# **Interpreting Graphs**

#### MathLinks 9, pages 220-230

# Suggested Timing

#### 80–100 minutes

#### Materials

- grid paper
- ruler
- spreadsheet program (optional)
- globe (optional)

## **Blackline Masters**

Master 2 Communication Peer Evaluation Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper BLM 6–3 Chapter 6 Warm-Up BLM 6–7 Section 6.2 Extra Practice BLM 6–8 Section 6.2 Math Link

#### **Mathematical Processes**

- Communication (C)
- Connections (CN)
- Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- 🖌 Reasoning (R)
- Technology (T)
- ✓ Visualization (V)

#### **Specific Outcomes**

**PR2** Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2, 4, 7, 8, 11, 14, 15, Math Link
Typical	#1, 2, 4, 7, 8, 11, 13–15, Math Link
Extension/Enrichment	#1, 3, 10, 11, 15–19

# **Planning Notes**

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce material learned in previous sections.

Use the cartoon and the opening text to introduce the Explore. Discuss the terms *salary* and *commission* and ensure that students understand the linear relationship between daily sales and Richard's pay for the day.

Have students identify the variables on the graph and discuss other information the graph provides. Have them refer to the graph during the Explore.



**Literacy Link** Direct students to the Literacy Link on page 220 about *commission*. Have students discuss examples of sales jobs that offer commission and brainstorm pros and cons of working for commission. For instance, students might mention the advantage of working for commission in a business with high sales.

Direct students to the second Literacy Link on the page, which explains that a line joining points on a graph indicates that the data are continuous. You might explain that the opposite of continuous data is discrete data. Discrete data points do not represent a continuous relationship, and therefore it is not reasonable to have values between given data points. For example, it is not reasonable to count fractions of people in data involving numbers of people. You might have students explain if a value such as (1000, 150) on the given graph makes sense.



# **Explore Using a Graph to Solve Problems**

In this Explore, students analyse a table of values, use a graph to estimate unknown values, and use an equation to determine unknown values.

Remind students that the table of values and the graph represent the same information. Ask students to think about how they could use the graph to determine the missing values in the table. Students should be able to make reasonable interpolations from the graph given either coordinate point.

Point out that the riverboat on page 221 is the Edmonton Queen, which runs tours on the river through downtown Edmonton, Alberta.

**Method 1** Have students work individually to complete #1 to 3 of the Explore. Consider providing students with Master 8 Centimetre Grid Paper or Master 9 0.5 Centimetre Grid Paper to create their own graph. As students work, ask:

- How accurate do you think the estimated values are?
- Why might students have different estimated values?
- How could you check that your estimated values are close to the actual values?

Have students compare their solutions with those of a classmate. For #3, ask them how the numerical coefficient and the constant were derived in the equation. You might have students discuss the connections between the graph, table of values, and linear equation.

Have students complete #4 to 6, and then discuss their answers in small groups. Ask students how they might improve on the accuracy of their estimated values on the graph.

Method 2 Have students work in pairs or small groups, and then have the pairs or groups compare their results with those of another pair or group. Discuss #6 as a class.

#### **Meeting Student Needs**

- Some students may benefit from exploring a relationship that involves multiples (e.g., commission) without a base value (e.g., salary). Consider using a context that is familiar to students such as the cost of fabric per metre or fuel consumption per kilometre. For example, consider buying duffle fabric at \$35 per metre to make kamiks. Work with students to create a table of values for the cost of fabric, and then graph the data. Ask questions that require students to interpolate values such as the following: - How much will 3.25 m of fabric cost?

  - How much fabric can be purchased for \$154?

Or, consider a snow machine that uses 1 L of fuel for every 5 km travelled. Have students graph the fuel consumption of a snow machine for 5 km, 10 km, 15 km, 20 km, and 25 km. Ask students to extrapolate values such as the following:

- How much fuel is needed for 100 km?
- How far can the snow machine travel on a tank that holds 32 L?

#### ELL

- Teach the following terms in context: *daily salary*, riverboat cruises, and graph.
- For #6, ensure students understand the meaning of effective.

#### Answers

#### **Explore Using a Graph to Solve Problems**

- **1.** b) Example: Locate the sales amount on the *x*-axis and draw a vertical line to the point where the line intersects the graph. From the intersection point, draw a horizontal line over to the *y*-axis to find the corresponding pay.
- **2.** a) Example: \$2400; Using a ruler, extend the graph upwards. Then, draw a horizontal line from \$300 on the *y*-axis to the point where this line intersects the graph. From the intersection point, draw a vertical line down to the *x*-axis and read the value where it touches the axis.
  - **b)** Example: \$175; Locate the sales amount of \$1150 on the *x*-axis, then draw a vertical line up to where this line intersects the graph. From the intersection point, draw a horizontal line to the *y*-axis and read the value where it touches the axis.

- **3.** Substitute the values into the equation and solve for the unknown.
- **4.** a) Example: By locating the known coordinate on the graph, the corresponding coordinate could be read.
- **5.** a) Example: \$300 b) Example: \$1150
- **6.** a) Example:
  - For a graph: An advantage is that you can see at a glance a wide range of possible income and sales scenarios. A disadvantage is that the values are estimates, not accurate.
  - For an equation: An advantage is that the values are accurate. A disadvantage is the need to calculate the values.
  - **b)** Example: Using a graph is more effective when an exact value is not necessary. Using an equation is more effective when a precise value is required.

Assessment	Supporting Learning
Assessment as Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Encourage students to discuss the strategies they used. Check that students can identify when a graph or equation is more appropriate to determine values.	<ul> <li>Encourage discussion and sharing of strategies with the class to benefit students who are uncertain about the conclusions.</li> <li>Discuss that estimating from a graph is useful when an exact value is not necessary, and using an equation is useful when a more precise answer is required.</li> <li>Some students may need coaching to determine the sales associated with a specific amount of pay.</li> <li>Encourage students who need help to interpolate and extrapolate values to use a ruler and draw a vertical line from the <i>x</i>-axis to the graphed line, and then draw a horizontal line to the <i>y</i>-axis.</li> <li>Encourage students to record a graph and related equation of their own in the section 6.2 booklet of the Foldable.</li> </ul>



# Link the Ideas

## Example 1

Example 1 illustrates solving a problem by interpolating data on a graph. Students graph the data provided in a table of values, and then interpolate values for altitude and temperature. Part c) explains why it is not possible to interpolate precise values within the problem context.

Have students complete the Show You Know before proceeding. As a class, discuss the answer to part c). Check that students understand when it is appropriate to join data points with a straight line and when it is not.

**Literacy Link** Direct students to the Literacy Link on page 222, which clarifies expressing the *y*-variable in terms of the *x*-variable. You might have students practise expressing the relationship between *x* and *y* on sample graphs you provide.

# Example 2

Example 2 illustrates solving a problem by extrapolating data. The example uses latitude and longitude on a map as a coordinate grid and asks students to extrapolate the values of the coordinates for latitude and longitude where Anna will land. Note that the solution grid provides grid lines between the values for latitude and longitude to improve precision.

Point out to students the convention of extending a graph by using a dashed line as shown on the solution grid.

Have students complete the Show You Know. Check students' answers for precision and their understanding of whether or not to connect the data points with a straight line.





Literacy Link As a class, read and discuss the Literacy Link on page 224. Compare the grid of latitude and longitude to a more typical graph. Point out the location of the zero axes and relate them to the *x*- and *y*-axes. For example, the Equator translates to the x-axis and the Prime Meridian translates to the *v*-axis. Consider displaying a globe so that students can see how the lines of longitude converge at the poles. This convergence is maintained in the projection shown in the visual. Explain that most maps that students are familiar with have been flattened so the lines of longitude appear parallel. Though this map and most globes do not show negative values, negative values are assigned to points west of the Prime Meridian and south of the equator, similar to a graph.

## **Key Ideas**

The Key Ideas summarize using a line to interpolate and extrapolate values between and beyond known values respectively. You might have students walk and talk through the procedure for interpolating a value starting with a known value for x. Use the first graph titled Temperature Change. For example, a student may say to use a finger on a value for t (in this case), such as 4.5. They might say, "Push your finger vertically up from 4.5 to the point where the line intersects the graph. From the intersection point, slide your finger horizontally over to the y-axis (C, in this case) to determine the y-value, which is 5.5. The point where the horizontal line meets the y-axis gives an approximate value." Alternatively, students could use a right angle from a geometry set instead of a finger.

Ask students if they can extrapolate a value of t = 0or C = 1 on the graph titled Temperature Change. Help them realize they can extrapolate values beyond either end of a graphed line.

Students should be able to interpolate and extrapolate values for either axis. Have students explain their understanding of when it is appropriate to use interpolation and extrapolation. Have them brainstorm scenarios in which it would not be reasonable to interpolate data using an example such as price increases (e.g., price increases that occur only at the end of a year; price increases in increments of tens of dollars only).

Have students develop and record their own example of interpolation and extrapolation in their Foldable.

## **Meeting Student Needs**

- Some students may benefit from working with a partner and solving additional problems similar to the Show You Knows.
- Ensure that students understand the similarities and differences between interpolation and extrapolation.

### ELL

- English language learners may have difficulty remembering the meaning of *interpolate* and *extrapolate*. You might point out that *interpolate* begins with *in*, as in inside a line. *Extrapolate* begins with *extra* (as in more than the line).
- Teach the following terms in context: *weather* balloon, altitudes, vertical, horizontal, axis, kayaking, present course, latitude, longitude, coordinates, and decreases.

#### **Gifted and Enrichment**

• Remind students that it is not always reasonable to interpolate or extrapolate values. For example, it does not make sense to interpolate values for fractions of a heart beat. Or, it does not make sense to extrapolate values for distance and time for a sprinter since a sprinter cannot maintain his or her speed over a long distance. Challenge students to brainstorm scenarios that can be graphed but for which it would not be reasonable to use interpolation or extrapolation. Have them present their work in the form of an entry for a math blog. • The Example 2: Show You Know features the loss in value of a computer. For income tax purposes, a computer can be expensed or depreciated over five years and therefore represents a linear relation. In daily life though, an object such as a computer or a car often loses up to half its value the instant it is bought, and therefore its depreciated value cannot be represented by a linear relation. The graph implies that a computer is not worthless until it is five years old. However, the faster technology changes, the shorter technology's lifespan becomes. Have students research the current useful life of a computer and consider how disposal costs might affect any equation that represents its value.



For an interactive graphing application corresponding to the cost of DVD rentals, go to www.mathlinks9.ca and follow the links.

## Answers

#### **Example 1: Show You Know**

- **a)** 10 km **b)** 48 s
- c) Example: Yes, because any distance or time is possible

#### **Example 2: Show You Know**

- a) 5 years
- b) 4 years
- c) Example: Yes, because any value or time is possible

Assessment	Supporting Learning
Assessment for Learning	
<b>Example 1</b> Have students do the Show You Know related to Example 1.	<ul> <li>Encourage students to verbalize their thinking.</li> <li>You may wish to have students work with a partner.</li> <li>Ensure that students understand the difference between discrete and continuous data. Have students record an explanation and an example in their Foldable.</li> <li>Check that students understand when it is reasonable to use interpolation and when it is not reasonable. Remind students that interpolation is used to estimate values.</li> </ul>
<b>Example 2</b> Have students do the Show You Know related to Example 2.	<ul> <li>Encourage students to verbalize their thinking.</li> <li>You may wish to have students work with a partner.</li> <li>Check that students understand when it is reasonable to use extrapolation and when it is not reasonable. Remind students that extrapolation is used to estimate values.</li> </ul>



# **Check Your Understanding**

## **Communicate the Ideas**

These questions allow students to express their understanding of interpolation and extrapolation.

In #1, students explain how interpolation and extrapolation are similar and how they are different.

In #2, they apply their understanding of interpolation. They should observe that Graph A involves discrete data (for which interpolation is not appropriate) and Graph B involves continuous data (for which interpolation is appropriate).

In #3, students develop a situation for which extrapolation can be used appropriately.

## Practise

These questions give students an opportunity to interpolate and extrapolate values on a graph.

The following pairs of questions are very similar. Some students may be given a choice of completing any two questions in each set: #4 to 6 and #8 to 10.

Consider assigning #7 and 11 in addition to the previous question sets as these require students to use a table of values and create a graph before interpolating or extrapolating values.

# Apply

The Apply questions provide a range of contexts that involve interpolation or extrapolation. You might assign questions based on student interest and/or familiarity with the contexts.

Assign at least one of #13 or 16 as these questions ask students to justify whether or nor interpolation or extrapolation is appropriate in the given context.

For #13, you might point out that the submarine HMCS *Victoria* was named after the city of Victoria, British Columbia. It has been monitoring the waters of the Pacific since 2002.

## Extend

The Extend questions involve non-linear data, but solutions can be estimated using interpolation and extrapolation. This can be done either by assuming the data are approximately linear or by using a curve rather than a straight line. Ask students why interpolation and extrapolation are inadvisable for non-linear graphs.

In #18, direct students to the Did You Know? to help them realize that the sky diver will never go faster than 54 m/s. In #18b), in order to calculate the distance travelled on the graph, students will need to calculate the area under the graphed line (base  $\div 2 \times$  height).



Literacy Link Using their sequence chart, have students complete the fifth and sixth boxes for interpolating and extrapolating values on a graph. Have them develop an example and a graph and explain how to interpolate and extrapolate values on the graph.

# **Math Link**

The Math Link allows students to apply extrapolation. Students will need to research the ITCZ to help them develop their response.

For part c), they will need to select a location and relevant width of the ITCZ. Alternatively, have students use the width of 1100 km provided in the Did You Know? to make the calculation.

In the past, either paddles or sails were required to move boats. Today, motors can be used to cross the ITCZ, but sailors often study seasonal data to find the narrowest part and watch weather reports to find the area with the most wind.

Encourage students to convert metres to kilometres before graphing. This will make the process of labelling and plotting the graph easier.



#### Extend

18. The table shows the relationship between copping distance and speed of a vehicle. Speed, s 15 30 45 60 75

- Stopping Distance, d (m)
   6
   15
   28
   42
   65
   a) Plot the data on a graph. Draw a line to join the data points to best approximate the trend.
- b) What happens to stopping distance as speed increases?
- speed increases?
  c) Estimate the stopping distances for speeds of 5 km/h, 55 km/h, and 80 km/h.
  d) Estimate the speed before a driver applied the brake for stopping distances of 10 m, 50 m, and 100 m.
- About how much farther is the stopping distance at 50 km/h than it is at 30 km/h? at 70 km/h than at 50 km/h?
- Why do you think the graph is not a straight line?



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# **Meeting Student Needs**

per day? How do you know?

How long could you rent the p washer if you had \$420?

- Some students may benefit from coaching about choosing a scale on a graph. Have students check the maximum value needed on each axis, as well as the difference between data values to help them choose appropriate intervals for each axis. If a graph will be used to interpolate or extrapolate values, have students consider choosing a scale that makes the task easier.
- Provide BLM 6-7 Section 6.2 Extra Practice to students who would benefit from more practice.

## ELL

- Teach the following terms in context: *trail mix*, hockey game, programs, power washer, propellers, motors, and anchor.
- · Allow students to discuss their ideas in their first language in order to access prior knowledge. Then, have them express their thinking in English.
- Allow English language learners to answer fewer questions.



### **Gifted and Enrichment**

- For #18, ask students the following questions:
  - What is the speed of the skydiver at 8 s?
- What does the graph look like after the skydiver reaches terminal velocity?

Draw students attention to the Did You Know? to help them realize that the sky diver will never go faster than 54 m/s, so the graph becomes a horizontal line all of a sudden.

- For the Math Link, encourage students to research the history and folklore surrounding the ITCZ. For instance, the ITCZ has been called the *Doldrums* (a term whose meaning has spread to indicate a mood) and the *Horse Latitudes*. Alternatively, they might research why the ITCZ varies in size across the globe and through the seasons, or what methods in addition to kedging were used to cross the ITCZ.
- For the Math Link, most students may calculate crossing the ITCZ at a right angle. Have students research traditional shipping routes to determine where, and at what angle, ships would be crossing the ITCZ. Have them also consider the season, as this would also have an impact on the distance that ships need to cross.

#### **Answers**

#### **Communicate the Ideas**

**1.** Example: The dotted lines show how to interpolate values between known values. The dashed line shows how to extrapolate values beyond known values.



Look for at least one similarity and one difference. Examples: Similarities:

- · Both can be used to determine unknown values.
- Either should be used only when it is reasonable to have values between or beyond known values on a graph.

Differences:

- Interpolation is used to find values between known values.
- Extrapolation is used to find values beyond known values.

- **2.** Example: No, Grace is not correct. It may be reasonable to interpolate values on Graph B because partial kilometres or dollar values may be possible. It is not reasonable to interpolate values on Graph A because it is not possible to buy partial cartons of ice cream.
- **3.** Example: Question: How much will it cost to send an 800 g package? Answer: \$5



## Answers

## **Math Link**

a) Example:

Number of Kedges, <i>k</i>	Distance Travelled, d (km)
1	0.65
100	65
500	325
1000	650
2000	1300



- c) 16 932 kedges to cross the ITCZ
- d) As a class, have students describe the skills learned in Chapter 6.

Assessment	Supporting Learning	
Assessment as Learning		
<b>Communicate the Ideas</b> Have all students complete #1 and 2.	<ul> <li>Encourage students to verbalize their thinking.</li> <li>Have students work with a partner.</li> <li>Have students discuss their response to #1. Clarify any misunderstandings before having students complete #2.</li> <li>Have students discuss their answers for #3. You may wish to distribute Master 2 Communication Peer Evaluation, which students can use for peer evaluation.</li> </ul>	
Assessment <i>for</i> Learning		
<b>Practise and Apply</b> Have students do #4, 7, 8, 11, 14, and 15. Students who have no problems with these questions can go on to the remaining Apply questions.	<ul> <li>Provide additional coaching with Example 1 to students who need assistance with #4. Help students work through #4 and ask them to verbalize the process of interpolation. Have them try #5 or 6 on their own.</li> <li>Provide additional coaching with Example 2 to students who need assistance with #8. Help students work through #8 and ask them to verbalize the process of extrapolation. Have them try #9 or 10 on their own.</li> <li>Clarify any misunderstandings with #7 before having students try #11.</li> <li>Encourage students who need help with #14 and 15 to verbalize the information each respective graph provides. For #14, ask questions such as the following: <ul> <li>Is it reasonable to assume that there are more than 2000 people at a game? What assumptions are you making?</li> </ul> </li> </ul>	
Math Link The Math Link on page 230 gives students practice with linear relationships and extrapolating values on a graph and helps them prepare for the chapter problem titled Wrap It Up! on page 247.	<ul> <li>It is not necessary for students to do the Math Link as it is not required to complete the Wrap It Up! However, doing so may help students who would benefit from practice with extrapolation. It would also serve as an excellent example for students to place in their Foldable as an Assessment <i>as</i> Learning piece.</li> <li>Consider having a small group of students act out kedging while students create a table of values or a graph either individually or on the board.</li> <li>Students may wish to use this as an example of extrapolation in their Foldable.</li> <li>Students who need help getting started could use BLM 6–8 Section 6.2 Math Link, which provides scaffolding.</li> </ul>	
Assessment as Learning		
<b>Literacy Link</b> By the end of section 6.2, have students complete the fifth and sixth boxes of the sequence chart.	• Ask students to work with a partner and explain how to interpolate and extrapolate values on a graph using the situation they developed.	
<ul> <li>Math Learning Log</li> <li>Have students respond to the following prompts:</li> <li>The difference between discrete and continuous data is</li> <li>Interpolation can be used when</li> <li>Extrapolation can be used when</li> </ul>	<ul> <li>Students may find it easier to respond to the prompts by referring to sample graphs such as those in #13 and 14. Consider providing an example such as the following: You are managing a hamburger stand at a local event and decide to graph sales. Would the graph show continuous or discrete data? Is it reasonable to extrapolate or interpolate from the graph?</li> <li>Depending on students' learning styles, have them provide oral or written responses.</li> <li>Encourage students to add notes to the definitions, examples, and Key Ideas for this section in their Foldable.</li> <li>Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with.</li> </ul>	