Adding and Subtracting Polynomials

5.3

***MathLinks 9*, pages 190–199**

Suggested Timing
100–120 minutes

Materials
- concrete materials, such as algebra tiles

Blackline Masters
Master 11 Algebra Tiles (Positive Tiles)
Master 12 Algebra Tiles (Negative Tiles)
BLM 5–3 Chapter 5 Warm-Up
BLM 5–9 Section 5.3 Extra Practice
BLM 5–10 Section 5.3 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

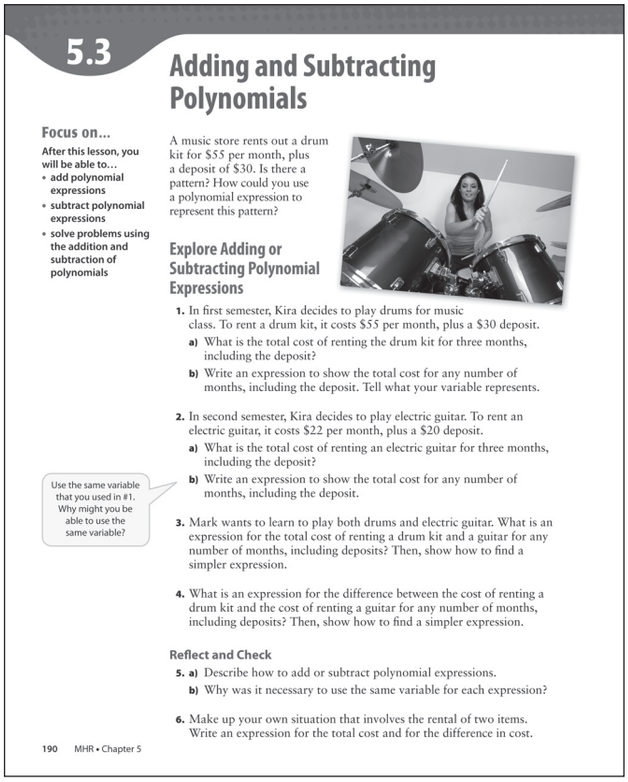
Specific Outcomes
PR6 Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2).

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 6, 8, 10, 12, 14, 16, 17, 18, Math Link
Typical	#1–3, 5, 6, 8, 10, 12, 14, 16–20, 24, Math Link
Extension/Enrichment	#1–4, 13, 19, 20, 22, 23, 26–30

Planning Notes

Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce material learned in previous sections.

As a class, read the opening paragraph for this section. Ask students what they know about patterns and how patterns can be represented by expressions. Encourage students to recall the work they did in grade 8 with linear equations.



5.3 Adding and Subtracting Polynomials

Focus on…
After this lesson, you will be able to…
- add polynomial expressions
- subtract polynomial expressions
- solve problems using the addition and subtraction of polynomials

A music store rents out a drum kit for $55 per month, plus a deposit of $30. Is there a pattern? How could you use a polynomial expression to represent this pattern?

Explore Adding or Subtracting Polynomial Expressions

1. In first semester, Kira decides to play drums for music class. To rent a drum kit, it costs $55 per month, plus a $30 deposit.
   a) What is the total cost of renting the drum kit for three months, including the deposit?
   b) Write an expression to show the total cost for any number of months, including the deposit. Tell what your variable represents.
2. In second semester, Kira decides to play electric guitar. To rent an electric guitar, it costs $22 per month, plus a $20 deposit.
   a) What is the total cost of renting an electric guitar for three months, including the deposit?
   b) Write an expression to show the total cost for any number of months, including the deposit.

Use the same variable that you used in #1. Why might you be able to use the same variable?

3. Mark wants to learn to play both drums and electric guitar. What is an expression for the total cost of renting a drum kit and a guitar for any number of months, including deposits? Then, show how to find a simpler expression.
4. What is an expression for the difference between the cost of renting a drum kit and the cost of renting a guitar for any number of months, including deposits? Then, show how to find a simpler expression.

Reflect and Check
5. a) Describe how to add or subtract polynomial expressions.
   b) Why was it necessary to use the same variable for each expression?
6. Make up your own situation that involves the rental of two items. Write an expression for the total cost and for the difference in cost.

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Explore Adding or Subtracting Polynomial Expressions

In this exploration, students use arithmetic and algebraic operations to solve problems to do with renting musical instruments. Students are required to add and subtract polynomial expressions. Students are not told what methods to use. However, students might model their expressions using algebra tiles and then combine like terms to simplify their expressions.

Method 1 Complete #1 as a class, eliciting responses from students. Then, have students work in small groups to complete #2 to 6. Have each small group present their findings relating to #6.

Method 2 Have students work in pairs. Circulate as students work on this Explore. If students are having difficulty, you may wish to ask:
- What variable did you use in #1? What does your variable represent? (Students should indicate that their variable represents the number of months the drum kit is rented.)

- What arithmetic operations do you need to find the answer in #1a) or #2a)?
- What is the thought bubble advising you to do? Why can you do this?
- Do the expressions in #1b) and 2b) use the same operations? Show me where/how?
- Why do you think models (algebra tiles) were not required in the Explore? (A large quantity of algebra tiles would be required.)
- In #4, what is the difference in the deposit for drums and the guitar? What is the difference between monthly charges? How do these differences help you find a simple expression for the difference?

Meeting Student Needs

- It may be better for your class for you to work through the Explore as a whole-class activity.

ELL

- Teach the following terms in context: *rents*, *deposit*, and *electric guitar*.
- Encourage students to write their answers in their first language to activate prior knowledge, and then to translate into English.

Answers

Explore Adding or Subtracting Polynomial Expressions

1. a) \$195 **b)** $55n + 30$, where n represents the number of months

2. a) \$86 **b)** $22n + 20$

3. $(55n + 30) + (22n + 20) = (55n + 22n) + (30 + 20) = 77n + 50$

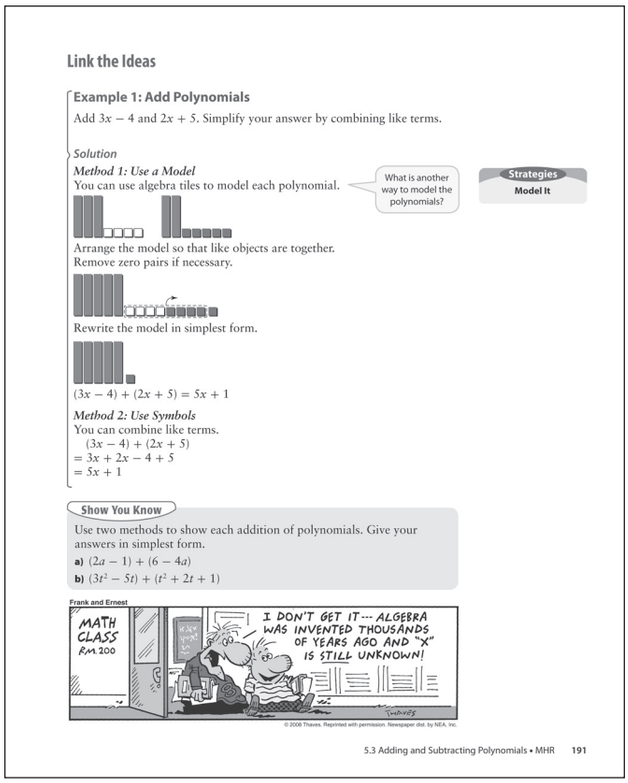
4. $(55n + 30) - (22n + 20) = (55n - 22n) + (30 - 20) = 33n + 10$

5. a) Add or subtract the coefficients of the like terms, and add or subtract the constants.

b) Example: Without the same variable, there would not be any like terms containing variables to simplify.

6. Example: A motorboat rents for \$200/h plus a \$500 deposit. A canoe rents for \$35/h plus a \$20 deposit. The total cost of renting both the motorboat and the canoe is
$(200n + 500) + (35n + 20) = 235n + 520$.
The difference in cost for renting a motorboat and a canoe is
$(200n + 500) - (35n + 20) = 165n + 480$.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss #4 in the Explore, and encourage them to generalize their findings. Check responses for conceptual understandings.	• Help students explore the concept using other items that could be rented such as a stereo system, television, or laptop. • Show that the expressions in #3 and #4 are quite similar and that you simplify the expressions by combining like terms. • Holding a class discussion about #5 would be beneficial. It would allow students to compare results and verbalize their thinking. • Students should complete #6 and have their partner either complete the problem or have their partner check their work.



Link the Ideas

Example 1: Add Polynomials

Add $3x - 4$ and $2x + 5$. Simplify your answer by combining like terms.

Solution

Method 1: Use a Model

You can use algebra tiles to model each polynomial.

What is another way to model the polynomials?

Strategies
Model It

Arrange the model so that like objects are together.
Remove zero pairs if necessary.

Rewrite the model in simplest form.

$(3x - 4) + (2x + 5) = 5x + 1$

Method 2: Use Symbols

You can combine like terms.

$(3x - 4) + (2x + 5)$
$= 3x + 2x - 4 + 5$
$= 5x + 1$

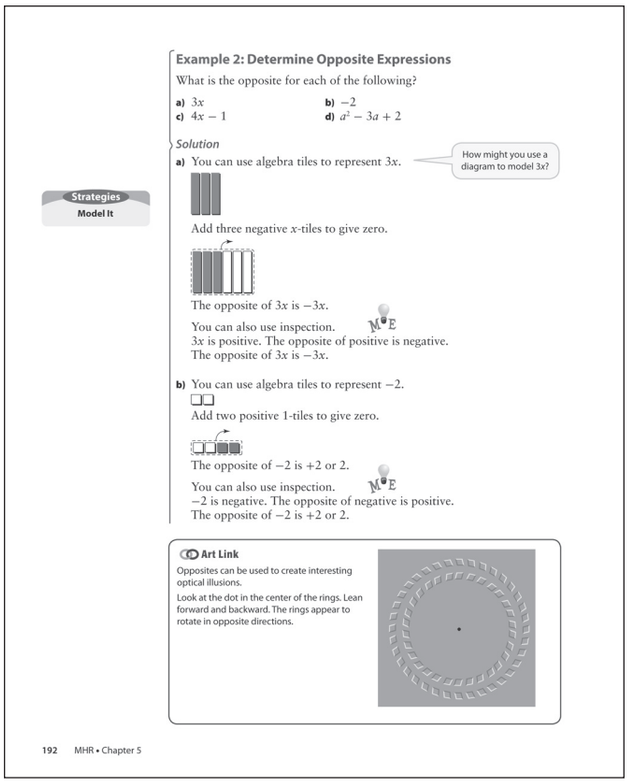
Show You Know

Use two methods to show each addition of polynomials. Give your answers in simplest form.

a) $(2a - 1) + (6 - 4a)$

b) $(3t^2 - 5t) + (t^2 + 2t + 1)$

Frank and Ernest



Example 2: Determine Opposite Expressions

What is the opposite for each of the following?

a) $3x$ **b)** -2

c) $4x - 1$ **d)** $a^2 - 3a + 2$

Solution

a) You can use algebra tiles to represent $3x$.

How might you use a diagram to model $3x$?

Strategies
Model It

Add three negative x-tiles to give zero.

The opposite of $3x$ is $-3x$.

You can also use inspection.
$3x$ is positive. The opposite of positive is negative.
The opposite of $3x$ is $-3x$.

b) You can use algebra tiles to represent -2.

Add two positive 1-tiles to give zero.

The opposite of -2 is $+2$ or 2.

You can also use inspection.
-2 is negative. The opposite of negative is positive.
The opposite of -2 is $+2$ or 2.

Art Link

Opposites can be used to create interesting optical illusions.

Look at the dot in the center of the rings. Lean forward and backward. The rings appear to rotate in opposite directions.

Link the Ideas

Example 1

This example illustrates the use of algebra tiles to model polynomial addition. Reinforce that the example uses a familiar problem-solving strategy (Model It). Ask students what other strategy they could use (Draw a Diagram). You may also want to remind student that there are other ways of modelling equations. Diagrams are one such model. Perhaps have students create diagrams to model and solve the equations in the examples.

Emphasize the relationship between the algebra tiles used. Encourage students to use symbols concurrently with their tile models. Perhaps have students set up the two methods side by side.

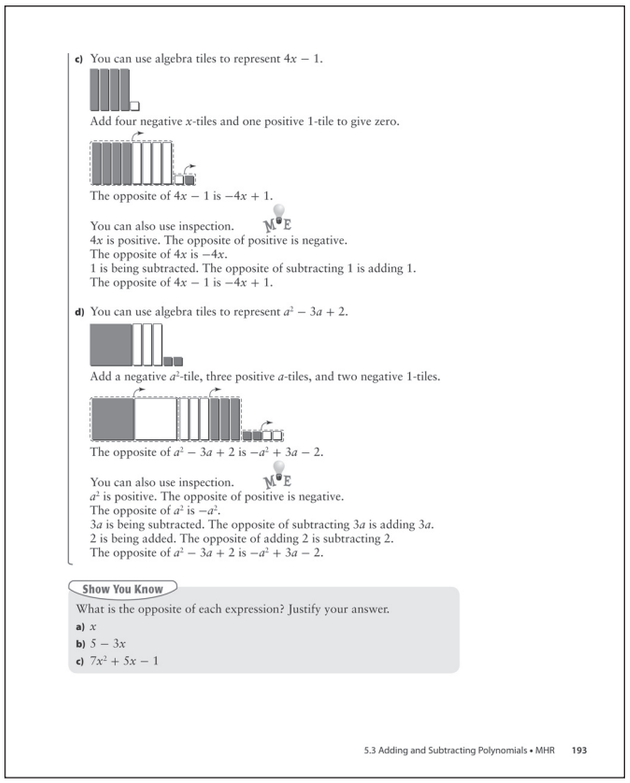
Have students complete the Show You Know. Students could do part a) using an algebra-tile model, and then check using symbols. Then, do part b) using symbols, and check using an algebra-tile model.

Example 2

This example uses models and symbols to introduce the concept of opposite term. The solutions again show the use of a familiar problem-solving strategy (Model It) involving algebra tiles. Emphasize that opposite expressions combine to give zero. This reintroduces the idea of a zero pair. Connect this to the idea of zero pairs in integers and fractions. You may wish to ask questions, such as the following:

- When in your mathematics learning has the word *opposite* appeared previously?
- Is the term $3x$ positive or negative?
- What operation symbol or sign precedes 3 in the expression $3x$?
- What is the opposite of this sign?
- How many terms are in the expression $4x - 1$?
- What is the opposite of each term in $4x - 1$? What is the opposite expression?

Rather than spending considerable time going over each part of Example 2, you might have a brief discussion about parts a) and b) and then work through part c). Then, students could attempt part d) on their own. If they have no difficulty, they should proceed to the Show You Know.



c) You can use algebra tiles to represent $4x - 1$.

Add four negative x-tiles and one positive 1-tile to give zero.

The opposite of $4x - 1$ is $-4x + 1$.

You can also use inspection.
$4x$ is positive. The opposite of positive is negative.
The opposite of $4x$ is $-4x$.
1 is being subtracted. The opposite of subtracting 1 is adding 1.
The opposite of $4x - 1$ is $-4x + 1$.

d) You can use algebra tiles to represent $a^2 - 3a + 2$.

Add a negative a^2-tile, three positive a-tiles, and two negative 1-tiles.

The opposite of $a^2 - 3a + 2$ is $-a^2 + 3a - 2$.

You can also use inspection.
a^2 is positive. The opposite of positive is negative.
The opposite of a^2 is $-a^2$.
$3a$ is being subtracted. The opposite of subtracting $3a$ is adding $3a$.
2 is being added. The opposite of adding 2 is subtracting 2.
The opposite of $a^2 - 3a + 2$ is $-a^2 + 3a - 2$.

Show You Know

What is the opposite of each expression? Justify your answer.

a) x

b) $5 - 3x$

c) $7x^2 + 5x - 1$

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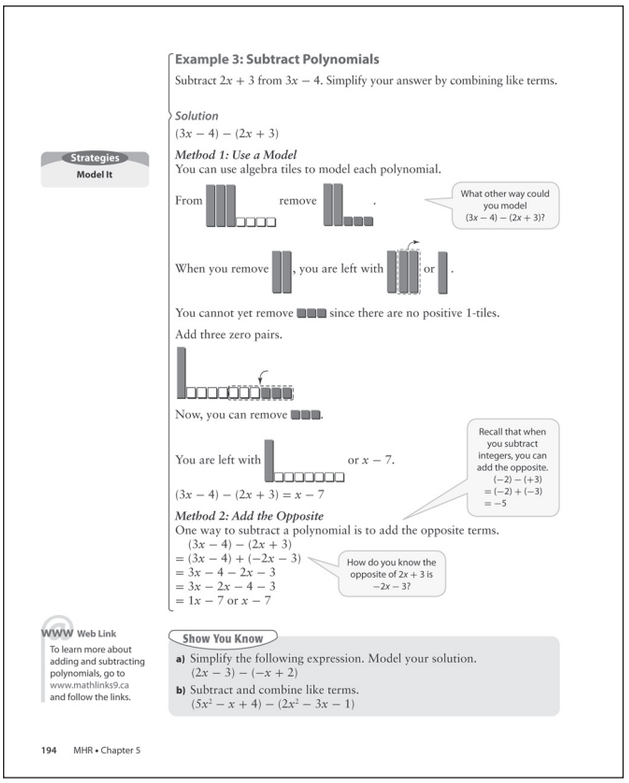

Example 3: Subtract Polynomials

Subtract $2x + 3$ from $3x - 4$. Simplify your answer by combining like terms.

Solution

$(3x - 4) - (2x + 3)$

Strategies: Model It

Method 1: Use a Model

You can use algebra tiles to model each polynomial.

From remove .

What other way could you model $(3x - 4) - (2x + 3)$?

When you remove , you are left with or .

You cannot yet remove since there are no positive 1-tiles.

Add three zero pairs.

Now, you can remove .

You are left with or $x - 7$.

$(3x - 4) - (2x + 3) = x - 7$

Recall that when you subtract integers, you can add the opposite.
$(-2) - (+3)$
$= (-2) + (-3)$
$= -5$

Method 2: Add the Opposite

One way to subtract a polynomial is to add the opposite terms.

$(3x - 4) - (2x + 3)$
$= (3x - 4) + (-2x - 3)$
$= 3x - 4 - 2x - 3$
$= 3x - 2x - 4 - 3$
$= 1x - 7$ or $x - 7$

How do you know the opposite of $2x + 3$ is $-2x - 3$?

WWW Web Link
To learn more about adding and subtracting polynomials, go to www.mathlinks9.ca and follow the links.

Show You Know

a) Simplify the following expression. Model your solution.
$(2x - 3) - (-x + 2)$

b) Subtract and combine like terms.
$(5x^2 - x + 4) - (2x^2 - 3x - 1)$

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Example 3

The example shows subtraction of polynomials using models and by adding the opposite. Encourage students to discuss other times they have subtracted in mathematics. Make the connection to the subtraction of integers, and talk about how it could be interpreted as "adding the opposite." With this fresh in students' memories, point out the first thought bubble on page 194. Have students model the question by adding the opposite. This may be easier than the take-away model shown in Method 1. Some questions you could ask are:

- What does a model for $2x + 3$ look like? Show me.
- How is a model of the opposite of $2x + 3$ different? Show me.
- What does a model for $3x - 4$ look like?
- Combine like terms using models for $3x - 4$ and the opposite of $2x + 3$.

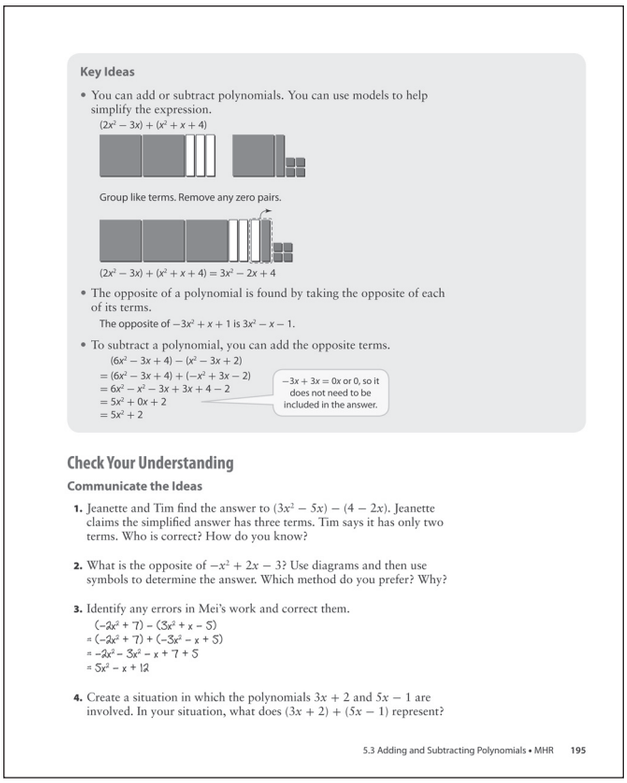
Encourage students to think of Method 2 as a way to eliminate the parentheses from the question, and then to combine like terms as in section 5.2

Key Ideas

Encourage students to write in their Foldable their own summary for section 5.3 prior to looking at the Key Ideas in the student resource. Then, have students compare their summaries with other students and with those in the student resource.

Meeting Student Needs

- It may be better for your class for you to teach this section over two periods.
- Consider first focusing on Example 1 and the related questions from the Check Your Understanding. Then, encourage students who are not having any difficulty to proceed to Examples 2 and 3 on their own.
- Depending on your class, it may be beneficial to your students to work through the Examples as a whole-class activity. Where appropriate, assign the first question or two of the Show You Know as a small-group or pairs activity. Then, have students complete the rest individually.
- Students often find addition to be easier than subtraction of polynomials. Work with them on only addition questions until they feel comfortable. Then, move on to subtraction.
- Model for students to show that all subtraction questions can be changed to "adding the opposite."



Key Ideas

- You can add or subtract polynomials. You can use models to help simplify the expression.

$(2x^2 - 3x) + (x^2 + x + 4)$

Group like terms. Remove any zero pairs.

$(2x^2 - 3x) + (x^2 + x + 4) = 3x^2 - 2x + 4$

- The opposite of a polynomial is found by taking the opposite of each of its terms.

The opposite of $-3x^2 + x + 1$ is $3x^2 - x - 1$.

- To subtract a polynomial, you can add the opposite terms.

$(6x^2 - 3x + 4) - (x^2 - 3x + 2)$
$= (6x^2 - 3x + 4) + (-x^2 + 3x - 2)$
$= 6x^2 - x^2 - 3x + 3x + 4 - 2$
$= 5x^2 + 0x + 2$
$= 5x^2 + 2$

$-3x + 3x = 0x$ or 0, so it does not need to be included in the answer.

Check Your Understanding

Communicate the Ideas

1. Jeanette and Tim find the answer to $(3x^2 - 5x) - (4 - 2x)$. Jeanette claims the simplified answer has three terms. Tim says it has only two terms. Who is correct? How do you know?

2. What is the opposite of $-x^2 + 2x - 3$? Use diagrams and then use symbols to determine the answer. Which method do you prefer? Why?

3. Identify any errors in Mei's work and correct them.

$(-2x^2 + 7) - (3x^2 + x - 5)$
$= (-2x^2 + 7) + (-3x^2 - x + 5)$
$= -2x^2 - 3x^2 - x + 7 + 5$
$= 5x^2 - x + 12$

4. Create a situation in which the polynomials $3x + 2$ and $5x - 1$ are involved. In your situation, what does $(3x + 2) + (5x - 1)$ represent?

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Gifted and Enrichment

For question #18, you could have students check the pricing for renting similar equipment in their community. Have them complete the question using the figures they researched.

Common Errors

- Some students may not know the correct zero pairs to introduce when modelling subtraction with algebra tiles.

R_x Prior to Method 1, introduce the concept of subtracting by adding the opposite. That way, students may then combine like terms using models as in section 5.2.

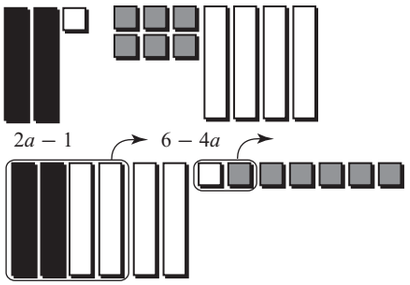
- Some students may miss a term when writing the opposite expression.

R_x Emphasize that each term of the polynomial must be expressed as an opposite. To clarify, indicate that a polynomial and its opposite always combine to zero. Demonstrate with models.

Answers

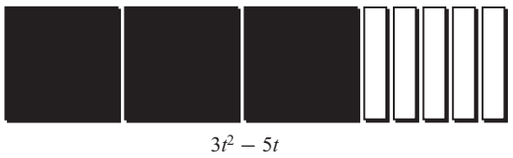
Example 1: Show You Know

a) $(2a - 1) + (6 - 4a) = 2a - 1 + 6 - 4a = -2a + 5$



Rewrite the model as $-2a + 5$.
Group together like terms and remove zero models.

b) $(3t^2 - 5t) + (t^2 + 2t + 1) = 3t^2 - 5t + t^2 + 2t + 1 = 4t^2 - 3t + 1$



$3t^2 - 5t$

$t^2 + 2t + 1$

Group together like terms and remove zero models.
Rewrite the model as $4t^2 - 3t + 1$.

Example 2: Show You Know

a) $-x$; $x + (-x) = 0$

b) $-5 + 3x$; $5 - 3x + (-5 + 3x) = 0$

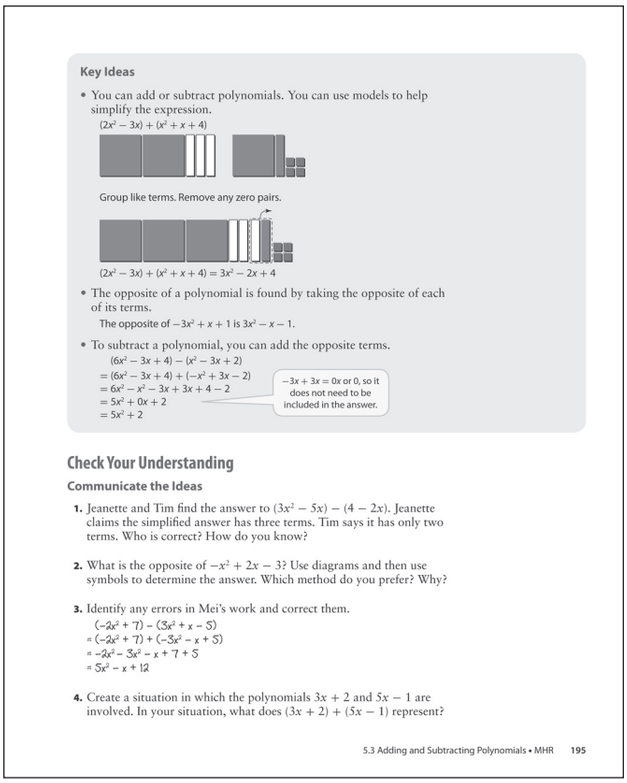
c) $-7x^2 - 5x + 1$; $7x^2 + 5x - 1 + (-7x^2 - 5x + 1) = 0$

Example 3: Show You Know

a) $3x - 5$

b) $3x^2 + 2x + 5$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • During class discussions, have students model how to solve the problems. Encourage students to explain in simple language what they did in each step. • As students complete their models, have them generalize to symbols before moving on to the next question. • Encourage students to model all questions using actual algebra tiles. If algebra tiles are not available, provide students with **Master 11 Algebra Tiles (Positive Tiles) and Master 12 Algebra Tiles (Negative Tiles)**. Ensure that all students understand the relationship between the algebra-tile model and the symbolic approach.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students of the importance of quickly identifying the number of terms in an expression. • Once students understand the concept of opposite polynomials, it is not necessary for students to continue using models to justify that two expressions are opposites. Simply let students use models as a method of finding opposite terms. • Make sure that students understand that they are finding the opposite of each term in the expression. The new expression + the old expression can be checked by adding like terms and obtaining an answer of zero.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Show students that they can check subtraction using addition ($a - b = c$ can be checked using the addition $c + b = a$). This technique is preferable to repeating the subtraction. • Some students find it easier to change the question from subtraction by adding the opposite before using either models or symbols. • Remind students that they are adding the opposite of each of the terms, not just the first, which is a common mistake.



Key Ideas

- You can add or subtract polynomials. You can use models to help simplify the expression.

$(2x^2 - 3x) + (x^2 + x + 4)$

Group like terms. Remove any zero pairs.

$(2x^2 - 3x) + (x^2 + x + 4) = 3x^2 - 2x + 4$

- The opposite of a polynomial is found by taking the opposite of each of its terms.

The opposite of $-3x^2 + x + 1$ is $3x^2 - x - 1$.

- To subtract a polynomial, you can add the opposite terms.

$(6x^2 - 3x + 4) - (x^2 - 3x + 2)$
$= (6x^2 - 3x + 4) + (-x^2 + 3x - 2)$
$= 6x^2 - x^2 - 3x + 3x + 4 - 2$
$= 5x^2 + 0x + 2$
$= 5x^2 + 2$

$-3x + 3x = 0x$ or 0, so it does not need to be included in the answer.

Check Your Understanding

Communicate the Ideas

1. Jeanette and Tim find the answer to $(3x^2 - 5x) - (4 - 2x)$. Jeanette claims the simplified answer has three terms. Tim says it has only two terms. Who is correct? How do you know?

2. What is the opposite of $-x^2 + 2x - 3$? Use diagrams and then use symbols to determine the answer. Which method do you prefer? Why?

3. Identify any errors in Mei's work and correct them.

$(-2x^2 + 7) - (3x^2 + x - 5)$
$= (-2x^2 + 7) + (-3x^2 - x + 5)$
$= -2x^2 - 3x^2 - x + 7 + 5$
$= 5x^2 - x + 12$

4. Create a situation in which the polynomials $3x + 2$ and $5x - 1$ are involved. In your situation, what does $(3x + 2) + (5x - 1)$ represent?

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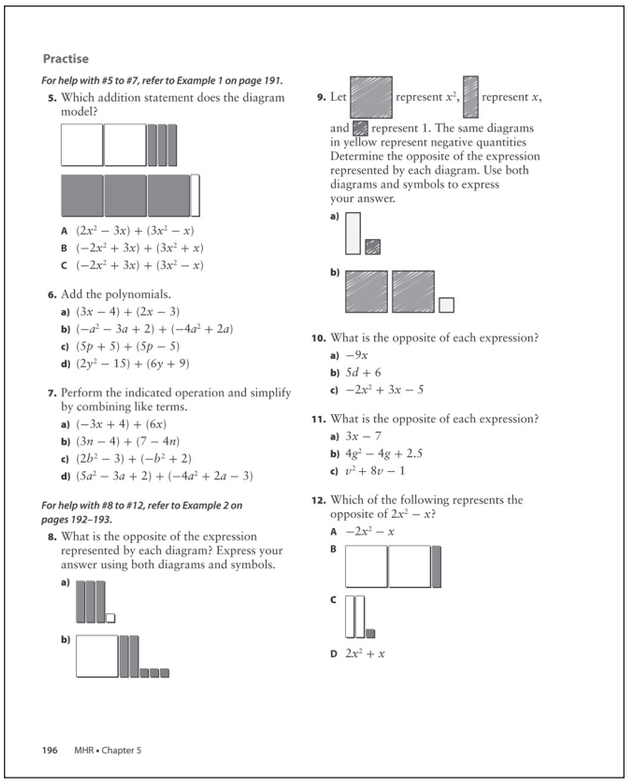

Practise

For help with #5 to #7, refer to Example 1 on page 191.

5. Which addition statement does the diagram model?

A $(2x^2 - 3x) + (3x^2 - x)$
B $(-2x^2 + 3x) + (3x^2 + x)$
C $(-2x^2 + 3x) + (3x^2 - x)$

6. Add the polynomials.
a) $(3x - 4) + (2x - 3)$
b) $(-a^2 - 3a + 2) + (-4a^2 + 2a)$
c) $(5p + 5) + (5p - 5)$
d) $(2y^2 - 15) + (6y + 9)$

7. Perform the indicated operation and simplify by combining like terms.
a) $(-3x + 4) + (6x)$
b) $(3n - 4) + (7 - 4n)$
c) $(2b^2 - 3) + (-b^2 + 2)$
d) $(5a^2 - 3a + 2) + (-4a^2 + 2a - 3)$

For help with #8 to #12, refer to Example 2 on pages 192–193.

8. What is the opposite of the expression represented by each diagram? Express your answer using both diagrams and symbols.

a)

b)

9. Let represent x^2, represent x, and represent 1. The same diagrams in yellow represent negative quantities. Determine the opposite of the expression represented by each diagram. Use both diagrams and symbols to express your answer.

a)

b)

10. What is the opposite of each expression?
a) $-9x$
b) $5d + 6$
c) $-2x^2 + 3x - 5$

11. What is the opposite of each expression?
a) $3x - 7$
b) $4g^2 - 4g + 2.5$
c) $v^2 + 8v - 1$

12. Which of the following represents the opposite of $2x^2 - x$?
A $-2x^2 - x$
B

C

D $2x^2 + x$

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Check Your Understanding

Communicate the Ideas

Have students work individually to answer #2 and 3. Then, have a brief class discussion following the individual work time. Draw out conceptual issues relating to opposite expressions and how they are helpful when subtracting. Then, students may complete #1.

Some students may find #4 difficult because they cannot quickly think of a reasonable situation matching the given expressions. (One idea is that Tom has a number of baseball cards with a value of $3x + 2$, and Tanya has cards worth $5x - 1$, where x is the number of cards.) You may wish to have students work with a partner on this question.

Practise

Students benefit from making choices so you may wish to let students choose one question from each of the following similar pairs of questions: #6 and 7, #8 and 9, #10 and 11, and #14 and 15. Students who feel they can make choices tend to take more responsibility for their homework assignments.

Apply

Have students complete #16, and then have them select #18 or 20. Alternatively, assign your choice of questions.

Question #17 is a different way to ask students to add a number of polynomials. Note that if a student gets the value in the top block correct, the other blocks must also be correct. This makes it easy to check students' solutions quickly.

Questions #23 and 24 include diagrams and may benefit visual learners.

Extend

Questions #26 to 29 are applications similar to those found in Apply. In #28, students must realize that Profit = revenues − expenses.

Question #30 follows a pattern: each pair of terms combines to give x ($2x - x = x$, $4x - 3x = x$, $6x - 5x = x$, etc.). To find the final answer, students need to find the correct number of terms (1004) and multiply that number by x ($1004x$).

For help with #13 to #15, refer to Example 3 on page 194.

13. Draw a diagram to model the subtraction statement $(-3x^2 + 4x) - (-2x^2 - x)$.

14. Simplify by combining like terms.

a) $(2x - 3) - (5x - 1)$

b) $(-3b^2 - 5b) - (2b^2 + 4b)$

c) $(5 - 6w) - (-2 - 3w)$

d) $(m + 7) - (m^2 + 7)$

15. Subtract.

a) $(8c - 3) - (-5c)$

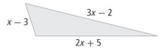
b) $(-3r^2 - 5r - 2) - (r^2 - 2r + 4)$

c) $(y^2 - 5y) - (2y - y^2)$

d) $(6j^2 - 4j + 3) - (-2j^2 - 5)$

Apply

16. A triangle has the dimensions shown.

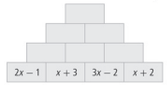


a) What does $(x - 3) + (3x - 2) + (2x + 5)$ represent?

b) Simplify the expression in part a).

c) If x has a value of 5, what is the perimeter of the triangle? Did you use the expression in part a) or part b) to find this answer? Why?

17. Complete the addition pyramid. Find the value in any box by adding the expressions in the two boxes immediately below it.



18. In Langley, British Columbia, you can rent a backhoe for $399 per day and a bulldozer for $550 per day. It costs $160, round trip to move each piece of equipment back and forth to the job site.

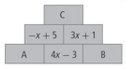
a) Write an expression for the total cost of renting the backhoe, before tax. Include transportation to and from the job site. What does your variable represent?

b) What is an expression for the total cost of renting and moving the bulldozer? Use the same variable as in part a).

c) What is an expression for the cost of renting both a backhoe and a bulldozer? Give your answer in its simplest form.

d) What is an expression for the difference in cost between renting the backhoe and the bulldozer? Give your answer in simplest form.

19. Consider the addition pyramid shown below.



a) Write an expression for box C. Do not simplify.

b) Show how you can find the value for boxes A and B. Simplify your answers.

20. The cost to print n copies of a book is $15n + 2000$ dollars. The cost to ship n copies of the book is $2n + 150$ dollars.

a) What is an expression for the total cost to print and ship n copies of the book?

b) What is the actual cost to print and ship 600 copies of the book?

c) What does $(15n + 2000) - (2n + 150)$ represent? Find a simpler expression for this subtraction statement.

21. Describe any errors in Jorge's work and how you would correct each one.

$(4p^2 - p + 3) - (p^2 + 3p - 2)$
$= 4p^2 - p + 3 - p^2 - 3p - 2$
$= (4p^2 - p^2) + (-p - 3p) + (3 - 2)$
$= 3p^2 - 3p + 1$

22. Simplify by combining like terms.

a) $(6x - 7) + (3x - 1) + (x - 4)$

b) $(3a^2 - 4a) + (3a - 5) - (a^2 - 1)$

c) $(4t^2 - t + 6) - (t^2 + 2t - 4) + (2t^2 - 3t - 1)$

d) $(2x - 1.8) - (3.4x - 2.1) - (0.9x - 0.1)$

23. Replace each question mark with algebra tiles to make a true statement.

a)

b)

24. The perimeter of the triangle shown is $12x^2 + 6x$, in metres. Find a polynomial representing the missing side length.

25. Your student council plans to thank 25 students and staff who have made special contributions to the school. Wooden plaques cost $17.95. It costs $0.12 per letter to engrave a message on the plaque. All costs are before tax.

a) Write an expression for the cost of engraving 25 plaques with the following message and the name of your school. *Thank you for your generous support.*

b) Write an expression for the cost of buying and engraving the plaques.

c) Write an expression for the cost of buying and engraving 25 plaques with an unknown number of letters.

d) Show how to use the addition of polynomials to find the cost of 50 plaques if each plaque has the same number of letters and numbers.

Extend

26. Kiesha's dad is a Haisla artist. He makes his own prints and sells them on the Internet. He will ship the prints to purchasers anywhere in Canada. For large prints, he charges $30 to ship one print plus $7 for each additional print. For small prints, he charges $20 for one print plus $5 for each additional print.

a) How much does her dad charge to ship two large prints?

b) How much does he charge to ship four small prints?

c) Write an expression to show how much he charges to ship an unknown number of large and small prints.

Math Link

This Math Link gives students an opportunity to explore an easy number trick to see if they can use algebra to help them explain it.

The following number trick is more difficult since it involves multiplying and dividing. You may wish to use it as enrichment or after studying Chapter 7.

1. Pick any number.
2. Add 3.
3. Multiply your result by 10.
4. Multiply your result by your original number.
5. Subtract 10.
6. Divide by 10.
7. Add 1.
8. Divide by your original number.
9. Subtract 3 from the number.
10. Subtract your original number.

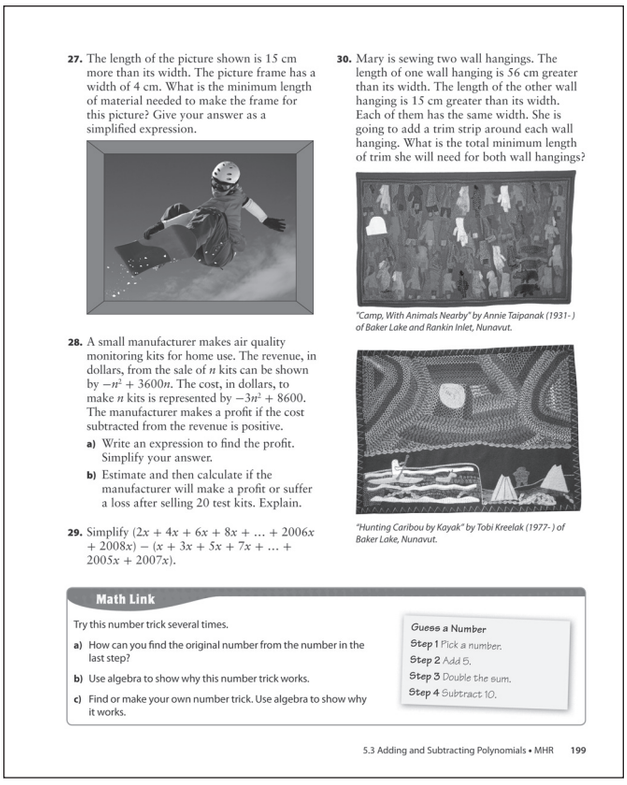
What is the answer?

Meeting Student Needs

- Consider having students add an oval to their concept map and label it *opposite term*. Have them discuss with a partner the information needed to complete this oval. You might also wish to have them record methods for determining opposite expressions.
- Provide **BLM 5–9 Section 5.3 Extra Practice** to students who would benefit from more practice.
- Consider having students work in pairs. They might work on one question together and then work individually on the next one. Ensure that each student completes a number of questions individually.
- Students should be able to add/subtract polynomials using more than one method. If some students restrict themselves to only one method, ask them to provide two methods for solving a specific question.

ELL

Teach the following terms in context: *claims*, *machinery*, *rented*, *contractors*, *backhoe*, *bulldozer*, *flat rate*, *print n copies of a book*, *ship*, *business cards*, *frame* (for a picture), *carbon dioxide test kits*, *sewing*, and *trim strip*.



27. The length of the picture shown is 15 cm more than its width. The picture frame has a width of 4 cm. What is the minimum length of material needed to make the frame for this picture? Give your answer as a simplified expression.

28. A small manufacturer makes air quality monitoring kits for home use. The revenue, in dollars, from the sale of n kits can be shown by $-n^2 + 3600n$. The cost, in dollars, to make n kits is represented by $-3n^2 + 8600$. The manufacturer makes a profit if the cost subtracted from the revenue is positive.

a) Write an expression to find the profit. Simplify your answer.

b) Estimate and then calculate if the manufacturer will make a profit or suffer a loss after selling 20 test kits. Explain.

29. Simplify $(2x + 4x + 6x + 8x + \ldots + 2006x + 2008x) - (x + 3x + 5x + 7x + \ldots + 2005x + 2007x)$.

30. Mary is sewing two wall hangings. The length of one wall hanging is 56 cm greater than its width. The length of the other wall hanging is 15 cm greater than its width. Each of them has the same width. She is going to add a trim strip around each wall hanging. What is the total minimum length of trim she will need for both wall hangings?

"Camp, With Animals Nearby" by Annie Taipanak (1931-) of Baker Lake and Rankin Inlet, Nunavut.

"Hunting Caribou by Kayak" by Tobi Kreelak (1977-) of Baker Lake, Nunavut.

Math Link

Try this number trick several times.

a) How can you find the original number from the number in the last step?

b) Use algebra to show why this number trick works.

c) Find or make your own number trick. Use algebra to show why it works.

Guess a Number
Step 1 Pick a number.
Step 2 Add 5.
Step 3 Double the sum.
Step 4 Subtract 10.

Gifted and Enrichment

- Ask students to investigate the development of polynomials for real-life applications.
- Have students complete question #18 using data they research in their community.
- Encourage students to go online and view mathematics involving polynomials in a language they are unfamiliar with. Challenge them to see how much of the text they can understand using the language of algebra only.
- For question #28, discuss what aspects of air quality might be monitored. For example, carbon monoxide, carbon dioxide, mould, asbestos, etc. Have students research the types of air quality issues that can exist in a home. This can be a particularly interesting discussion in light of allergies, sick building syndrome, and the risks involved with doing home renovations.

Common Errors

- Some students may use only symbolic methods to add/subtract polynomials.

R_x Require students to use two methods of solution. Most students will use one symbol method and one diagram/model method, rather than figuring out two symbol methods.

- Some students may consistently make mistakes in subtraction questions when they "add the opposite."

R_x Show students another way to deal with the negative subtraction sign before the second polynomial. You may wish to have students distribute the negative sign using the distributive property:

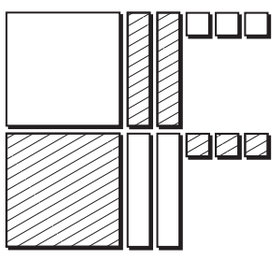
$$\begin{aligned} -(3x - 1) &= -(3x) - (-1) \\ &= -3x + 1 \end{aligned}$$

Answers

Communicate the Ideas

1. Jeanette is correct. Example: $(3x^2 - 5x) - (4 - 2x)$ simplifies to $3x^2 - 3x - 4$, which has three terms.

2. $x^2 - 2x + 3$



Example: Symbols may be preferred because you can visualize changing signs without the need for diagrams.

3. There should be a negative coefficient on the x^2-term in the final answer. The final answer should be $-5x^2 - x + 12$.

4. Example: $3x + 2$ represents the length of a driveway and $5x - 1$ represents the width of the driveway. The expression $(3x + 2) + (5x - 1)$ represents half the perimeter of the driveway.

Math Link

a) Divide the number in the last step by 2.

b) $2(n + 5) = 2n + 10$. When 10 is subtracted, the result is $2n$, and $2n$ divided by 2 is n.

c) Example: The following trick always results in the number 3.

• Choose a number from 1 to 20.	n
• Double it.	$2n$
• Add six.	$2n + 6$
• Divide by two.	$n + 3$
• Subtract the number you originally started with.	$n + 3 - n$

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1–3. Check students' responses for understanding.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • In #2, check whether students identify the given expression as having three terms and then emphasize that to find an opposite expression, it is necessary to give the opposite of each of its terms. • Check each student's response to #3 to see where they identified errors in the given solution. This shows if students can correctly combine like terms. • Have students who finish quickly work on #4 with a partner. Only students who have a good understanding of polynomials should be assigned #4.
Assessment *for* Learning	
Practise and Apply Have students do questions #5, 6, 8, 10, 12, 14, and #16–18. Students who have no problems with these questions can go on to the remaining Apply questions.	• Provide additional coaching for Example 1 to students who need help with #5 and 6. Remind them that with models for polynomials, each term has a separate representation and only like terms can be combined. Struggling and visual learners can be encouraged to identify the like terms by circling or boxing like values. Allow students to use algebra tiles if needed, but have them write the symbolic notation at the same time. • Provide additional coaching for Example 2 to students who struggle with #8 and 10. Have them explain their thinking on these questions and then assign #9 and 11. • #12 is a good opportunity for students to check their understanding when a choice of answers is presented. This question combines both the modeling and symbolic representations of opposites. • Encourage students to solve the questions in #14 using several different methods. Reinforce the idea that alternative methods enhance conceptual understanding. • #16 and 18 are basic questions allowing students to apply their understanding to problem situations. Remind students about the importance of using the same variable when representing the same value in two or more expressions. Review the concept of like terms if needed.
Math Link The Math Link on page 199 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 207.	• Students who need help getting started could use **BLM 5–10 Section 5.3 Math Link**, which provides scaffolding. • It is recommended that all students complete the Math Link. • Provide additional number tricks for students who are unable to find their own. Sources include puzzle books, airline magazines, and the Internet. • Remind students not to pick a number trick or puzzle that is long or complex, since part of the task is to explain how the trick works using polynomial addition or subtraction.
Assessment *as* Learning	
Math Learning Log Have students respond to the following prompts: • Two methods that can be used to add/subtract polynomials are … • The addition and subtraction of polynomials are the same in the following ways … • The addition and subtraction of polynomials are different in the following ways …	• Encourage students to use the What I Need to Work On section of their Foldable to note the concepts with which they continue to have difficulties. • Depending on students' learning styles, have them provide oral or written answers. • Encourage kinesthetic learners to use manipulatives and to draw diagrams as part of their solution strategies. • Have students support their statements with examples and diagrams.