6.1 Representing Patterns

***MathLinks 9*, pages 210–219**

Suggested Timing
80–100 minutes

Materials
- grid paper
- coloured pencils
- ruler
- algebra tiles

Blackline Masters
Master 8 Centimetre Grid Paper
Master 9 0.5 Centimetre Grid Paper
Master 11 Algebra Tiles
BLM 6–3 Chapter 6 Warm-Up
BLM 6–5 Section 6.1 Extra Practice
BLM 6–6 Section 6.1 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
PR1 Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.

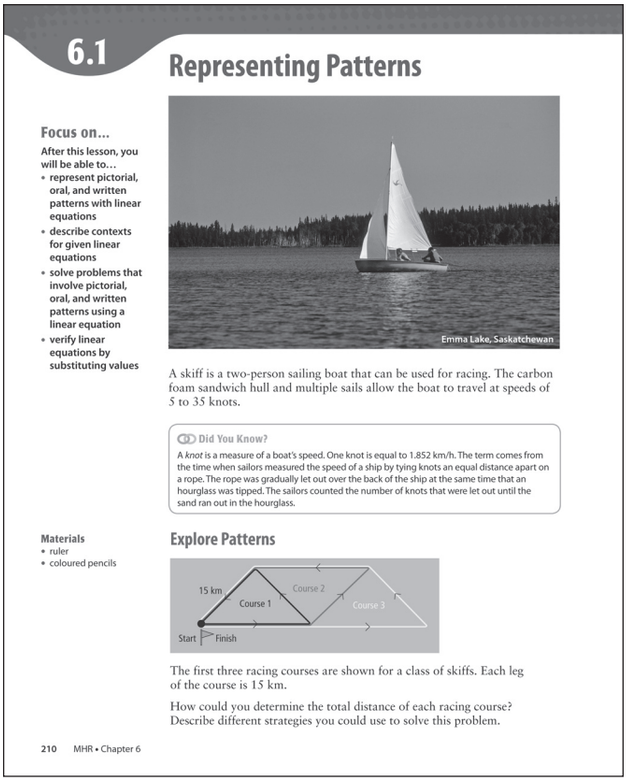
Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2, 4, 7, 10a), c), 11, 12, Math Link
Typical	#1, 2, 4, 6, 7, 10a), c), 11, 13, Math Link
Extension/Enrichment	#1, 3, 8, 9, 14–18

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce material learned in previous sections.

As a class, have students read and discuss the opening text and the photo that was taken at Emma Lake in Saskatchewan. You might have students share what they know about boat races and using knots as a measure of boat speed.

As students organize the course information, encourage them to think of different methods. Ask them to explain how the various methods relate the information to each other.



6.1 Representing Patterns

Focus on...
After this lesson, you will be able to...
- represent pictorial, oral, and written patterns with linear equations
- describe contexts for given linear equations
- solve problems that involve pictorial, oral, and written patterns using a linear equation
- verify linear equations by substituting values

Emma Lake, Saskatchewan

A skiff is a two-person sailing boat that can be used for racing. The carbon foam sandwich hull and multiple sails allow the boat to travel at speeds of 5 to 35 knots.

Did You Know?
A *knot* is a measure of a boat's speed. One knot is equal to 1.852 km/h. The term comes from the time when sailors measured the speed of a ship by tying knots an equal distance apart on a rope. The rope was gradually let out over the back of the ship at the same time that an hourglass was tipped. The sailors counted the number of knots that were let out until the sand ran out in the hourglass.

Materials
- ruler
- coloured pencils

Explore Patterns

15 km
Course 1
Course 2
Course 3
Start
Finish

The first three racing courses are shown for a class of skiffs. Each leg of the course is 15 km.

How could you determine the total distance of each racing course? Describe different strategies you could use to solve this problem.

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Explore Patterns

In this exploration, students identify a pattern in a series of race courses, and then represent the pattern graphically, using a table, and verbally before developing an equation to represent the pattern.

As many students may not have seen a sailboat race, consider showing aerial photos or a video to help them gain understanding. (You might search for images or videos using the search terms *regatta*, *sailing*, and *buoy*.) Explain that to complete a course a sailboat must go around each buoy placed at the ends of the course legs (sides). Unlike channel markers, racing buoys are often enormous yellow or orange inflated pyramids, standing taller than an adult. On a short course, sailors are able to see the next buoy. On a long course such as the one shown in the student resource, sailors would also be given directions (e.g., headings/bearings), since they would be unable to see the buoys from the race start. They would use navigational aids such as maps, a compass, and GPS to stay on course.

1. Draw what you think the next two courses might look like.

2. Organize the information for the first five courses so that you can summarize the results.

3. a) Describe the pattern in the race course lengths. Then, check that your Courses 4 and 5 fit the pattern.
   b) Describe the relationship between the course number and the length of the course.

4. Write an equation that can be used to model the length of the course in terms of the course number. Explain what your variables represent.

Reflect and Check

5. a) What are two methods you could use to determine the length of Course 9?
   b) Which method do you prefer? Explain why.

6. a) Determine which course is 135 km long.
   b) Determine the length of Course 23.
   c) How did you determine the answers to parts a) and b)?
   d) Discuss your solutions with a classmate.

Help students recognize each course shown in the Explore by identifying the shapes they make: Course 1 forms a triangle; Course 2 is a rhombus; and Course 3 is a trapezoid. For #2, students may choose to represent the relationship between the course number and a course distance using a table of values, a statement, or a graph.

Students should be able to use any two methods to represent the pattern and to make predictions concerning the length of additional courses.

Method 1 Have students work in pairs. Make **Master 8 Centimetre Grid Paper** and **Master 9 0.5 Centimetre Grid Paper** available. As student pairs work, ask the following questions:

- How are the different ways of recording the data for the first five courses connected?
- What variables could you use to represent each part of the pattern?
- How can you check that the equation models the pattern accurately?

Have pairs discuss their solution with another pair of students before having student pairs discuss their findings with the class. Discuss which methods are more effective and less effective for representing the pattern. You might ask about the strengths and weaknesses of each method.

As a class, discuss the responses to #5. Have students describe the connections between the table of values, the graph, and the equation to encourage understanding. Have students complete #6 individually or in pairs.

Method 2 Have students work in small groups. Encourage each small group to check with another group and discuss their solution for each step of the Explore.

Meeting Student Needs

- For the Explore, students who are not familiar with sailing may draw meaningful analogies to race courses for snow machines or animal competitions.
- Provide scaffolding to help students organize the information for #2, such as using a table of values and providing the first two coordinate pairs.

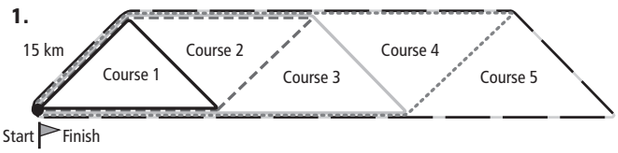
ELL

- Use the image of the *skiff* to help explain this term.
- Teach the following terms in context: *carbon foam sandwich hull*, *multiple*, *knots*, *hourglass*, and *racing course*.
- Allow students to explain their process in their first language. This may help them to ask for the missing vocabulary in English.

Answers

Explore Patterns

1.



2. Example:

Course Number, *n*	Length of Course, *d* (km)
1	45
2	60
3	75
4	90
5	105

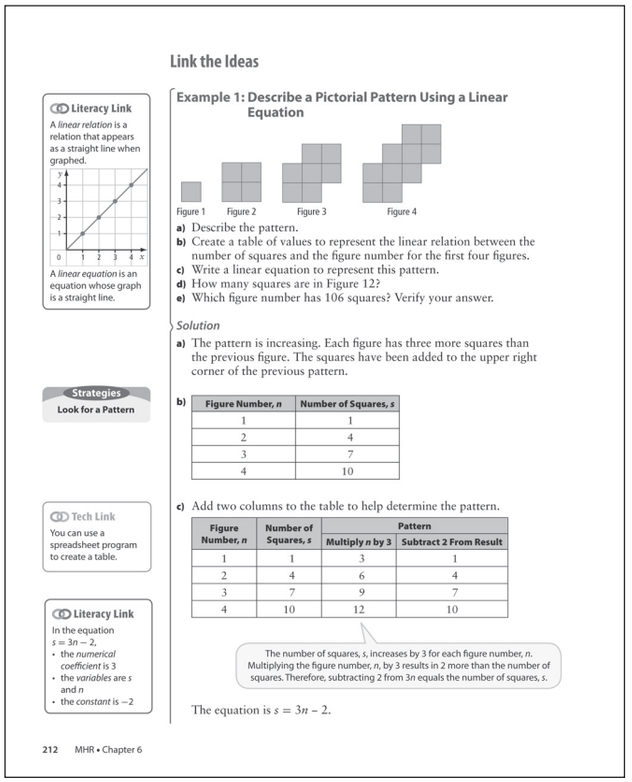
3. a) The race course length increases by 15 km with each course number. Example: Yes, Courses 4 and 5 fit this pattern.
b) The course distance is 30 km more than 15 times the course number.

4. $d = 15n + 30$, where d is the course length in kilometres and n is the course number

5. a) Example: Graph the coordinate pairs and extrapolate, or substitute 9 for n in the equation and solve for d.
b) Example: The equation is preferred because it takes less time and gives an accurate answer.

6. a) Course 7 **b)** 375 km
c) Example: By substituting values into the equation

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Have students explain how the methods of representing patterns are similar and different.	• As a class, discuss the connections between the table of values, the graph, and the equation. • Explain that methods of representing patterns are equivalent, but that some may be more useful depending upon what is being asked.



Link the Ideas

Literacy Link
A *linear relation* is a relation that appears as a straight line when graphed.

A *linear equation* is an equation whose graph is a straight line.

Example 1: Describe a Pictorial Pattern Using a Linear Equation

a) Describe the pattern.
b) Create a table of values to represent the linear relation between the number of squares and the figure number for the first four figures.
c) Write a linear equation to represent this pattern.
d) How many squares are in Figure 12?
e) Which figure number has 106 squares? Verify your answer.

Solution

a) The pattern is increasing. Each figure has three more squares than the previous figure. The squares have been added to the upper right corner of the previous pattern.

Strategies
Look for a Pattern

b)

Figure Number, *n*	Number of Squares, *s*
1	1
2	4
3	7
4	10

Tech Link
You can use a spreadsheet program to create a table.

c) Add two columns to the table to help determine the pattern.

Figure Number, *n*	Number of Squares, *s*	Pattern	
		Multiply *n* by 3	Subtract 2 From Result
1	1	3	1
2	4	6	4
3	7	9	7
4	10	12	10

Literacy Link
In the equation $s = 3n - 2$,
- the *numerical coefficient* is 3
- the *variables* are s and n
- the *constant* is -2

The number of squares, s, increases by 3 for each figure number, n. Multiplying the figure number, n, by 3 results in 2 more than the number of squares. Therefore, subtracting 2 from $3n$ equals the number of squares, s.

The equation is $s = 3n - 2$.

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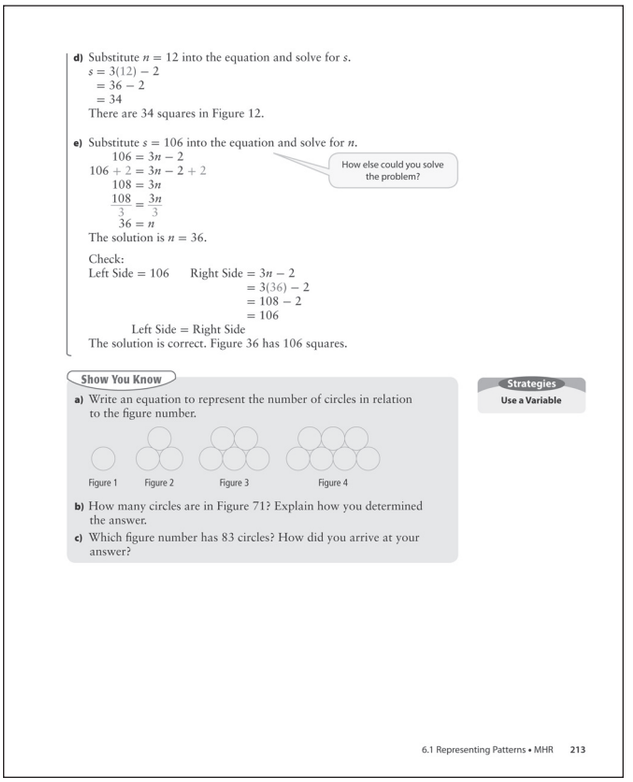


d) Substitute $n = 12$ into the equation and solve for s.
$s = 3(12) - 2$
$= 36 - 2$
$= 34$
There are 34 squares in Figure 12.

e) Substitute $s = 106$ into the equation and solve for n.
$106 = 3n - 2$
$106 + 2 = 3n - 2 + 2$
$108 = 3n$
$\frac{108}{3} = \frac{3n}{3}$
$36 = n$
The solution is $n = 36$.

How else could you solve the problem?

Check:
Left Side $= 106$ Right Side $= 3n - 2$
$= 3(36) - 2$
$= 108 - 2$
$= 106$
Left Side = Right Side
The solution is correct. Figure 36 has 106 squares.

Show You Know

a) Write an equation to represent the number of circles in relation to the figure number.

b) How many circles are in Figure 71? Explain how you determined the answer.
c) Which figure number has 83 circles? How did you arrive at your answer?

Strategies
Use a Variable

6.1 Representing Patterns • MHR 213

Link the Ideas

Literacy Link Before beginning the Link the Ideas, read and discuss the Literacy Links on page 212. The first link reviews the definitions for *linear relation* and *linear equation*. Provide several examples of graphs and equations and have students identify those that depict linear relationships.

The second link reviews *numerical coefficient*, *variables*, and *constant*. Use sample equations and have students identify the components in each one. As students work through the chapter, encourage them to use these terms when discussing linear equations.

Example 1

This example uses a pictorial pattern to illustrate how to develop a linear equation. In the solution, some students may find it easier to describe the pattern after creating the table of values in part b). Or, you may have students recreate the pattern using algebra tiles to help them see the pattern. Provide students with **Master 11 Algebra Tiles**, if algebra tiles are not available.

The table of values in part b) is used to create an equation that models the pattern. You may need to help students recall identifying the x- and y-variable and how to organize the variables in a table of values.

Example 2: Describe a Written Pattern Using a Linear Equation

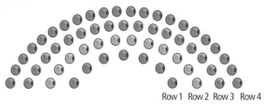
A bead design for a necklace has an arc shape:
- Row 1 has seven red beads.
- Row 2 has five additional beads and all the beads are green.
- Row 3 has five additional beads and all the beads are blue.
- The pattern repeats. Five beads are added to each successive row.

a) Draw the pattern for the first four rows.
b) Make a table of values showing the number of beads in relation to the row number.
c) What equation shows the pattern between the row number and the number of beads in the row?
d) How many beads are in Row 4? Explain how to check your answer.
e) How many beads are in Row 38?
f) If the bead pattern were continued, which row number would have 92 beads? How did you determine the answer?

Solution

Strategies
Draw a Diagram

a)



Strategies
Make an Organized List or Table

b)

Row Number, n	Number of Beads, b
1	7
2	12
3	17
4	22

c) Add two columns to the table to help determine the pattern.

Row Number, n	Number of Beads, b	Pattern	
		Multiply n by 5	Add 2 to Result
1	7	5	7
2	12	10	12
3	17	15	17
4	22	20	22

The equation is $b = 5n + 2$.

Look at the diagram of the pattern. In the equation, what does the 5 mean? What does the 2 mean?

d) Count the number of beads in Row 4. There are 22 beads. You can check this by substituting $n = 4$ into the equation and solving for b.
$b = 5n + 2$
$= 5(4) + 2$
$= 20 + 2$
$= 22$
There are 22 beads in Row 4.

e) Substitute $n = 38$ into the equation and solve for b.
$b = 5n + 2$
$= 5(38) + 2$
$= 190 + 2$
$= 192$
There are 192 beads in Row 38.

How else could you solve the problem?

What colour are the beads in Row 38? Explain how you know.

f) Substitute $b = 92$ into the equation and solve for n.
$92 = 5n + 2$
$92 - 2 = 5n + 2 - 2$
$90 = 5n$
$\frac{90}{5} = \frac{5n}{5}$
$18 = n$
The solution is $n = 18$.

Check:
Left Side = 92
Right Side $= 5n + 2$
$= 5(18) + 2$
$= 90 + 2$
$= 92$
Left Side = Right Side
The solution is correct. Row 18 has 92 beads.

Show You Know

In a banquet hall, a single rectangular table seats six people. Tables can be connected end to end as shown. Four additional people can be seated at each additional table of the same size.

a) What linear equation could represent this situation? Share with a classmate how you determined the equation.
b) How many tables connected together will seat 26 people?

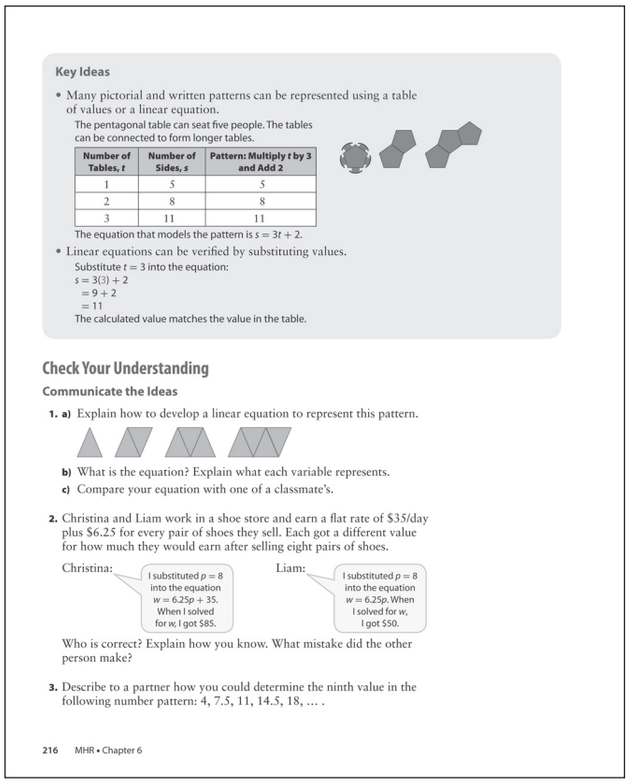
You might explain that the x-value in a coordinate pair is the independent variable and the y-value is the dependent variable. Ask:
- What are the two variables in this pattern? (figure number and number of squares)
- Which is the independent variable? Or, which variable is changed? (number of squares)
- Which is the dependent variable? Or, which variable changes as a result of the other variable? (figure number)

In part c) of the solution, students consider the relationship between the figure number, n, and the number of squares, s. Ensure they recognize that the four-column table is the part b) table repeated, with the addition of columns to help analyse the pattern. Draw students' attention to the thought bubble in part c) and help them identify where the coefficient and constant come from in the table of values. Help students develop a clear understanding of the solution by asking questions such as the following:
- What is the difference between consecutive values of s?
- What is the relationship between s and the numerical coefficient?
- What is the difference between $3s$ and s?
- What is the relationship between the variables and the constant?

You might consider asking students to develop a different equation to represent the pattern and explain how it does. For example, $s = 3(x - 1) + 1$ is a valid equation. Have students who develop this equation explain why the numerical coefficients make sense within the context of the squares design in the pattern. Alternatively, provide this equation and ask students to explain why the numerical coefficients make sense within the context of the pattern design.

Have students share their observations about the pattern. Instead of analysing the integers in the table of values, students might visually recognize that each figure increases by three squares (figure number $\times$ 3), but that, for example, Figure 1 has only one square, or two less than an increase of three (–2).

Have students describe how the equation is used to answer parts d) and e). For part d), the figure number is known and for part e), the number of squares is known. Ask them to share other methods for solving these questions. For example, students may make multiple additions of 3 and subtract 2, or draw a diagram and count squares. For part e), have them discuss how else they might verify the solution. You might have students discuss the more efficient and less efficient methods for solving the problems.



Key Ideas

- Many pictorial and written patterns can be represented using a table of values or a linear equation.

The pentagonal table can seat five people. The tables can be connected to form longer tables.

Number of Tables, t	Number of Sides, s	Pattern: Multiply t by 3 and Add 2
1	5	5
2	8	8
3	11	11

The equation that models the pattern is $s = 3t + 2$.

- Linear equations can be verified by substituting values.

Substitute $t = 3$ into the equation:
$s = 3(3) + 2$
$= 9 + 2$
$= 11$
The calculated value matches the value in the table.

Check Your Understanding

Communicate the Ideas

1. a) Explain how to develop a linear equation to represent this pattern.

b) What is the equation? Explain what each variable represents.

c) Compare your equation with one of a classmate's.

2. Christina and Liam work in a shoe store and earn a flat rate of $35/day plus $6.25 for every pair of shoes they sell. Each got a different value for how much they would earn after selling eight pairs of shoes.

Christina: I substituted $p = 8$ into the equation $w = 6.25p + 35$. When I solved for w, I got $85.

Liam: I substituted $p = 8$ into the equation $w = 6.25p$. When I solved for w, I got $50.

Who is correct? Explain how you know. What mistake did the other person make?

3. Describe to a partner how you could determine the ninth value in the following number pattern: 4, 7.5, 11, 14.5, 18, ...

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Have students complete the Show You Know before continuing. Check that each student understands how to develop the linear equation for part a).

Example 2

This example uses a written pattern to illustrate how to develop a linear equation. In the solution, the written pattern is represented in a diagram. Stress how the diagram helps clarify the problem. From part b) on, the solution process is the same as for Example 1.

For part c), you might ask students to describe verbally the pattern in the table. (The number of beads, b, increases by 5 for each row number. Multiplying the row number, n, by 5 results in 2 less than the number of beads. Therefore, adding 2 to $5n$ equals the number of beads.) Help students make connections between the diagram, verbal description, table of values, and the equation by asking them to explain why the equation makes sense within the context of the problem. (The 5 shows that there is a constant increase of 5 beads in each new row; the 2 shows that the first row had an extra 2 beads (for a total of 7) compared to starting with 5.)

Ask students why it might be preferable to calculate the answer to part e) rather than count beads as in part d). Ask students what other methods might be used for solving part e). For example, students may make multiple additions of 5 and add 2, or draw a diagram and count beads. Have students discuss the more efficient and less efficient methods for solving the problem.

The thought bubble in part e) asks students to consider the repeating colours in the pattern. Survey students for solutions, look for a consensus, and then solve. (Row 38 is green.)

Key Ideas

The Key Ideas summarize using a table of values or a linear equation to represent a pictorial or written pattern. Linear equations are emphasized as a problem-solving tool because they are easy to solve. Ask students about the connections between the two representations. Students should be able to create a table of values and use it to identify the numerical coefficient and constant for an equation that models a linear pattern. They should be able to use the equation to solve problems related to the pattern it represents. Have students prepare their own summary of the Key Ideas in the section 6.1 booklet in their Foldable.

Meeting Student Needs

- Help students explicitly make connections between the patterns in a table of values and the related linear equation. Have students use manipulatives to represent a pattern, identify relevant data in the related table of values, and substitute values into the related equation.
- For part b) in Example 1, some students may need to recall the conventions for organizing a table of values. In a vertical table, place x in the left column and y in the right column. In a horizontal table, place x in the top row and y in the bottom row.
- For part d) in Example 1, some students may benefit from drawing or modelling Figure 12 and counting squares to verify the answer.
- Some students may benefit from additional questions similar to the Show You Know before proceeding to Example 2.
- Some students may benefit from representing the pattern in Example 2 and the Show You Knows using algebra tiles. Provide students with **Master 11 Algebra Tiles**, if algebra tiles are not available.

- Bring in examples of beadwork to provide a context for Example 2. Many cultures traditionally adorn clothing and other items with beadwork.
- Alternatively for Example 2, use an analogy of an igloo constructed in a spiral of snow blocks, and pose a similar series of questions. The spirals get smaller as the igloo gets taller. There are three fewer blocks in each consecutive spiral. For example, spiral 7 has 21 blocks, spiral 6 has 18 blocks, and spiral 5 has 15 blocks.
- Some students may benefit from a more explicit method for deriving the numerical coefficient and constant in an equation from a table of values. Tell students:
  – The numerical coefficient is the difference between any two consecutive y-values.
  – The constant is the difference between a y-value and the product of the corresponding x-value and the coefficient. Use an example to illustrate how it works.

x	y
0	1
1	3
2	5
3	7

  Using an example of y-values 3 and 1, the numerical coefficient is $3 - 1 = 2$.
  Using an example of (2, 5), the constant is $5 - (2 \times 2) = 1$
  Therefore, the equation is $y = 2x + 1$.

ELL

- Allow English language learners to describe the patterns in their first language, and then discuss their ideas in English.
- Paraphrase the instructions in Example 2, part b) so that students understand that *in relation* to means the relationship between the number of beads and the row number. Students are to draw the number of beads in each row.
- Teach the following terms in context: *increasing*, *previous pattern*, *column*, *row*, *beads*, *additional*, *substitute*, *banquet hall*, *connected end to end*, *pictorial*, and *pentagonal table*.
- Model an equation and pattern to ensure students understand the terms *equation*, *represent*, *relation*, *figure number*, and *pattern*.

Gifted and Enrichment

- For Example 2, challenge students to develop a different equation to represent the pattern and explain how it does. For example, $b = 5(n + 1) + 1$ represents the same pattern. Have students who develop this equation explain why the numerical coefficients make sense within the context of the pattern.

Common Errors

- Some students may have difficulty deriving the numerical coefficient and constant in a linear equation from the table of values.

R_x Remind students that the numerical coefficient in an equation (i.e., the number that is multiplied by x) is the difference between any two consecutive y-values in the table of values. The constant is the difference between a y-value and the product of the corresponding x-value and the coefficient.

- Some students may have difficulty using an equation to solve a problem involving a linear pattern.

R_x Have students state what each variable represents and identify the values they know. Then, have them substitute the known values into the equation. Encourage them to check their solution using another method (e.g., drawing a diagram).

Answers

Example 1: Show You Know

a) $c = 2n - 1$

b) 141; Example: Substitute $n = 71$ into the equation and solve for c.

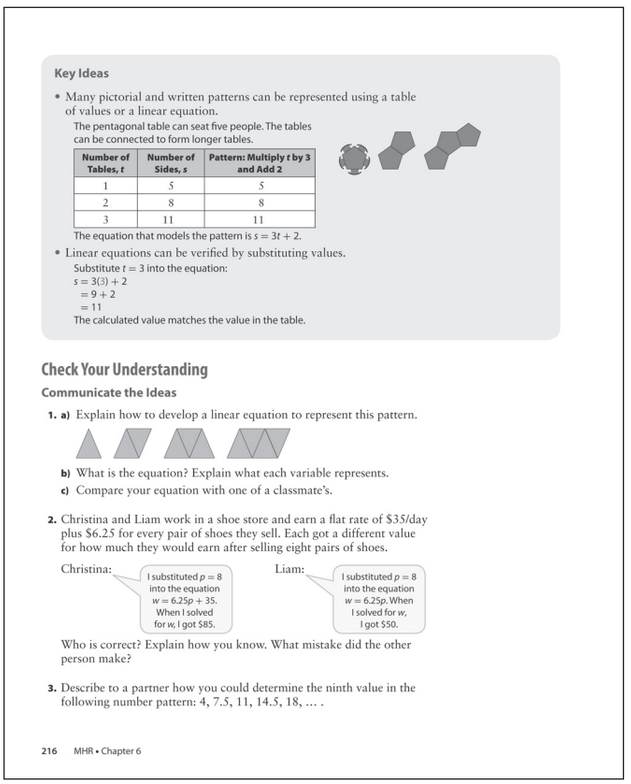
c) Figure 42; Example: Substitute $c = 83$ into the equation and solve for n.

Example 2: Show You Know

a) $p = 4t + 2$; Example: Each time a table is added, four more people can be seated.

b) 6 tables

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students develop an example in their Foldable that shows how to use a table of values to determine the numerical coefficient and constant for the related equation. Encourage them to model the pattern using a diagram.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to create a table of values to help them develop the linear equation. • Encourage students to draw a diagram of the pattern. Check that they can relate the description of the written pattern to the diagram they draw. • Refer students to Example 1, which poses similar questions.



Key Ideas

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1	5	5
2	8	8
3	11	11

The equation that models the pattern is $s = 3t + 2$.

- Linear equations can be verified by substituting values.

Substitute $t = 3$ into the equation:
$s = 3(3) + 2$
$= 9 + 2$
$= 11$
The calculated value matches the value in the table.

Check Your Understanding

Communicate the Ideas

1. a) Explain how to develop a linear equation to represent this pattern.

b) What is the equation? Explain what each variable represents.

c) Compare your equation with one of a classmate's.

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Who is correct? Explain how you know. What mistake did the other person make?

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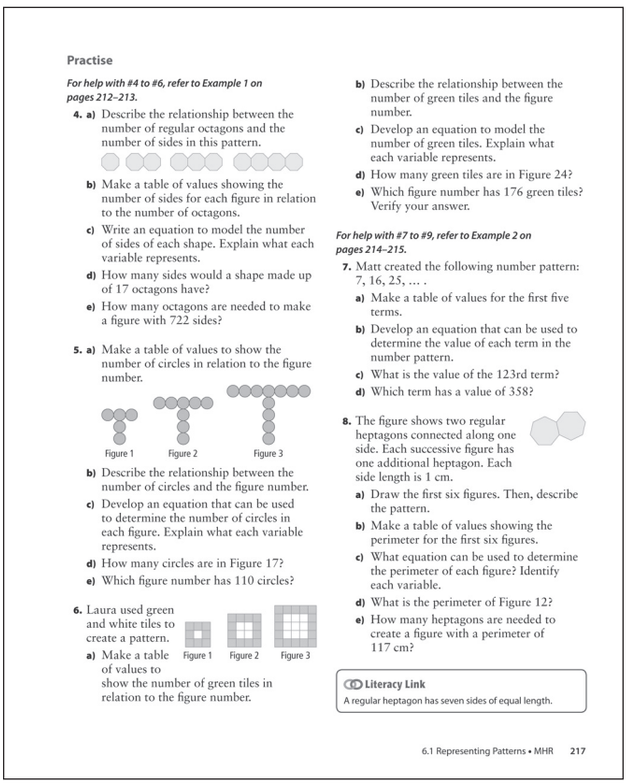


Practise

For help with #4 to #6, refer to Example 1 on pages 212–213.

4. a) Describe the relationship between the number of regular octagons and the number of sides in this pattern.

b) Make a table of values showing the number of sides for each figure in relation to the number of octagons.

c) Write an equation to model the number of sides of each shape. Explain what each variable represents.

d) How many sides would a shape made up of 17 octagons have?

e) How many octagons are needed to make a figure with 722 sides?

5. a) Make a table of values to show the number of circles in relation to the figure number.

Figure 1 Figure 2 Figure 3

b) Describe the relationship between the number of circles and the figure number.

c) Develop an equation that can be used to determine the number of circles in each figure. Explain what each variable represents.

d) How many circles are in Figure 17?

e) Which figure number has 110 circles?

6. Laura used green and white tiles to create a pattern.

Figure 1 Figure 2 Figure 3

a) Make a table of values to show the number of green tiles in relation to the figure number.

b) Describe the relationship between the number of green tiles and the figure number.

c) Develop an equation to model the number of green tiles. Explain what each variable represents.

d) How many green tiles are in Figure 24?

e) Which figure number has 176 green tiles? Verify your answer.

For help with #7 to #9, refer to Example 2 on pages 214–215.

7. Matt created the following number pattern: 7, 16, 25, ...

a) Make a table of values for the first five terms.

b) Develop an equation that can be used to determine the value of each term in the number pattern.

c) What is the value of the 123rd term?

d) Which term has a value of 358?

8. The figure shows two regular heptagons connected along one side. Each successive figure has one additional heptagon. Each side length is 1 cm.

a) Draw the first six figures. Then, describe the pattern.

b) Make a table of values showing the perimeter for the first six figures.

c) What equation can be used to determine the perimeter of each figure? Identify each variable.

d) What is the perimeter of Figure 12?

e) How many heptagons are needed to create a figure with a perimeter of 117 cm?

Literacy Link
A regular heptagon has seven sides of equal length.

6.1 Representing Patterns • MHR 217

Check Your Understanding

Communicate the Ideas

These questions allow students to express their understanding of developing a linear equation from a pictorial or written pattern, and substituting values into an equation.

In #1 and 3, students develop an equation from a pictorial or written pattern and show their understanding of the numerical coefficient and constant in the equation.

In #2, it is important for students to recognize that the constant (35) must be included in the equation.

Practise

The following pairs of questions are very similar. Some students may be given a choice of completing any two questions in each set: #4 to 6 and #7 to 9.

For #5, have students explain the connection between at least two representations, or why their equation makes sense within the context of the problem.

For #6, you might coach students who observe different patterns to develop the related equations. You might point out how the different equations simplify to the same equation.

9. Jessica created a number pattern that starts with the term −5. Each subsequent number is 3 less than the previous number.

a) Make a table of values for the first five numbers in the pattern.

b) What equation can be used to determine each number in the pattern? Verify your answer by substituting a known value into your equation.

c) What is the value of the 49th term?

d) Which term has a value of −119? Verify your answer.

10. What linear equation models the relationship between the numbers in each table?

a)

x	y
0	13
1	16
2	19
3	22

b)

r	p
0	17
1	24
2	31
3	38

c)

k	t
1	−1.3
2	1.4
3	4.1
4	6.8

d)

f	w
1	−0.5
2	−4
3	−7.5
4	−11

12. A school pays a company $125 to design gym T-shirts. It costs an additional $15 to make each T-shirt.

a) Copy and complete the table of values.

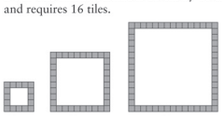
Number of T-Shirts	Cost ($)
0	125
5	200
10	
15	
35	
	950

b) Develop an equation to determine the cost of the T-shirts. Explain the meaning of the numerical coefficient.

c) What would it cost to make 378 T-shirts?

d) If the school store has a budget of $2345 for T-shirts, how many T-shirts can be ordered?

13. An art store sells square picture frames with a border of tiles that each measure 2 cm by 2 cm. The smallest frame is 10 cm by 10 cm and requires 16 tiles.



a) Develop an equation to determine the number of tiles required for each size of frame.

b) How many tiles are needed to make a frame that is 30 cm by 30 cm?

c) What are the dimensions of a square frame made with 196 tiles?

Apply

11. Rob is in charge of arranging hexagonal tables for a parent-night presentation. The tables, which can seat six people, can be connected to form longer tables.

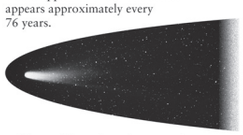
a) Develop an equation to model the pattern. Identify each variable.

b) How many parents can be seated at a row of five tables?

c) Check your answer for part b). Show your work.

d) A group of 30 people want to sit together. How many tables must be joined together to seat them?

14. Edmund Halley, after whom Halley's comet was named, predicted that the comet would appear in 1758. The comet appears approximately every 76 years.



a) Use a table to show the years of the next six sightings after 1758.

b) When will Halley's comet appear in your lifetime?

c) Write an equation that can be used to predict the years when Halley's comet will appear.

d) Will Halley's comet appear in the year 2370? How did you arrive at your answer?

Science Link

A comet, which is made of frozen gas and dust, orbits around the sun. The dust tail of a comet can be up to 10 000 000 km long. This is 2.5 times as great as the average distance from Earth to the moon.

Extend

15. Find the pattern that expresses all the numbers that are 1 more than a multiple of 3.

a) What is the 42nd number?

b) How can your pattern test to see if 45 678 is 1 more than a multiple of 3?

16. a) A landscaper is planting elm trees along a street in a new subdivision. If elm trees need to be spaced 4.5 m apart, then how long is a row of n elm trees? Write the equation.

b) The street is 100 m long. If the landscaper wants to line the street on both sides with elm trees, how many trees will be needed? Will the trees be evenly spaced along the entire street?

17. A ball is dropped from a height of 2 m. The ball rebounds to $\frac{2}{3}$ of the height it was dropped from. Each subsequent rebound is $\frac{2}{3}$ of the height of the previous one.

a) Make a table of values for the first five rebound heights in the pattern.

b) What is the height of the fourth rebound bounce?

c) Is this a linear relation? Explain how you know.

Math Link

You are in charge of developing a racing course for a sailboat race on Lake Diefenbaker, in Saskatchewan. Five classes of sailboats will race on courses that are the same shape, but different lengths.

a) Design a racing course based on a regular polygon. The shortest course must be at least 5 km long. The longest course must be no longer than 35 km.
- Draw and label a diagram of the racing course. Show at least the first four courses. Record the total length of each course.

b) Develop a linear relation related to your racing course.
- Make a table of values.
- Develop a linear equation that represents the relationship between the course number and the course distance.

c) Develop a problem related to your racing course. Provide the solution and verify it.

For example, students might note the following:

- The number of green squares on the sides is the same as the length of the white squares ($4x$, where x is the side length of the white square), and then add four corners for an equation of $y = 4x + 4$.
- The top and bottom green columns are each two squares longer than the white section ($2(x + 2)$ or $2x + 4$), and then the green column is the same length as the corresponding white column ($2x$), for an equation $y = 2x + 4 + 2x$.
- If you keep one corner with each green side ($x + 1$), and there are four sides, the equation is $y = 4(x + 1)$.

Literacy Link For #8, point out the definition for heptagon. *Hepta* comes from Greek and means seven. Ask students to discuss other numerical prefixes they know, such as *tri*, *quad*, *penta*, *hexa*, *octa*, or even *triskaideka* (from Greek and means 13). Invite English language learners to share different numerical prefixes they are familiar with.

Apply

The Apply questions provide a range of contexts for solving problems related to linear equations. You might assign questions based on student interest and/or familiarity with the contexts. Consider allowing students to have some choice in the questions they complete. For example, you might assign #11 and 12, and then have students select one additional question.

For #14, have students read and discuss the Science Link about a comet.

Extend

The Extend questions range from abstract mathematical questions to science-related questions. Students could be asked to select one question based on their interest. Consider using #17 as a classroom activity related to a physics unit.

Literacy Link Using their sequence chart, have students complete the second, third, and fourth boxes for representing patterns, solving equations, and describing patterns respectively. In the second box, students list ways to represent patterns. Have them develop an example that shows their understanding. In the third box, students show how to solve a linear equation by substituting values. In the fourth box, students describe a pattern. Have them develop a pattern and use it to create a table of values and a linear equation.

Math Link

The Math Link provides students with an opportunity to apply their understanding of linear relations by designing a sailboat race course within a given a set of parameters. The question implies that each class of boat will race a different course. Since the point is to explore linear relations, instruct students to design a series of courses that have a linear relationship to each other.

Encourage students to reflect on what they learned in the Explore in this section. This Math Link differs in that the race courses must all be the same shape, rather than different shapes added together. You may wish to sketch an example to ensure students understand this difference.

Students may find it easier to begin with part b), and then determine appropriate values for the race courses.

For part c), students develop and solve their own problem, which requires a deeper understanding of what is involved in solving a problem based on a linear pattern. As a class, have students discuss and justify the problems they developed. You might ask them how their solution to part c) demonstrates what they learned about representing patterns.

Meeting Student Needs

- Encourage students who would benefit to use manipulatives such as algebra tiles to model patterns.
- For #2, you may wish to provide similar problems for students who need practice determining a constant for a linear equation.
- Provide **BLM 6–5 Section 6.1 Extra Practice** to students who would benefit from more practice.

ELL

- Review that a *regular polygon* is a two-dimensional closed figure that has all sides and angles the same. You might show some examples including triangles, squares, pentagons, and hexagons.
- Teach the following terms in context: *flat rate*, *perimeter*, *budget*, *Halley's comet*, *your lifetime*, *elm trees*, *driveway*, and *rebound*.
- Allow students to do the calculations only and provide explanations for only a few questions.

Gifted and Enrichment

- Challenge students to complete all the Extend questions, and create their own question and solution.

Common Errors

- Some students may have difficulty visualizing a pattern from a written description.

R_x Remind students to create a table of values with several coordinate pairs in order to develop the pattern. Alternatively, have them draw a diagram or use manipulatives.

Answers

Communicate the Ideas

1. a) Example: Use a table of values to find a pattern.

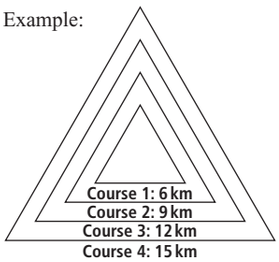
b) Example: $t = n$, where t represents the number of triangles and n represents the figure number

2. Christina; Example: The flat rate must be included. Liam forgot the daily flat rate of $35 in his equation.

3. Example: Create a table and determine a pattern and equation that represents the data, $v = 3.5n + 0.5$. Then, substitute $n = 9$ and solve for v.

Math Link

a) Example:



b) Example:

Course Number, n	Course Distance, d (km)
1	6
2	9
3	12
4	15

$d = 3n + 3$

c) Example: Problem: How long is Course 5? Solution: 18 km; $18 = 3(5) + 3$

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to visualize the pattern in #1 by developing a table of values.
Assessment *for* Learning	
Practise and Apply Have students do #4, 7, 10a), 10c), 11, and 12. Students who have no problems with these questions can go on to the remaining Apply questions.	• Provide additional coaching with Example 1 to students who need assistance with #4. Help students work through #4, and then have them try #5 on their own. Check back with students to ensure understanding. • Provide additional coaching with Example 2 to students who need assistance with #7. Help students work through #7, and then have them try #9 on their own. • Encourage students to refer to the notes in their Foldable to help them develop equations. • Assigning #10a) and c) will provide sufficient indication of student understanding. Provide coaching to students who would benefit. Work through the solution to #10a) and c), and then have students attempt #10b) or d) before proceeding. • Direct students who find #11 challenging to the Show You Know for Example 2 on page 215, which is similar. Have students review their approach and solution to the Show You Know to help them.
Math Link The Math Link on page 219 gives students practice with linear relationships and helps them prepare for the chapter problem titled Wrap It Up! on page 247.	• It is not necessary for students to do the Math Link in order to complete the Wrap It Up! However, doing so may benefit students who need practice with developing linear equations. • Explain that *classes of sailboats* means sailboats grouped by length or design. • You may wish to adapt the sailing scenario to one that is more familiar to students such as a stock car race, a swimming competition, or a triathlon. • Students who need help getting started could use **BLM 6–6 Section 6.1 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Literacy Link By the end of section 6.1, have students complete the second, third, and fourth boxes of the sequence chart.	• For the second box, refer students to Examples 1 and 2, which demonstrate different ways of representing patterns. You might encourage students to develop an example to demonstrate their understanding. • For the third box, allow students to use a variation of an existing equation. Have students avoid copying an equation verbatim. An example of an equation is $s = 9n - 7$, and substitute $s = 47$. • For the fourth box, encourage students to draw a pictorial pattern and describe it verbally before creating a table of values and a linear equation.
Math Learning Log Have students respond to the following prompts: • I can use a table of values to determine the numerical coefficient in an equation by … • I can use a table of values to determine the constant in a linear equation by …	• Encourage concrete and kinesthetic learners to use manipulatives such as algebra tiles to model patterns. • Depending on students' learning styles, have them provide oral or written responses. • Encourage students to add notes to the definitions, examples, and Key Ideas for this section in their Foldable. • Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with.