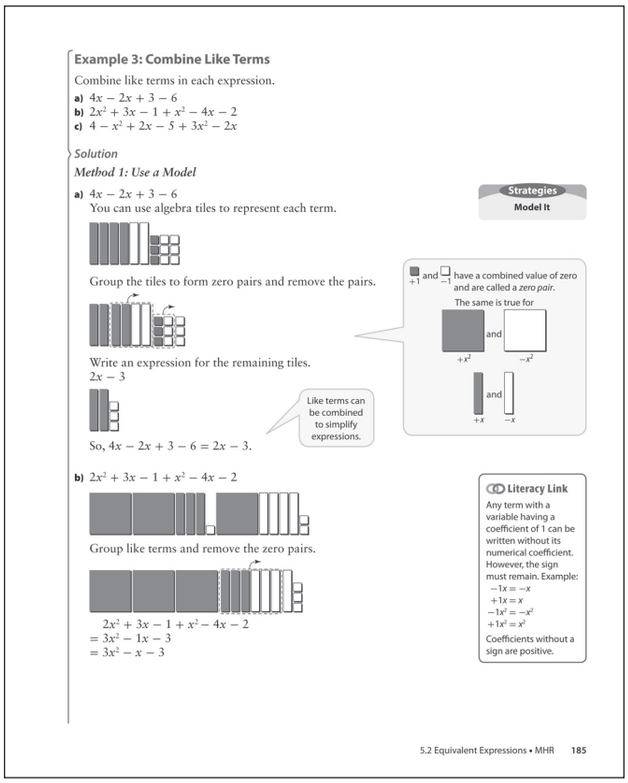

Example 3: Combine Like Terms

Combine like terms in each expression.

a) $4x - 2x + 3 - 6$
b) $2x^2 + 3x - 1 + x^2 - 4x - 2$
c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$

Solution

Method 1: Use a Model

a) $4x - 2x + 3 - 6$
You can use algebra tiles to represent each term.

Strategies
Model It

Group the tiles to form zero pairs and remove the pairs.

+1 and −1 have a combined value of zero and are called a *zero pair*.
The same is true for
$+x^2$ and $-x^2$
$+x$ and $-x$

Write an expression for the remaining tiles.
$2x - 3$

Like terms can be combined to simplify expressions.

So, $4x - 2x + 3 - 6 = 2x - 3$.

b) $2x^2 + 3x - 1 + x^2 - 4x - 2$

Group like terms and remove the zero pairs.

$2x^2 + 3x - 1 + x^2 - 4x - 2$
$= 3x^2 - 1x - 3$
$= 3x^2 - x - 3$

Literacy Link
Any term with a variable having a coefficient of 1 can be written without its numerical coefficient. However, the sign must remain. Example:
$-1x = -x$
$+1x = x$
$-1x^2 = -x^2$
$+1x^2 = x^2$
Coefficients without a sign are positive.

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Link the Ideas

Example 1

Have students try Example 1 based on their prior knowledge. For each expression in the example, you might ask the following:

- What number is in front of each term? What mathematical name is given to this number?
- What operation separates each number or variable in a term?
- What is another name for a one term expression?
- Which given terms are positive?

Literacy Link Draw students' attention to the Literacy Link on page 184. Remind students that each expression in Example 1 is a polynomial, but that it has only one term and could be called a monomial. Single-term expressions have only one coefficient, but may have one or more variables. For example, $8ab$ can be thought of as 8 multiplied by a, which is multiplied by b. It could also be written as $8ba$, but $8ab$ is preferred because variables are usually put in alphabetical order. This term cannot be written as $4a2b$.

Have students complete the Show You Know to ensure that they can identify coefficients and variables for single-term expressions.

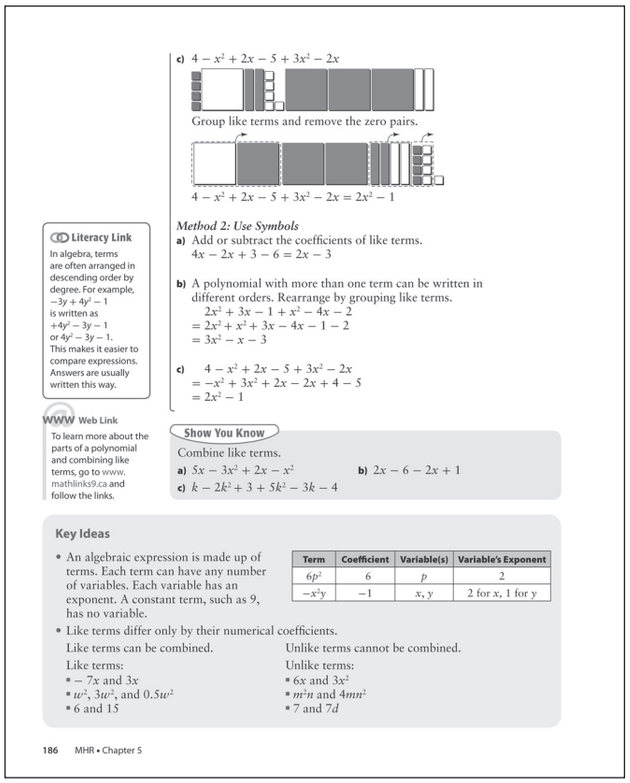
Example 2

Emphasize that like terms are determined by their variable or variables, and the exponents for the variables. Discuss that the variable(s) of a term and the exponents for the variable(s) must be identical for terms to be considered "like." Ensure that students understand the following:

- $7x$ and $5x$ are like terms whose variables are each x.
- $4xy$ and $5x$ are not like terms.
- $4xy$ and xy^2 are not like because one term has a variable y and the other a variable y^2. The fact that each expression contains the variable x does not make them like terms. Any difference in the variable or exponent will cause the terms to be unlike.
- $3xy^2$ and y^2x are considered like terms. Both contain the variables x and y^2, although not in the same order. Since the variables are separated by multiplication, y^2x can be expressed as xy^2.

It might work well to complete one part of Example 2 with the entire class and then have students attempt the remaining parts individually. Emphasize that they are expected to identify all the like terms in each group. Consider asking the following questions:

- How many terms are in each expression?
- What causes terms to be considered "like"?
- How many like terms are in part b)? What are they?



c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$

Group like terms and remove the zero pairs.

$4 - x^2 + 2x - 5 + 3x^2 - 2x = 2x^2 - 1$

Method 2: Use Symbols

a) Add or subtract the coefficients of like terms.
$4x - 2x + 3 - 6 = 2x - 3$

b) A polynomial with more than one term can be written in different orders. Rearrange by grouping like terms.
$2x^2 + 3x - 1 + x^2 - 4x - 2$
$= 2x^2 + x^2 + 3x - 4x - 1 - 2$
$= 3x^2 - x - 3$

c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$
$= -x^2 + 3x^2 + 2x - 2x + 4 - 5$
$= 2x^2 - 1$

Literacy Link
In algebra, terms are often arranged in descending order by degree. For example, $-3y + 4y^2 - 1$ is written as $+4y^2 - 3y - 1$ or $4y^2 - 3y - 1$. This makes it easier to compare expressions. Answers are usually written this way.

WWW Web Link
To learn more about the parts of a polynomial and combining like terms, go to www.mathlinks9.ca and follow the links.

Show You Know

Combine like terms.

a) $5x - 3x^2 + 2x - x^2$ **b)** $2x - 6 - 2x + 1$

c) $k - 2k^2 + 3 + 5k^2 - 3k - 4$

Key Ideas

- An algebraic expression is made up of terms. Each term can have any number of variables. Each variable has an exponent. A constant term, such as 9, has no variable.

Term	Coefficient	Variable(s)	Variable's Exponent
$6p^2$	6	p	2
$-x^2y$	-1	x, y	2 for x, 1 for y

- Like terms differ only by their numerical coefficients.

Like terms can be combined.
Like terms:
- $-7x$ and $3x$
- w^2, $3w^2$, and $0.5w^2$
- 6 and 15

Unlike terms cannot be combined.
Unlike terms:
- $6x$ and $3x^2$
- m^2n and $4mn^2$
- 7 and $7d$

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- How many terms in part c) are unlike all the others?
- What are the coefficient and variable(s) for the term pq?

Have students work on the Show You Know individually or with a partner. Encourage students to explain why their examples in part a) are considered like terms.

Example 3

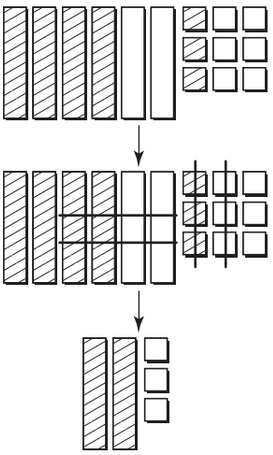
In Method 1, algebra tiles are used to model combining like terms to simplify expressions. In Method 2, simplification is completed using symbols and the addition/subtraction of like terms.

Have students model each question, checking their models against those of other students. Consider using questions such as the following:

- What colour represents positive x^2-tiles or positive x-tiles?
- Show me what $x^2 + x$ would look like using algebra tiles.
- What is the combined value of $+x$ and $-x$? Show this with tiles.
- What is the combined value of $2x$ and $+x$?
- What is the simplified value for $6x - 4x + 3x$?

It is important for all students to understand how to use manipulatives to represent equivalent expressions as this will set the foundation for understanding how to address equivalent expressions algebraically.

Remind student that they may want to use diagrams to model the equations. For example, part a) could be drawn as follows:



The black lines show cancelling of opposite values. Parts b) and d) can be similarly diagrammed.

Literacy Link Draw students attention to the Literacy Links found on pages 185 and 186.

The first Literacy Link shows how to express terms with coefficients of 1. Remind students that even though it appears a term has no coefficient, it has a coefficient of 1. For example, in $3xy + x$, the coefficient of the x-term is $+1$.

The second Literacy Link encourages students to use the convention of writing expressions in descending order by degree. For example $3x - 1 + x^2$ would be written in descending order as $x^2 + 3x - 1$. If students use this convention, it will be much easier to check and compare answers.

Have students complete the Show You Know using two methods for each part.

Key Ideas

The Key Ideas provide a summary of critical terminology from section 5.2. Students should not simply copy the Key Ideas as notes, but rather identify vocabulary, concepts, and methodology that are important to them. Have students record these items in their Foldable.

Meeting Student Needs

- It may be better for your class to work through the examples as a whole-class activity. You could then assign the Show You Know activity as a small-group or pairs activity.
- It may be beneficial to complete part a) of Example 3 using both Method 1 and Method 2 before proceeding to part b), and then follow the same process with part b) before moving on to part c). This approach will provide a closer connection between the model and the algebraic symbols.
- In Example 3, Method 2, some students may need to be reminded that $+2 - 2$ is the same as $+2 + (-2) = 0$. Ask them, "How does knowing this help you to know that $+2x - 2x = 0x$ or 0?
- Encourage all students to use concrete materials and diagrams. This will strengthen their understanding of combining like terms and will encourage students who may experience difficulty to use materials.
- When students move from algebra tiles to solving an expression algebraically, they may need to be reminded that when grouping like terms, the sign in front of a coefficient determines the sign of the term. For example, $3x - 6x = 3x + (-6x)$.
- Some students will benefit from changing all subtraction signs to adding the opposite value.
- In the Show You Know, you may want to ask students if they would use algebra tiles differently when the variable is k instead of x?

Gifted and Enrichment

- Ask students to show that unlike terms cannot be added and to prove their thinking. Further, challenge them to find situations where adding unlike terms apparently produces a correct result and to explain what that means.

Common Errors

- Some students may apply rules incorrectly for adding or subtracting integers.

R_x Have students work with concrete materials such as two colour counters to correct their understanding of the operations. Then, follow up with practice exercises.

- Some students may combine unlike terms when simplifying expressions.

R_x Stress that only like terms can be combined using addition or subtraction. Have students explain, in their own words, the difference between like and unlike terms. It may be helpful to use an analogy involving family names to ensure students understand the concept of like terms.

Answers

Example 1: Show You Know

a) coefficient: 3; variable: c; exponent: 2

b) coefficient: -1; variable: x; exponent: 1

c) coefficient: 1; variable: b; exponent: 1

d) coefficient: 7; variables: s and t; exponent: 1 and 2

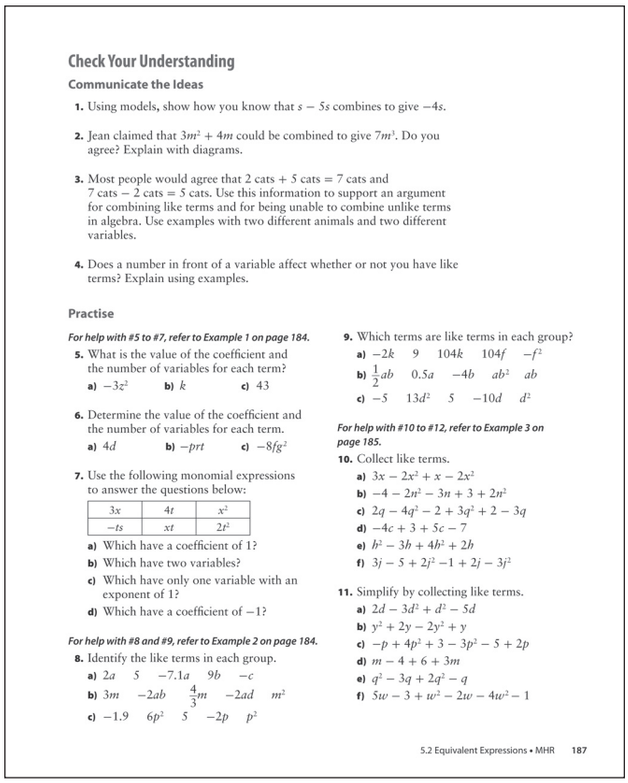
Example 2: Show You Know

a) Examples: $9y$, $-y$, and $16y$ **b)** $3s$ and $-8s$

Example 3: Show You Know

a) $-4x^2 + 7x$ **b)** -5 **c)** $3k^2 - 2k - 1$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • For variation, give students the values of the coefficient, variables, and exponents for each variable, and have them generate the monomial expression. • Some students may benefit from first circling the coefficient and the variable in each term before responding. Struggling visual learners may do well to use a convention of circling coefficients and underlining variables until such time that they can simply respond by looking at the terms.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Give students one term and have them create a like term. • Remind students that understanding like terms is critical in the study of algebra and that they may wish to keep examples and non-examples in their Foldable. • Some students may benefit from underlining the variables that are matched in like terms. Struggling visual learners may do well to use a convention of circling coefficients and underlining variables until such time that they can simply respond by looking at the terms. • Indicate that part a) of the Show You Know is different from Example 2 and is intended to deepen their understanding.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students explain the connection between algebra tiles or diagrams, and the terms of an expression. • Encourage students to model the questions using actual algebra tiles. If algebra tiles are not available, provide students with **Master 11 Algebra Tiles (Positive Tiles)** and **Master 12 Algebra Tiles (Negative Tiles)**. Ensure that all students understand the relationship between the algebra-tile model and the symbolic approach. • It is important that students learn that the algebra tiles can represent any variable, so it may be beneficial to discuss the response to part c) as a class. • Ask students to show with materials why $5x - 3x = 2x$ and $5 - 9 = -4$.



Check Your Understanding

Communicate the Ideas

1. Using models, show how you know that $s - 5s$ combines to give $-4s$.

2. Jean claimed that $3m^2 + 4m$ could be combined to give $7m^3$. Do you agree? Explain with diagrams.

3. Most people would agree that 2 cats + 5 cats = 7 cats and 7 cats − 2 cats = 5 cats. Use this information to support an argument for combining like terms and for being unable to combine unlike terms in algebra. Use examples with two different animals and two different variables.

4. Does a number in front of a variable affect whether or not you have like terms? Explain using examples.

Practise

For help with #5 to #7, refer to Example 1 on page 184.

5. What is the value of the coefficient and the number of variables for each term?
a) $-3z^2$ **b)** k **c)** 43

6. Determine the value of the coefficient and the number of variables for each term.
a) $4d$ **b)** $-prt$ **c)** $-8fg^2$

7. Use the following monomial expressions to answer the questions below:

$3x$	$4t$	x^2
$-ts$	xt	$2t^2$

a) Which have a coefficient of 1?
b) Which have two variables?
c) Which have only one variable with an exponent of 1?
d) Which have a coefficient of −1?

For help with #8 and #9, refer to Example 2 on page 184.

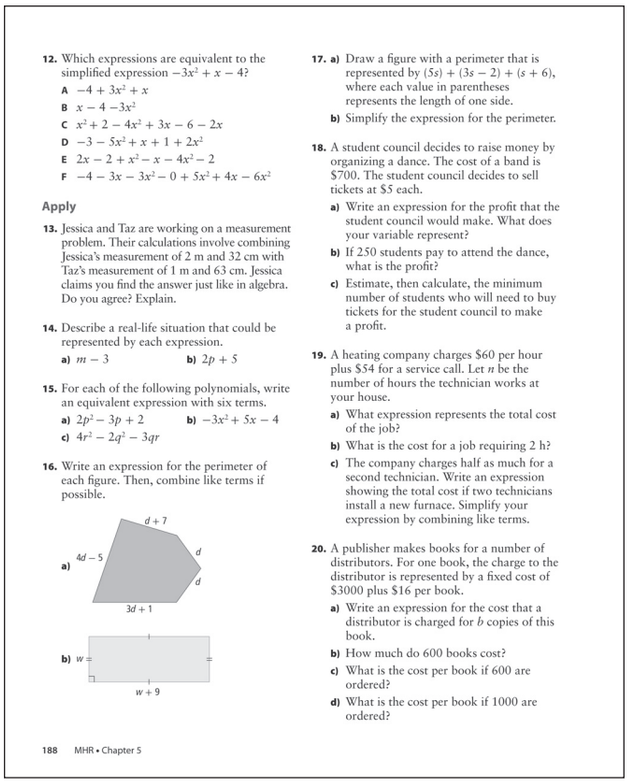
8. Identify the like terms in each group.
a) $2a$ 5 $-7.1a$ $9b$ $-c$
b) $3m$ $-2ab$ $\frac{4}{3}m$ $-2ad$ m^2
c) -1.9 $6p^2$ 5 $-2p$ p^2

9. Which terms are like terms in each group?
a) $-2k$ 9 $104k$ $104f$ $-f^2$
b) $\frac{1}{2}ab$ $0.5a$ $-4b$ ab^2 ab
c) -5 $13d^2$ 5 $-10d$ d^2

For help with #10 to #12, refer to Example 3 on page 185.

10. Collect like terms.
a) $3x - 2x^2 + x - 2x^2$
b) $-4 - 2n^2 - 3n + 3 + 2n^2$
c) $2q - 4q^2 - 2 + 3q^2 + 2 - 3q$
d) $-4c + 3 + 5c - 7$
e) $b^2 - 3b + 4b^2 + 2b$
f) $3j - 5 + 2j^2 - 1 + 2j - 3j^2$

11. Simplify by collecting like terms.
a) $2d - 3d^2 + d^2 - 5d$
b) $y^2 + 2y - 2y^2 + y$
c) $-p + 4p^2 + 3 - 3p^2 - 5 + 2p$
d) $m - 4 + 6 + 3m$
e) $q^2 - 3q + 2q^2 - q$
f) $5w - 3 + w^2 - 2w - 4w^2 - 1$



12. Which expressions are equivalent to the simplified expression $-3x^2 + x - 4$?
A $-4 + 3x^2 + x$
B $x - 4 - 3x^2$
C $x^2 + 2 - 4x^2 + 3x - 6 - 2x$
D $-3 - 5x^2 + x + 1 + 2x^2$
E $2x - 2 + x^2 - x - 4x^2 - 2$
F $-4 - 3x - 3x^2 - 0 + 5x^2 + 4x - 6x^2$

Apply

13. Jessica and Taz are working on a measurement problem. Their calculations involve combining Jessica's measurement of 2 m and 32 cm with Taz's measurement of 1 m and 63 cm. Jessica claims you find the answer just like in algebra. Do you agree? Explain.

14. Describe a real-life situation that could be represented by each expression.
a) $m - 3$ **b)** $2p + 5$

15. For each of the following polynomials, write an equivalent expression with six terms.
a) $2p^2 - 3p + 2$ **b)** $-3x^2 + 5x - 4$
c) $4r^2 - 2q^2 - 3qr$

16. Write an expression for the perimeter of each figure. Then, combine like terms if possible.

a)

b)

17. **a)** Draw a figure with a perimeter that is represented by $(5s) + (3s - 2) + (s + 6)$, where each value in parentheses represents the length of one side.
b) Simplify the expression for the perimeter.

18. A student council decides to raise money by organizing a dance. The cost of a band is $700. The student council decides to sell tickets at $5 each.
a) Write an expression for the profit that the student council would make. What does your variable represent?
b) If 250 students pay to attend the dance, what is the profit?
c) Estimate, then calculate, the minimum number of students who will need to buy tickets for the student council to make a profit.

19. A heating company charges $60 per hour plus $54 for a service call. Let n be the number of hours the technician works at your house.
a) What expression represents the total cost of the job?
b) What is the cost for a job requiring 2 h?
c) The company charges half as much for a second technician. Write an expression showing the total cost if two technicians install a new furnace. Simplify your expression by combining like terms.

20. A publisher makes books for a number of distributors. For one book, the charge to the distributor is represented by a fixed cost of $3000 plus $16 per book.
a) Write an expression for the cost that a distributor is charged for b copies of this book.
b) How much do 600 books cost?
c) What is the cost per book if 600 are ordered?
d) What is the cost per book if 1000 are ordered?

Check Your Understanding

Communicate the Ideas

Have students work in pairs to answer the assigned questions.

In #1, students are asked to use models to show that $s - 5s = -4s$. An incorrect answer could be an indication that students are having difficulty adding/subtracting integers.

In #2, students are asked to agree or disagree with a student answer. This question serves as a check to see if students understand combining like terms.

In #3, students are to use animals and variables to support the fact that like terms can be combined and to show that unlike terms cannot be combined.

In #4, students explain whether the number in front of a variable (the coefficient) is of any significance when determining if terms are like. This question helps to show whether students have an understanding of like terms.

Practise

The following pairs of questions are similar, but with slightly different wording: #5 and 6, #8 and 9, and #10 and 11. Students should be given a choice of one question from each pair of questions.

Students may work individually on these Practise questions and then check their answers in the back of the student resource, or they may work with a partner.

Apply

Questions #13 to 17 and #21 require students to use their terminology and understanding of concepts and skills from section 5.2 in a variety of ways.

Questions #18 to 20 and #22 provide real-world contexts.

Extend

In #23, students must realize that y must equal w, and then demonstrate why.

Question # 24, part b), challenges students to find the answer two ways. The two methods could include following the pattern in the table and finding 40% of $x + 10$.

Literacy Link At the end of section 5.2, have students fill in the oval labelled *like terms* on their concept map. Brainstorm and discuss as a class the information needed to complete this oval.

21. Raj was told to write an expression equivalent to $3x - 8 - 5x + 9$.

- $3x - 8 - 5x + 9$
- $3x - 5x - 8 + 9$
- $2x - 1$

a) What errors did he make?

b) Show the correct response.

22. The diagram represents a piece of string.

a) What is an expression for the total length of the string?

b) Combine like terms to get the simplest expression possible for the length of the string.

Extend

23. When would the expressions $x + y + 3$ and $x + w + 3$ be equal? How do you know?

24. A department store marks up wholesale prices 40% to get its retail or selling price.

a) Complete the following table. The first row has been done for you.

Wholesale Price ($)	Expression For Retail Price	Retail Price ($)
8.00	8 + (0.4)(8)	11.20
12.00		
30.00		
x		

b) How could you find the retail price if the wholesale price is $x + 10$ dollars? Show two ways to find the answer.

25. Zip Publishers will print posters for fundraising events for an initial cost of $100 plus $2 per poster. Henry's Printers charges $150 plus $1 per poster.

a) Write an expression for each company, showing the total cost for any number of posters.

b) What is the cost of 125 posters from each company?

c) What is the total cost if you print 200 posters at each company? Show two different ways to find the answer.

Math Link

Refer to the Math Link for section 5.1 on page 182. Represent each item with a variable:

a = blender, b = watch, c = book, d = soccer ball, e = drum, f = coffeemaker

a) Rewrite all your combinations that add to $100, using the letters a to f. Arrange each combination in alphabetical order. For example, $a + e + d + 3c$ would be written as $a + 3c + d + e$.

b) The example in part a) can be used to find other combinations. Notice that e has a value of $40. What other items from the list have a value of $40? By substituting into e the letter or letters that combine to a total value of $40, you arrive at another answer. Do not forget to combine like terms and arrange each expression in alphabetical order. What other combinations can you find using substitution?

c) If you were asked to find combinations of the items that add to $101, how could you use algebra to help you? Give two ways that algebra could help you.

Math Link

This Math Link is a continuation of the one from section 5.1. It is designed to show how to use algebra to determine combinations of prices that add to exactly $100. It provides an opportunity to use substitution and addition/subtraction of polynomials to find other correct combinations.

In part b), students need to find combinations with a value of $40. These are $a + b$, $5c$, or $d + f$. Each of these can be substituted for e in $a + 3c + d + e$ to get

$a + 3c + d + (a + b) = 2a + b + 3c + d$

$a + 3c + d + (5c) = a + 8c + d$

$a + 3c + d + (d + f) = a + 3c + 2d + f$

Other useful combinations are

$b + c = 25$, so $4b + 4c = 100$

$a + f = 50$, so $2a + 2f = 100$

Give students some time to explore winning combinations, reminding them there are at least 19 combinations that total to $100. Then, ask different students to explain their thinking in finding a correct combination.

Meeting Student Needs

- Provide **BLM 5–7 Section 5.2 Extra Practice** to students who would benefit from more practice.
- Give students as much variety and choice as possible to encourage them to take responsibility for their learning. Variety can provide differentiated learning opportunities.
- All students should attempt the Math Link as it is a fast way for them to explore combining like terms.
- Students may work in small groups to see if they can find all the combinations totalling $100.
- You may wish to reword some of the Apply questions to make them more relevant to the lives of your students. For example, #18 could be reworded to put it into the context of a powwow: Some First Nations communities hold celebrations, called powwows. People from all directions come to dance, drum, and visit. For one powwow, the cost of running the event for the day would be $700. Each dancer will be charged a registration fee of $5.

ELL

- Teach the following terms in context: *wholesale price*, *retail price*, and *posters*.
- If it seems necessary, model for English language learners the first part of each Practise question to get them started and to help them understand what each question is asking.
- For #19, define the term *service call*. Also, help students to understand what is meant by *half as much for a second technician*.

Common Errors

- Some students may make mistakes when substituting letters into an expression in the Math Link.

R_x Encourage students to check each combination to see if it generates a total of $100.

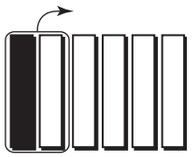
- Some students may combine all terms with the same variables.

R_x Remind students that like terms require two conditions. First, they must have the same variables. Second, the variables must have the same exponents. For example, $3xy$ and $-2xy$ are like terms each with two variables, x and y, both having exponents of 1 (x^1y^1). On the other hand, $3xy$ and $3x^2y$ are not like since the variable x has exponents of 1 and 2, respectively.

Answers

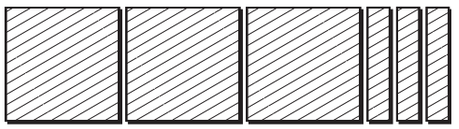
Communicate the Ideas

1. Example:



Remove one zero model. -4s remains.

2.



Example: No, they cannot be combined. The m^2-tiles and m-tiles cannot be combined because they have a different size and shape.

3. Example: 2 cats + 5 dogs represent a statement with two different animals. If c represents cats and d represents dogs, the expression is $2c + 5d$. These two terms are unlike terms and cannot be combined.

4. No. Example: The definition of like terms is that they are terms that differ only by their coefficients. Therefore, the coefficients do not affect whether or not terms are like terms. For example, $3x$ and $-2x$ have different coefficients, but they are like terms.

Math Link

a) Answers are dependent on Math Links on page 189.
Example: $4b + 4c$, $b + 2c + d + 2f$, $2b + 2d + e$, $a + 3b + 2d$, $a + 2b + 2c + f$, $2b + 3d + f$

b) Example: Take the expression $2b + 2d + e$. Since $e = a + b$, substitute to get $2b + 2d + a + b$, which simplifies to $a + 3b + 2d$. Take this expression and replace a with $e - b$. This becomes: $e - b + 3b + 2d$, which simplifies to $2b + 2d + e$.

c) Example: Find an expression that adds up to 101, and replace some variables for their equivalents as done in b), above. Then, combine like terms to obtain new expressions.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1, either 2 or 3, and 4. Check for understanding by having students explain their solutions to each question.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Check # 1 to ensure students can demonstrate combining two like terms, one of which is negative. If students have difficulty, they should be given similar questions to complete using both models and symbols. • If students seem unable to get started, give specific examples and ask what bearing the coefficients have on the terms being "like."
Assessment *for* Learning	
Practise and Apply Have students do #6, 10, 12, 16, 17, 21. Students who have no problems with these questions can go on to the remaining Apply questions.	• If students have difficulty with #6, coach them on identifying coefficients and variables. Then, have them complete #5 and #7 on their own. • If students struggle with #8, have them show what they believe *like terms* means. Correct any misconceptions and have students complete #9. • If students have difficulty with #10, check to see if the difficulty is due to a lack of understanding like terms or an inability to add/subtract integers. Allow students to use algebra tiles, or another similar form of manipulative, as needed. After some work on the area(s) of difficulty, assign #11. • For #12, students may be successful in simplifying the expressions, but may make sign errors in writing the final expression. It may be beneficial to review the effects the operational sign has on a term that is being rewritten. Use $4 - 3x$ as an example (rewrites to $-3x + 4$). • Encourage students to write their expression in #16 in a similar format as #17. Show them how brackets can be used to describe a side of a polygon and to help organize their thinking.
Math Link The Math Link on page 189 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 207.	• Students who need help getting started could use **BLM 5–8 Section 5.2 Math Link**, which provides scaffolding. • Show students that algebra can help them organize their expressions for items that have a total value of $100. Have all students use the same variable for each item and then arrange each expression in alphabetical order. • Show students that substitution and combining like terms may be useful for finding other combinations that add to $100.
Assessment *as* Learning	
Literacy Link (page 171) At the end of section 5.2, have students work in pairs to complete the next oval on the concept map labelled like terms.	• Have students add a definition and examples of *like terms*. Encourage students to write the definition in their own words and provide examples other than the ones in the student resource.
Math Learning Log Have students respond to the following prompts: • The mathematical terminology I can use with the expression $-4x^2$ is … • When I combine like terms, the method that I prefer is…	• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. • Have students present answers in oral and written form. Then, have them indicate which method they prefer and why. • Help students who need assistance by having them select an example from section 5.2 that involves combining like terms.