

**Challenges**

**Hot-Air Ballooning**

On January 15, 1991, the Pacific Flyer completed the longest flight ever made by a hot air balloon. The balloon flew 7671.91 km from Japan to northern Canada.

The balloon is designed to fly in the transoceanic jet streams. The Pacific Flyer hitched a ride on these strong winds and was swept high above the ocean. The balloon reached a ground speed of 394 km/h. This is the fastest ground speed ever achieved by a hot-air balloon!

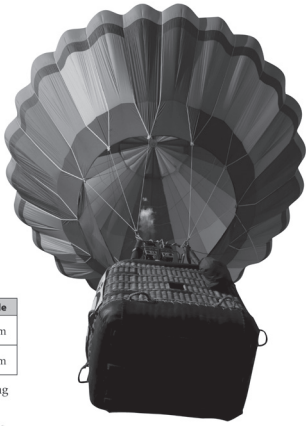
Thousands of Canadians enjoy a far less extreme ballooning experience each year. The following altitudes were recorded for two hot-air balloons at the indicated times.


|                   | Time      | Altitude | Time      | Altitude |
|-------------------|-----------|----------|-----------|----------|
| Hot-Air Balloon 1 | 8:00 a.m. | 100 m    | 9:00 a.m. | 5100 m   |
| Hot-Air Balloon 2 | 8:15 a.m. | 8100 m   | 8:45 a.m. | 6600 m   |

Assume that each balloon is ascending or descending at a steady rate.

Justify your answers to each of the following questions.

- How far did Balloon 1 ascend between 8 a.m. and 9 a.m.? Based on your answer, calculate the speed of ascent in metres per hour. Show your calculations.
- How far did Balloon 2 descend between 8:15 a.m. and 8:45 a.m.? Based on your answer, calculate the speed of descent in metres per hour. Show your calculations.
- At what time will both balloons be at the same altitude?
  - What is the altitude?
- What is the altitude of each balloon at 8:20 a.m.?
- At what time would you expect Hot Air Balloon 1 to reach 8100 m?



 **WWW Web Link**  
 For more information about hot-air balloon records, go to [www.mathlinks9.ca](http://www.mathlinks9.ca) and follow the links.

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## MathLinks 9, page 248

### Suggested Timing

60–80 minutes

### Materials

- grid paper
- ruler

### Blackline Masters

Master 1 Project Rubric

Master 9 0.5 Centimetre Grid Paper

### Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Mathematics and Estimation (ME)
- Problem Solving (PS)
- Reasoning (R)
- Technology (T)
- Visualization (V)

### Specific Outcomes

**PR1** Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.

**PR2** Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.

## Planning Notes: Hot-Air Ballooning

You may wish to use the following steps to introduce and complete this Challenge:

- Read through the Challenge as a class, ensuring that students recognize from the table of values that Balloon 1 is ascending and Balloon 2 is descending. Provide copies of **Master 9 0.5 Centimetre Grid Paper** to students for creating their graph.
- Encourage students to use different strategies to solve the problems. You might use the following prompts:
  - How can you use patterns?
  - How can you use linear equations?
  - Is there a formula you can use?

Challenge students to use different strategies, and then compare their answers.

- Encourage students to use different strategies for answering #3 to 5. If they graph the linear relation, encourage them to use increments of no more than 1000 on the vertical axis. This will make it easier to interpolate heights from the graph.
- Clarify that the task is to
  - determine the distance each balloon ascended or descended
  - calculate the speed of ascent or descent for each balloon
  - determine the time when both balloons will be at the same altitude
- Wrap up by having students account for any differences in their answers. For example, students might note that estimating using a graph was less accurate in comparison to substituting values into an equation.
- Review the **Master 1 Project Rubric** with students so that they will know what is expected.

### Meeting Student Needs

- Encourage students to use a linear graph. You may need to work with them to develop some coordinates for the graph.
- Students who are comfortable with technology may wish to use spreadsheet software or a graphing calculator.
- Students may find it easier to compare the two rates if they convert the data to minutes instead of hours, with the first time becoming zero.
- Have students calculate the altitude of each balloon at 8:30 a.m., which is a midpoint in the data for each. (Answer: Balloon 1: 2600 m; Balloon 2: 7350 m)

### ELL

- Ensure students understand the terms *transoceanic jet streams*, *ascending*, and *descending*.

### Gifted and Enrichment

- Have students calculate the exact time and altitude at which the two balloons meet. (Answer: 9:05:37.5 a.m. at 5568.75 m)
- Have students present more than one way to find the solution.

### Common Errors

- Students may misread times as decimal values.

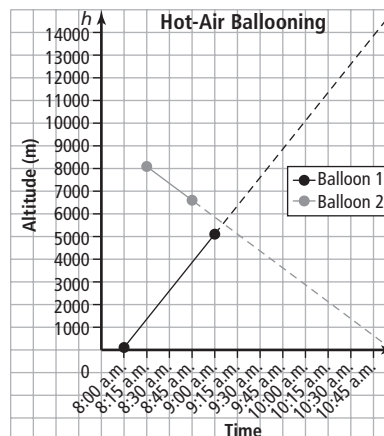
**R<sub>x</sub>** Remind students that hours are measured in 60 min so that the change from 8:15 to 8:45 is 0.5 h, rather than 0.3 h.

## Answers

### Hot-Air Ballooning

**Note:** Students may work in either m or km.

1. 5000 m; 5000 m/h
2. 1500 m; 3000 m/h
3. a),b) Example: According to the graph, the two balloons will meet around 9:07 a.m. at an altitude of approximately 5600 m.



### 4. Examples:

- Using the graph in #3, the altitude appears to be about 1700 m for Balloon 1 and almost 8000 m for Balloon 2.
- Alternatively, Balloon 1 is rising at 83.3 m/min. It is at 100 m at 8:00 a.m. At 8:20 a.m., it will be at  $(100 + 20 \times 83.3) = 1766$  m. Balloon 2 is descending at 50 m/min. At 8:15 a.m., it is at 8100 m. At 8:20 a.m., it will be at  $(8100 - 50 \times 5) = 7850$  m.

### 5. Examples:

- Using the graph from #3, the balloon should reach 8100 m a little before 9:45 a.m.
- By calculating, the balloon should reach 8100 m at approximately 9:36 a.m. ( $3000 \div 83.3$ ).

This Challenge can be used for either *Assessment for Learning* or *Assessment of Learning*.

| Assessment   | Supporting Learning  |
|--|--|
| <b>Assessment for Learning</b>   |  |
| <b>Hot-Air Ballooning</b><br>Discuss the Challenge as a class. Have students provide individual responses.     | <ul style="list-style-type: none"> <li>• Consider allowing students to work with a partner and then write individual responses.</li> </ul>   |
| <b>Assessment of Learning</b>  |  |
| <b>Hot-Air Ballooning</b><br>Introduce the Challenge to the class. Have students provide individual responses. | <ul style="list-style-type: none"> <li>• <b>Master 1 Project Rubric</b> provides a holistic descriptor that will assist you in assessing student work on this Challenge. Page 339 provides notes on how to use this rubric for the Challenge.</li> <li>• To view student exemplars, go to <a href="http://www.mathlinks9.ca">www.mathlinks9.ca</a>, access the Teacher Centre on the Online Learning Centre, go to Assessment, and then follow the links.</li> </ul> |

The chart below shows the **Master 1 Project Rubric** for tasks such as this Challenge, Hot-Air Ballooning, and provides notes that specify how to identify the level of specific answers for this project.

| Score/Level                          | Holistic Descriptor   | Specific Question Notes   |
|--------------------------------------|---|---|
| <b>5</b><br>(Standard of Excellence) | <ul style="list-style-type: none"> <li><input type="checkbox"/> Applies/develops <b>thorough</b> strategies and mathematical processes making <b>significant</b> comparisons/connections that demonstrate a <b>comprehensive</b> understanding of how to develop a complete solution</li> <li><input type="checkbox"/> Procedures are <b>efficient and effective</b> and may contain a <b>minor mathematical error</b> that does not affect understanding</li> <li><input type="checkbox"/> Uses <b>significant</b> mathematical language to explain their understanding and provides <b>in-depth</b> support for their conclusion</li> </ul> | <ul style="list-style-type: none"> <li>• provides a complete and correct solution</li> </ul>  |
| <b>4</b><br>(Above Acceptable)       | <ul style="list-style-type: none"> <li><input type="checkbox"/> Applies/develops <b>thorough</b> strategies and mathematical processes for making <b>reasonable</b> comparisons/connections that demonstrate a <b>clear</b> understanding</li> <li><input type="checkbox"/> Procedures are <b>reasonable</b> and may contain a <b>minor mathematical error</b> that may hinder the understanding in one part of a complete solution</li> <li><input type="checkbox"/> Uses <b>appropriate</b> mathematical language to explain their understanding and provides <b>clear</b> support for their conclusion</li> </ul>                          | Demonstrates one of the following: <ul style="list-style-type: none"> <li>• provides a complete response with weak communication in at most two questions</li> <li>• provides a complete and correct response based on an incorrect #1 <i>or</i> 2</li> <li>• provides a correct response with justification to any 4 out of 5 questions</li> </ul> |
| <b>3</b><br>(Meets Acceptable)       | <ul style="list-style-type: none"> <li><input type="checkbox"/> Applies/develops <b>relevant</b> strategies and mathematical processes making <b>some</b> comparisons/connections that demonstrate a <b>basic</b> understanding</li> <li><input type="checkbox"/> Procedures are <b>basic</b> and may contain a <b>major error or omission</b></li> <li><input type="checkbox"/> Uses <b>common</b> language to explain their understanding and provides <b>minimal</b> support for their conclusion</li> </ul>   | Demonstrates one of the following: <ul style="list-style-type: none"> <li>• correctly completes #1 to 3 and makes a start to #4 or 5 with some weak communication</li> <li>• correctly completes #1 to 5 without showing any work or justification</li> <li>• correctly completes #3 to 5</li> </ul>  |
| <b>2</b><br>(Below Acceptable)       | <ul style="list-style-type: none"> <li><input type="checkbox"/> Applies/develops <b>some relevant</b> mathematical processes making <b>minimal</b> comparisons/connections that lead to a <b>partial solution</b></li> <li><input type="checkbox"/> Procedures are <b>basic</b> and may contain <b>several major mathematical errors</b></li> <li><input type="checkbox"/> Communication is <b>weak</b></li> </ul>  | Demonstrates one of the following: <ul style="list-style-type: none"> <li>• correctly completes #1 and 2; communication for one question may be weak or absent</li> <li>• correctly completes #3 or 4</li> </ul>  |
| <b>1</b><br>(Beginning)              | <ul style="list-style-type: none"> <li><input type="checkbox"/> Applies/develops an <b>initial start</b> that may be <b>partially correct</b> or could have led to a correct solution</li> <li><input type="checkbox"/> Communication is <b>weak or absent</b></li> </ul>   | <ul style="list-style-type: none"> <li>• correctly completes #1 or 2</li> </ul>   |

For student exemplars, go to [www.mathlinks9.ca](http://www.mathlinks9.ca) and follow the links.