6.3 Graphing Linear Relations

***MathLinks 9*, pages 231–243**

Suggested Timing

80–100 minutes

Materials

- grid paper
- ruler

Blackline Masters

Master 8 Centimetre Grid Paper
Master 9 0.5 Centimetre Grid Paper
BLM 6–3 Chapter 6 Warm-Up
BLM 6–9 Method 3: Use a Graphing Calculator
BLM 6–10 Section 6.3 Extra Practice
BLM 6–11 Section 6.3 Math Link

Mathematical Processes

- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

Specific Outcomes

PR2 Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#2–4, 6–8, 10, 12a), b), 13, Math Link
Typical	#2–4, 6–8, 10, 12a), b), 13, Math Link
Extension/Enrichment	#2, 3, 7, 16–21, Math Link

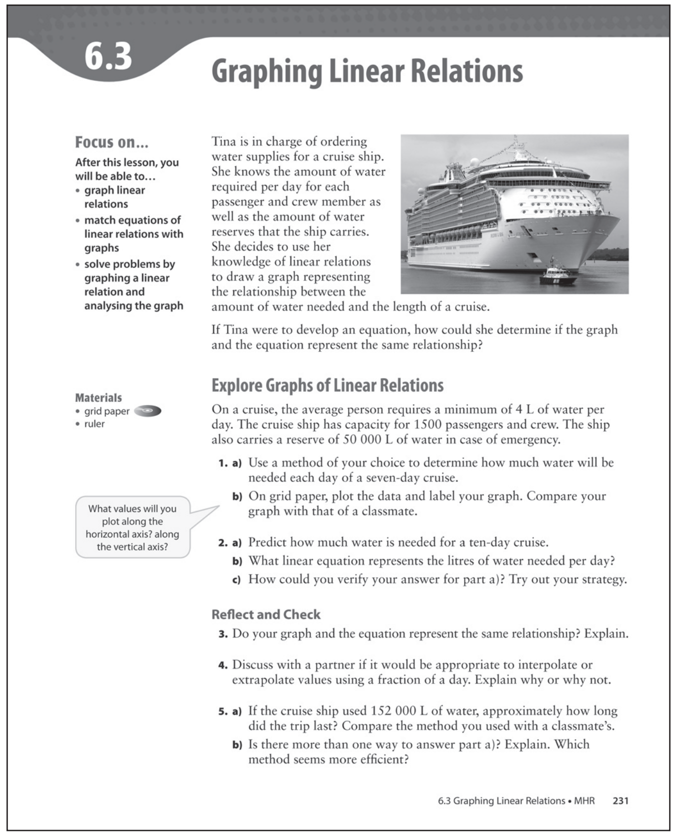
Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce material learned in previous sections.

As a class, read the opening paragraph as an introduction to the Explore.

Explore Graphs of Linear Relations

In this exploration, students graph a linear relation and develop an equation that models the water requirements for a cruise ship.



6.3 Graphing Linear Relations

Focus on…

After this lesson, you will be able to…
- graph linear relations
- match equations of linear relations with graphs
- solve problems by graphing a linear relation and analysing the graph

Tina is in charge of ordering water supplies for a cruise ship. She knows the amount of water required per day for each passenger and crew member as well as the amount of water reserves that the ship carries. She decides to use her knowledge of linear relations to draw a graph representing the relationship between the amount of water needed and the length of a cruise.

If Tina were to develop an equation, how could she determine if the graph and the equation represent the same relationship?

Explore Graphs of Linear Relations

Materials
- grid paper
- ruler

On a cruise, the average person requires a minimum of 4 L of water per day. The cruise ship has capacity for 1500 passengers and crew. The ship also carries a reserve of 50 000 L of water in case of emergency.

1. a) Use a method of your choice to determine how much water will be needed each day of a seven-day cruise.
b) On grid paper, plot the data and label your graph. Compare your graph with that of a classmate.

What values will you plot along the horizontal axis? along the vertical axis?

2. a) Predict how much water is needed for a ten-day cruise.
b) What linear equation represents the litres of water needed per day?
c) How could you verify your answer for part a)? Try out your strategy.

Reflect and Check

3. Do your graph and the equation represent the same relationship? Explain.

4. Discuss with a partner if it would be appropriate to interpolate or extrapolate values using a fraction of a day. Explain why or why not.

5. a) If the cruise ship used 152 000 L of water, approximately how long did the trip last? Compare the method you used with a classmate's.
b) Is there more than one way to answer part a)? Explain. Which method seems more efficient?

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Method 1 Have students work in pairs to complete the Explore. Make **Master 8 Centimetre Grid Paper** and **Master 9 0.5 Centimetre Grid Paper** available to students.

Some students may question whether exactly the same amount of water is used daily. This is an opportunity to discuss that the coefficient in this situation is based on average daily use. You might discuss other assumptions that were made in this scenario (e.g., assume no special situations will arise).

Encourage students to use a method of their choice for #1a). Note that #1b) requires them to use skills developed earlier in the chapter. Invite pairs to discuss the methods they used for #1a) and 5a) with another student pair. As a class, discuss the advantages and disadvantages of each method, and the most efficient methods.

Method 2 Divide the class into three sections and assign each section a different method to answer the questions. Compare the results and the ease of obtaining answers. As a class, answer #3 to 5.

Meeting Student Needs

- You might discuss why a cruise ship needs to carry water. What might the reserve water be used for? You might have students relate to the way water is supplied to some communities, such as homes in remote communities, or trips over land that require carrying water. Ask what happens if water reserves are not monitored. You might adapt the situation in the Explore by having students determine how much water is needed per person in a community each day for a week after a natural disaster, and include an amount for water reserves for other purposes.
- Coach students to recall how to use an equation to generate an x- or y-coordinate, given a known coordinate.
- For the graph, help students recall which variable to plot along the horizontal axis (day) and which variable to plot along the vertical axis (amount of water). For the equation, remind students to express the y-variable in terms of the x-variable.

ELL

- Teach the following terms in context: *cruise ship*, *passenger*, *crew member*, and *water reserves*.
- Ensure students understand all steps in the Explore. Allow students to discuss their ideas using their first language before expressing them in English.

Gifted and Enrichment

- Have students consider how the quantity of water required by the cruise ship compares to the needs on a cargo ship or commuter ferry, or to that required during an adventure in a remote area with limited drinking water.

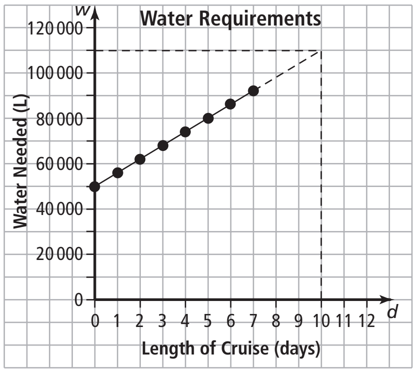
Answers

Explore Graphs of Linear Relations

1. a) Example:

Length of Cruise, d (days)	Water Needed, w (L)
0	50 000
1	56 000
2	62 000
3	68 000
4	74 000
5	80 000
6	86 000
7	92 000

b)



2. a) Example: 110 000 L

b) Example: $w = 4pd + 50\ 000$, where p is number of people and d is number of days

c) Substitute $d = 10$ into the equation and solve for w.
$110\ 000 = 4(1500)(10) + 50\ 000$

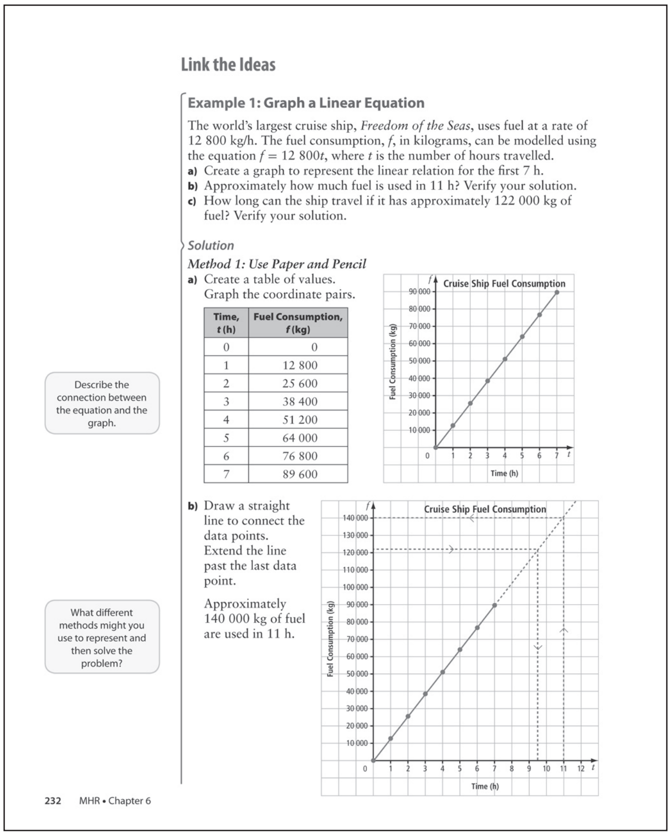
3. Yes, the graph of the line represents the equation.The numerical coefficient shows where the line goes and the constant shows the value of the graph at zero days.

4. Example: No, it is not appropriate to interpolate or extrapolate values using a fraction of a day because we do not know when during the day the water is used by the passengers or crew.

5. a) 17 days. Example: Substitute and solve.

b) Example: Yes, you could extrapolate the graph or use the equation to substitute and solve. Using the equation is more efficient.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Check that students are able to determine that a graph and an equation represent the same relationship.	• For #3, encourage students to explain how to determine whether a graph and an equation represent the same relationship. Have them develop an example and record the response in their Foldable. • Help students who would benefit to recall how to use the equation to generate the x- or y-coordinate, when given one coordinate. • For #5, have students explain their choice of methods and which method is more efficient. Encourage class discussion to benefit students who are uncertain about the conclusions.



Link the Ideas

Example 1: Graph a Linear Equation

The world's largest cruise ship, *Freedom of the Seas*, uses fuel at a rate of 12 800 kg/h. The fuel consumption, f, in kilograms, can be modelled using the equation $f = 12\ 800t$, where t is the number of hours travelled.

a) Create a graph to represent the linear relation for the first 7 h.
b) Approximately how much fuel is used in 11 h? Verify your solution.
c) How long can the ship travel if it has approximately 122 000 kg of fuel? Verify your solution.

Solution

Method 1: Use Paper and Pencil

a) Create a table of values. Graph the coordinate pairs.

Time, t (h)	Fuel Consumption, f (kg)
0	0
1	12 800
2	25 600
3	38 400
4	51 200
5	64 000
6	76 800
7	89 600

Describe the connection between the equation and the graph.

b) Draw a straight line to connect the data points. Extend the line past the last data point.

Approximately 140 000 kg of fuel are used in 11 h.

What different methods might you use to represent and then solve the problem?

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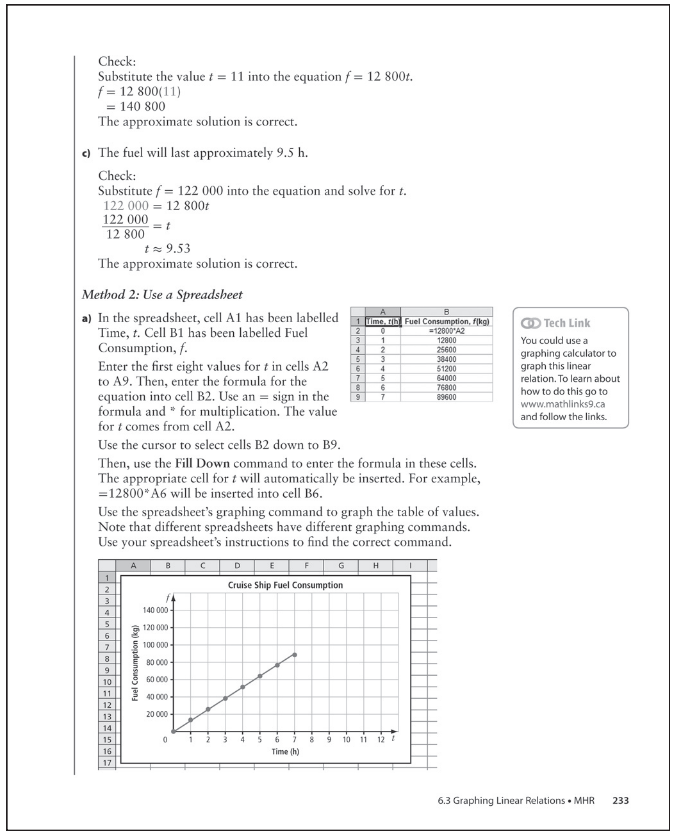

Check:
Substitute the value $t = 11$ into the equation $f = 12\ 800t$.
$f = 12\ 800(11)$
$= 140\ 800$
The approximate solution is correct.

c) The fuel will last approximately 9.5 h.

Check:
Substitute $f = 122\ 000$ into the equation and solve for t.
$122\ 000 = 12\ 800t$
$\frac{122\ 000}{12\ 800} = t$
$t \approx 9.53$
The approximate solution is correct.

Method 2: Use a Spreadsheet

a) In the spreadsheet, cell A1 has been labelled Time, t. Cell B1 has been labelled Fuel Consumption, f.

Enter the first eight values for t in cells A2 to A9. Then, enter the formula for the equation into cell B2. Use an = sign in the formula and * for multiplication. The value for t comes from cell A2.

	A	B
1	Time, t(h)	Fuel Consumption, f(kg)
2	0	=12800*A2
3	1	12800
4	2	25600
5	3	38400
6	4	51200
7	5	64000
8	6	76800
9	7	89600

Tech Link
You could use a graphing calculator to graph this linear relation. To learn about how to do this go to www.mathlinks9.ca and follow the links.

Use the cursor to select cells B2 down to B9.

Then, use the **Fill Down** command to enter the formula in these cells. The appropriate cell for t will automatically be inserted. For example, =12800*A6 will be inserted into cell B6.

Use the spreadsheet's graphing command to graph the table of values. Note that different spreadsheets have different graphing commands. Use your spreadsheet's instructions to find the correct command.

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Link the Ideas

Example 1

Example 1 demonstrates creating a graph to represent a linear relation involving fuel consumption on a cruise ship, solving problems involving extrapolation, and verifying the solutions.

As a class, read the introduction. Explain that *Freedom of the Seas* is a member of the Royal Caribbean cruise line. This cruise line does have scheduled departures out of Vancouver, BC. Consider discussing assumptions made in order for fuel consumption to represent a linear relation. For example, assumptions are made that the ship travels at the same constant speed all the time, the winds blow in the same direction and with the same intensity all the time, and the mass of the ship (which is actually decreasing as fuel is used) remains constant.

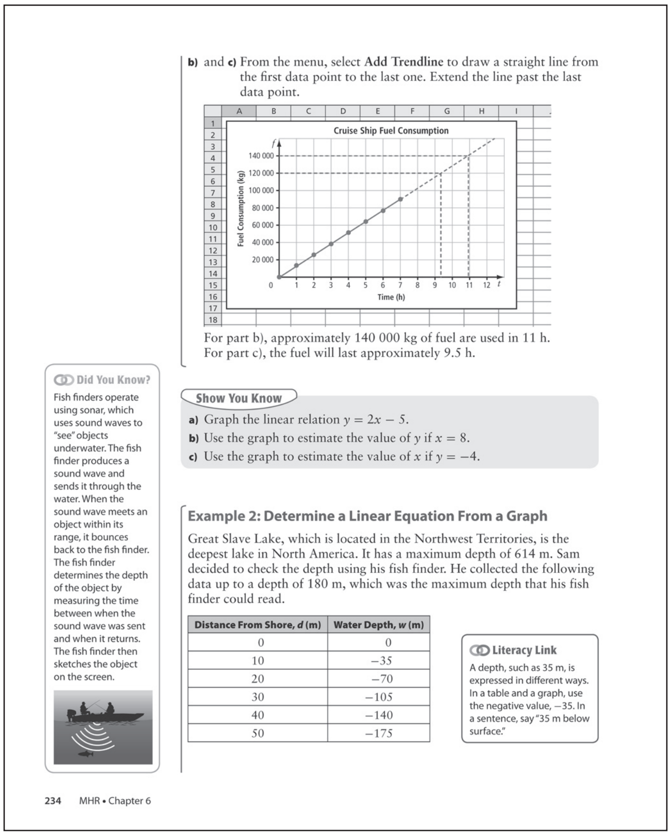
You might explain that the norm is to use kg/h as the units of measurement, similar to airplanes. These units make it easier to calculate the impact of changes on the overall mass of a ship. For instance, as a ship becomes lighter, it becomes faster and its centre of gravity changes. As a result, materials called ballast are shifted around to keep the ship stable.

Method 1 uses paper and pencil to solve the problem. For part a), ask students to describe the connection between the graph and the equation to help make sense of the problem context and the representations.

For part b), discuss how else students might solve the problem (e.g., substitute values into the equation). Have students explain why they would use the methods they suggested.

Method 2 uses a spreadsheet program to solve the problem. Note that instructions may vary among different versions of software. Ensure you are familiar with the software that students will use so that you can provide coaching when the method varies from the written instructions. Encourage students to use estimation skills to verify whether they set up the spreadsheet properly (i.e., the results look reasonable).

In addition to extending the trendline, students should recognize that they could continue the **Fill Down** command in part a) to automatically create a table of values that includes the unknown values. To find instructions on how to extend the trendline in the particular spreadsheet program students are using, they may have to search the Help file for a term such as *forecast*.



b) and **c)** From the menu, select **Add Trendline** to draw a straight line from the first data point to the last one. Extend the line past the last data point.

For part b), approximately 140 000 kg of fuel are used in 11 h. For part c), the fuel will last approximately 9.5 h.

Did You Know?

Fish finders operate using sonar, which uses sound waves to "see" objects underwater. The fish finder produces a sound wave and sends it through the water. When the sound wave meets an object within its range, it bounces back to the fish finder. The fish finder determines the depth of the object by measuring the time between when the sound wave was sent and when it returns. The fish finder then sketches the object on the screen.

Show You Know

a) Graph the linear relation $y = 2x - 5$.
b) Use the graph to estimate the value of y if $x = 8$.
c) Use the graph to estimate the value of x if $y = -4$.

Example 2: Determine a Linear Equation From a Graph

Great Slave Lake, which is located in the Northwest Territories, is the deepest lake in North America. It has a maximum depth of 614 m. Sam decided to check the depth using his fish finder. He collected the following data up to a depth of 180 m, which was the maximum depth that his fish finder could read.

Distance From Shore, *d* (m)	Water Depth, *w* (m)
0	0
10	−35
20	−70
30	−105
40	−140
50	−175

Literacy Link

A depth, such as 35 m, is expressed in different ways. In a table and a graph, use the negative value, −35. In a sentence, say "35 m below surface."

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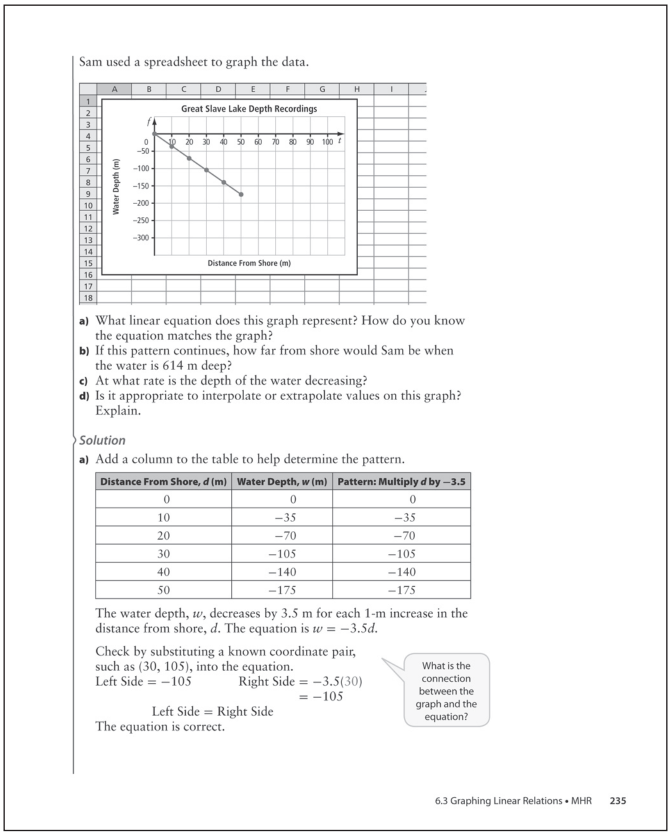

Sam used a spreadsheet to graph the data.

a) What linear equation does this graph represent? How do you know the equation matches the graph?
b) If this pattern continues, how far from shore would Sam be when the water is 614 m deep?
c) At what rate is the depth of the water decreasing?
d) Is it appropriate to interpolate or extrapolate values on this graph? Explain.

Solution

a) Add a column to the table to help determine the pattern.

Distance From Shore, *d* (m)	Water Depth, *w* (m)	Pattern: Multiply *d* by −3.5
0	0	0
10	−35	−35
20	−70	−70
30	−105	−105
40	−140	−140
50	−175	−175

The water depth, w, decreases by 3.5 m for each 1-m increase in the distance from shore, d. The equation is $w = -3.5d$.

Check by substituting a known coordinate pair, such as (30, 105), into the equation.
Left Side = −105 Right Side = −3.5(30) = −105
Left Side = Right Side
The equation is correct.

What is the connection between the graph and the equation?

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Note that the Tech Link provides students with a third method for solving the problem by using a graphing calculator. It is called Method 3: Use a Graphing Calculator. You may wish to distribute **BLM 6–9 Method 3: Use a Graphing Calculator** and have students use it to graph the linear relation.

Have students consider the (two or) three methods. Ask:

- Which method do you prefer? Why?
- Is each method equally convenient to use? Explain.

Consider having students work in pairs to do the Show You Know. You might encourage them to try using a spreadsheet program after they solve the problem using paper and pencil. Have student pairs compare their solution with another student pair. If the solutions differ, have them check each other's solutions to find out why.

Example 2

Example 2 demonstrates determining a linear equation from a graph of a real world situation. Read the problem as a class. Have students who may be unfamiliar with a *fish finder* read and discuss the information in the Did You Know?

For part a), have students connect the graph and the equation and explain where the numerical coefficient and the constant are derived from. Reinforce how to check that the equation matches the graph.

Prompt students to realize that it is more convenient to use the equation for answering part b). Ask why (graph provided does not extend to 614; 614 m is a precise value given the scale on the *y*-axis). You might have students try answering the question by drawing a graph.

Part c) asks students to determine a new value: rate. They might describe the difference between data points on the graph or in the table, refer to the coefficient in the equation, or examine the slope of the line (although slope is not required knowledge).

Part d) asks students to think about whether interpolation or extrapolation is appropriate for this type of data. Note that most lake bottoms are not even gradual descents, but are made up of levels and drop-offs.

Literacy Link Direct students to the Literacy Link on page 234 that explains how to express a depth. You might have students practise using a different example such as expressing a depth of 500 m below sea level.

Strategies
Solve an Equation

b) Substitute $w = 614$ into the equation and solve for d.

$$-614 = -3.5d$$
$$\frac{-614}{-3.5} = d$$
$$d \approx 175.4$$

Sam would be approximately 175.4 m from shore when the water is 614 m deep.

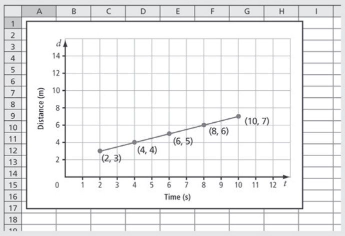
How else could you solve this problem?

c) The depth is decreasing at a rate of 3.5 m for each metre away from shore. The rate at which the water depth is decreasing is the coefficient of d in the equation.

d) Yes, it is reasonable to interpolate or extrapolate values between and beyond the given data points since the values for distance and depth exist. However, it is unreasonable to extrapolate values beyond the maximum depth of 614 m.

Show You Know

Identify the linear equation that represents the graph.



Example 3: Graph Horizontal and Vertical lines

For each table of values, answer the following questions:

Table 1

Time, t (s)	Distance, d (m)
0	6
30	6
60	6
90	6
120	6

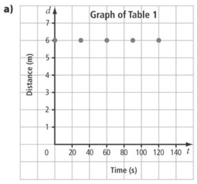
Table 2

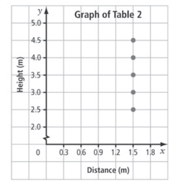
Distance, x (m)	Height, y (m)
1.5	2.5
1.5	3.0
1.5	3.5
1.5	4.0
1.5	4.5

a) Draw a graph to represent the table of values.
b) Describe a situation that the graph might represent.
c) Write the equation. Explain how you know the graph represents the equation.

Solution

a)





b) Table 1: The graph could show the relationship between distance and time when a pedestrian is waiting for a traffic light to change. The distance from the pedestrian to the opposite side of the road is constant.

Table 2: The graph could show the relationship between the height of a ladder and its distance from the wall where it is placed. The distance of the base of the ladder from the wall is constant as the ladder is extended.

Think of a different situation to represent each graph.

c) Table 1: The distance, d, remains constant for each interval of time. The equation is $d = 6$.
For each value of t in the table and the graph, the value of d is 6.

Table 2: The distance, x, remains constant for each interval of height. The equation is $x = 1.5$.
For each value of y in the table and the graph, the value of x is 1.5.

Have students work on their own to complete the Show You Know. Then, have students compare their equation with the one of a classmate. If there are discrepancies, have students discuss how to verify the equation, and then do so.

Example 3

Example 3 demonstrates graphs of horizontal and vertical lines. For part b), discuss with the class other possible situations where a linear graph is horizontal (e.g., value of money saved in a piggy bank over time as opposed to earning interest in an account) or vertical (e.g., force of gravity from first to tenth floor).

Have students work on their own to complete the Show You Know. You may wish to have students compare their equation with the one of a classmate, and then show how they know the graph matches the equation by substituting values.

Key Ideas

The Key Ideas summarize how to graph a linear relation represented by an equation. You might have students describe the relationship between the equation, the table of values, and the graph in the example.

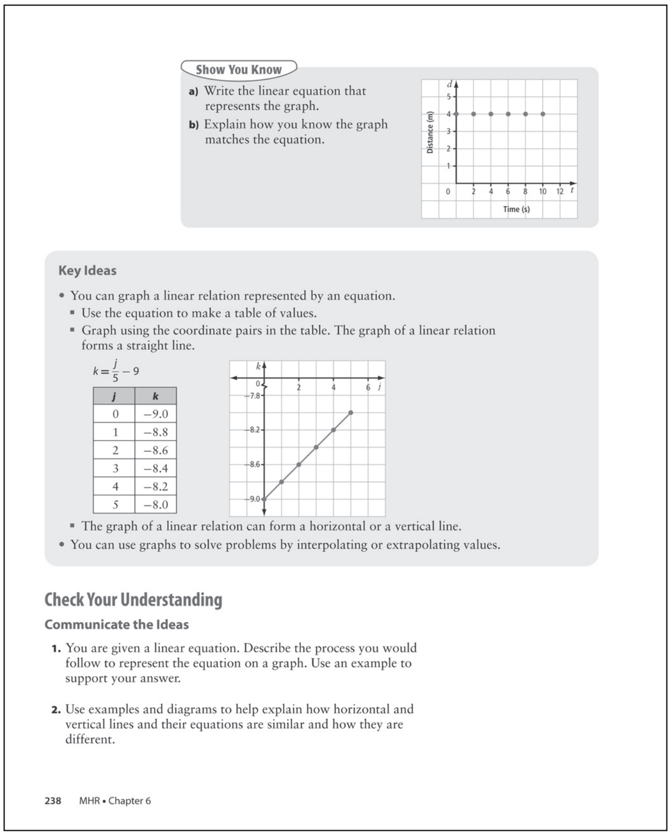
Have students prepare their own summary of the Key Ideas in the section 6.3 booklet of their Foldable.

Meeting Student Needs

- Some students may need additional time to complete this section. Consider a focus on Example 1 during one period and Examples 2 and 3 during a second period.
- Some students may benefit from additional examples to check their understanding of when it is reasonable to interpolate or extrapolate values.
- Assist students in solving equations by helping them to recall the order of operations and the corresponding inverse operations.

ELL

- Teach the following words in context: *fuel consumption*, *spreadsheet*, *cell*, *cursor*, *deepest lake*, and *pedestrian*.
- Ensure students understand all the steps in the Show You Knows. Allow them to use their first language to discuss ideas and then provide answers in English.



Show You Know

a) Write the linear equation that represents the graph.

b) Explain how you know the graph matches the equation.

Key Ideas

- You can graph a linear relation represented by an equation.
  - Use the equation to make a table of values.
  - Graph using the coordinate pairs in the table. The graph of a linear relation forms a straight line.

$k = \frac{j}{5} - 9$

j	k
0	−9.0
1	−8.8
2	−8.6
3	−8.4
4	−8.2
5	−8.0

  - The graph of a linear relation can form a horizontal or a vertical line.
- You can use graphs to solve problems by interpolating or extrapolating values.

Check Your Understanding

Communicate the Ideas

1. You are given a linear equation. Describe the process you would follow to represent the equation on a graph. Use an example to support your answer.

2. Use examples and diagrams to help explain how horizontal and vertical lines and their equations are similar and how they are different.

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Gifted and Enrichment

- For Example 1, while a linear equation provides a workable method of determining fuel needs, challenge students to find out how the actual fuel consumption would be affected by cruising speed, wind, and reduction in mass as fuel and other supplies are consumed during a cruise. You might have students research airplane fuel consumption to determine how these factors are considered.
- For Example 3, challenge students to represent their trip to school this morning using a distance–time graph. Encourage them to include all modes of travel such as the bus and walking from the bus stop. Use question prompts such as the following:
  - What does the time spent at a red light or stop sign look like?
  - What would the graph look like if you had to go back home to get something?
- Explain to students that because of Earth's gravity, a falling object gains speed steadily as it falls. Without air resistance, a falling object's speed increases by 9.8 m/s, every second. Ask them to plot a graph of speed for the first 10 s. Then, tell them the moon's gravity is only $\frac{1}{6}$ as strong as Earth's and there is no atmosphere to slow the object. Have them estimate how that would affect the speed over time. Challenge students to research the effect of gravity on other planets and create graphs to compare the same object free-falling on each planet.

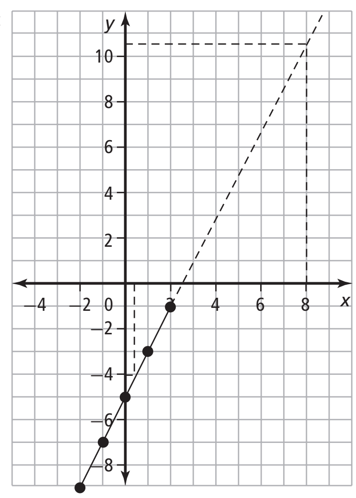
Web Link

For instructions and alternative resources for graphing software, go to www.mathlinks9.ca and follow the links.

Answers

Example 1: Show You Know

a) Example:



b) 11

c) 0.5

Example 2: Show You Know

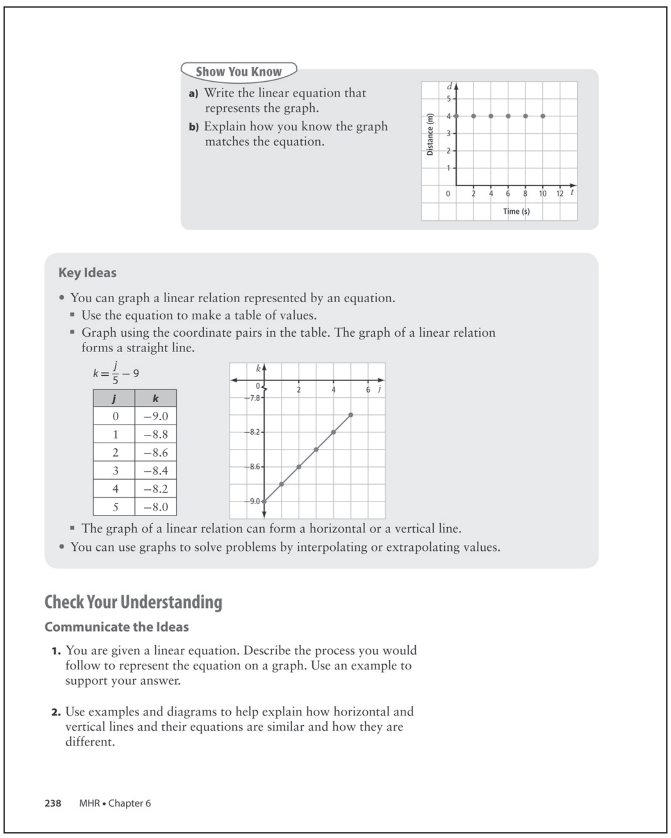
$d = 0.5t + 2$

Example 3: Show You Know

a) $d = 4$

b) Example: For each value of t, the value of d is always 4.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Check that students are able to use an equation to generate a table of values and the corresponding graph.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to develop a table of values from the graph in order to help determine the coefficient and constant values in the equation. Check that students know how to derive the coefficient and the constant in the equation from the table of values. • Encourage students to verify their solution.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Ensure that students understand the difference between the graphs for a vertical line and a horizontal line and the corresponding equations. • Help students generalize equations for a horizontal line ($y = \#$) and a vertical line ($x = \#$).


Show You Know

a) Write the linear equation that represents the graph.

b) Explain how you know the graph matches the equation.

Key Ideas

- You can graph a linear relation represented by an equation.
  - Use the equation to make a table of values.
  - Graph using the coordinate pairs in the table. The graph of a linear relation forms a straight line.

$k = \frac{j}{5} - 9$

j	k
0	−9.0
1	−8.8
2	−8.6
3	−8.4
4	−8.2
5	−8.0

  - The graph of a linear relation can form a horizontal or a vertical line.
- You can use graphs to solve problems by interpolating or extrapolating values.

Check Your Understanding

Communicate the Ideas

1. You are given a linear equation. Describe the process you would follow to represent the equation on a graph. Use an example to support your answer.

2. Use examples and diagrams to help explain how horizontal and vertical lines and their equations are similar and how they are different.

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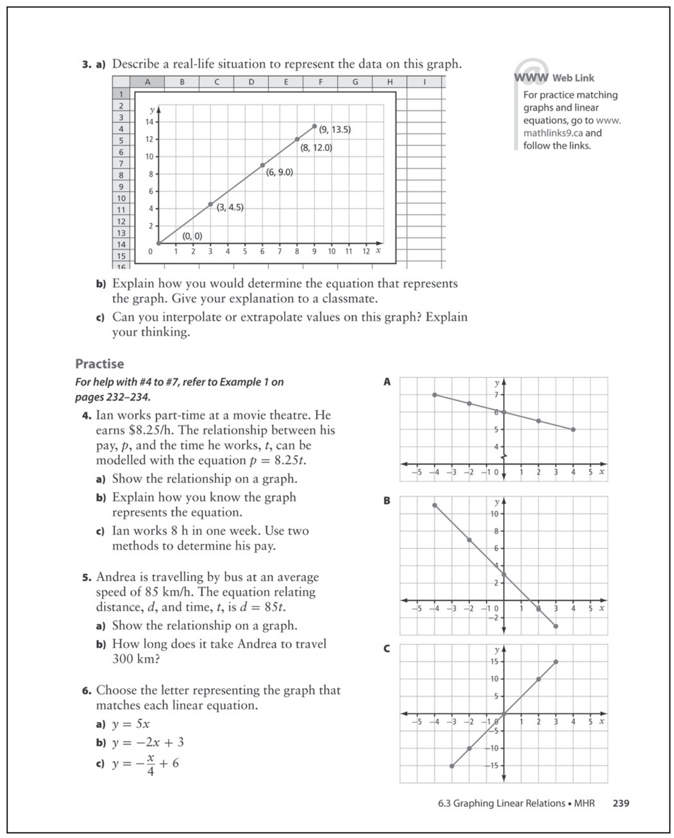
3. a) Describe a real-life situation to represent the data on this graph.

b) Explain how you would determine the equation that represents the graph. Give your explanation to a classmate.

c) Can you interpolate or extrapolate values on this graph? Explain your thinking.

WWW Web Link

For practice matching graphs and linear equations, go to www.mathlinks9.ca and follow the links.

Practise

For help with #4 to #7, refer to Example 1 on pages 232–234.

4. Ian works part-time at a movie theatre. He earns $8.25/h. The relationship between his pay, p, and the time he works, t, can be modelled with the equation $p = 8.25t$.
   a) Show the relationship on a graph.
   b) Explain how you know the graph represents the equation.
   c) Ian works 8 h in one week. Use two methods to determine his pay.

5. Andrea is travelling by bus at an average speed of 85 km/h. The equation relating distance, d, and time, t, is $d = 85t$.
   a) Show the relationship on a graph.
   b) How long does it take Andrea to travel 300 km?

6. Choose the letter representing the graph that matches each linear equation.
   a) $y = 5x$
   b) $y = -2x + 3$
   c) $y = -\frac{x}{4} + 6$

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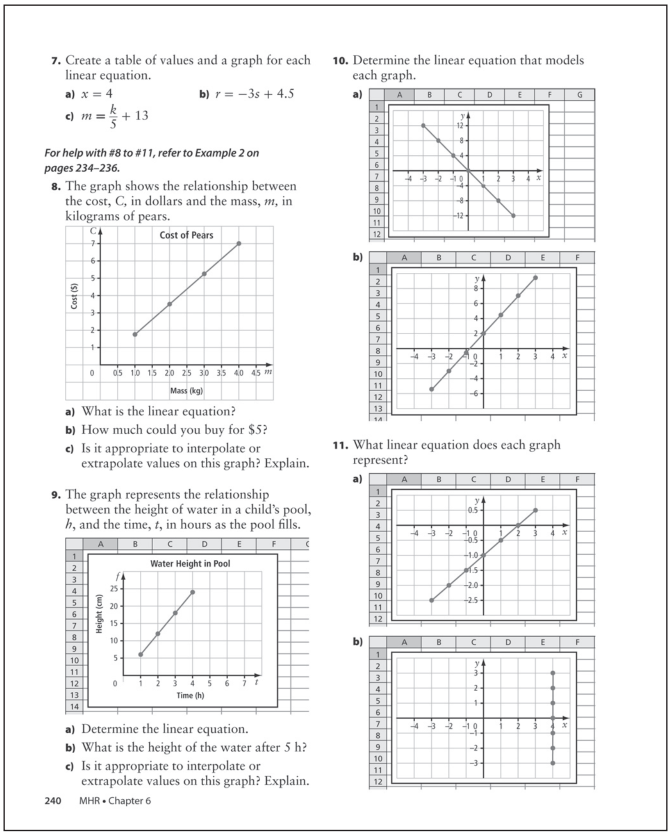
Check Your Understanding

Communicate the Ideas

These questions allow students to explain their understanding of graphing linear equations, analysing graphs and equations of horizontal and vertical lines, and developing linear equations from a graph.

In #1, students describe the process for developing a graph from an equation, which represents a high degree of conceptual understanding.

In #2, students explain similarities and differences between horizontal and vertical lines and their equations, which will reinforce their learning about the meaning of these graphs.



7. Create a table of values and a graph for each linear equation.

a) $x = 4$ **b)** $r = -3s + 4.5$

c) $m = \frac{k}{5} + 13$

For help with #8 to #11, refer to Example 2 on pages 234–236.

8. The graph shows the relationship between the cost, *C*, in dollars and the mass, *m*, in kilograms of pears.

Cost of Pears

a) What is the linear equation?

b) How much could you buy for $5?

c) Is it appropriate to interpolate or extrapolate values on this graph? Explain.

9. The graph represents the relationship between the height of water in a child's pool, *h*, and the time, *t*, in hours as the pool fills.

Water Height in Pool

a) Determine the linear equation.

b) What is the height of the water after 5 h?

c) Is it appropriate to interpolate or extrapolate values on this graph? Explain.

10. Determine the linear equation that models each graph.

a)

b)

11. What linear equation does each graph represent?

a)

b)

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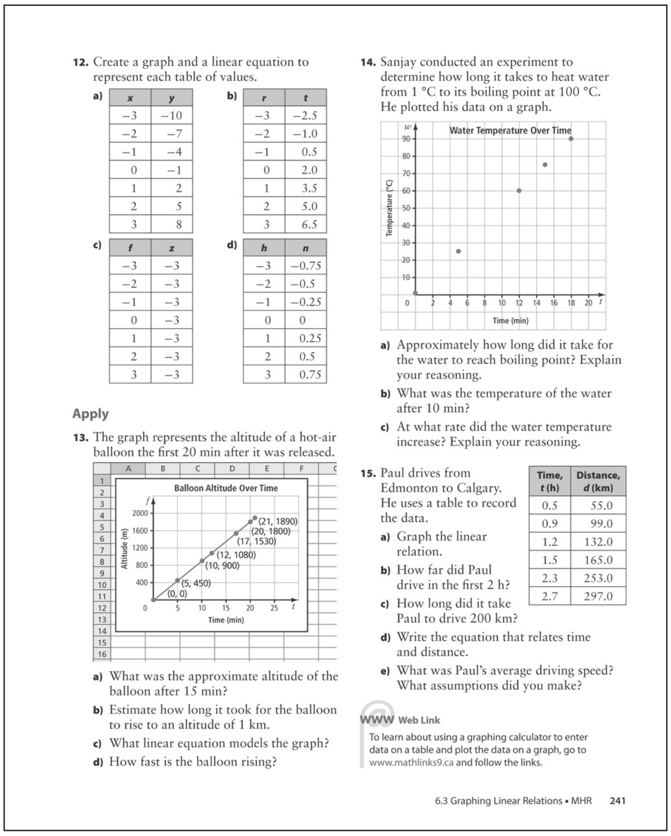

12. Create a graph and a linear equation to represent each table of values.

a)

x	*y*
−3	−10
−2	−7
−1	−4
0	−1
1	2
2	5
3	8

b)

r	*t*
−3	−2.5
−2	−1.0
−1	0.5
0	2.0
1	3.5
2	5.0
3	6.5

c)

f	*z*
−3	−3
−2	−3
−1	−3
0	−3
1	−3
2	−3
3	−3

d)

h	*n*
−3	−0.75
−2	−0.5
−1	−0.25
0	0
1	0.25
2	0.5
3	0.75

Apply

13. The graph represents the altitude of a hot-air balloon the first 20 min after it was released.

Balloon Altitude Over Time

a) What was the approximate altitude of the balloon after 15 min?

b) Estimate how long it took for the balloon to rise to an altitude of 1 km.

c) What linear equation models the graph?

d) How fast is the balloon rising?

14. Sanjay conducted an experiment to determine how long it takes to heat water from 1 °C to its boiling point at 100 °C. He plotted his data on a graph.

Water Temperature Over Time

a) Approximately how long did it take for the water to reach boiling point? Explain your reasoning.

b) What was the temperature of the water after 10 min?

c) At what rate did the water temperature increase? Explain your reasoning.

15. Paul drives from Edmonton to Calgary. He uses a table to record the data.

Time, *t* (h)	Distance, *d* (km)
0.5	55.0
0.9	99.0
1.2	132.0
1.5	165.0
2.3	253.0
2.7	297.0

a) Graph the linear relation.

b) How far did Paul drive in the first 2 h?

c) How long did it take Paul to drive 200 km?

d) Write the equation that relates time and distance.

e) What was Paul's average driving speed? What assumptions did you make?

WWW Web Link

To learn about using a graphing calculator to enter data on a table and plot the data on a graph, go to www.mathlinks9.ca and follow the links.

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In #3, students describe a situation modelled by a given graph. Have them present their explanation for parts b) and c) to a classmate.

Practise

The Practise questions give students practice with graphing equations and developing linear equations.

The following pairs of questions are very similar. Some students may be given a choice of completing any two questions in each set: #4 and 5 and #8 and 9.

Apply

The Apply questions provide a range of contexts for solving problems related to linear equations. You might assign questions based on student interest and/or familiarity with the contexts. Consider allowing students to have some choice in the questions they complete. For example, you might assign #13 and then have students select two additional questions.

For #12, consider allowing students to use spreadsheet software or a graphing calculator.

For #13, check students' understanding of extrapolation by asking if it would be reasonable to estimate values beyond what is shown. At some point, the increase in altitude will slow down and remain constant. (You might mention that passengers in a hot-air balloon are in the open atmosphere.)

For #14, students may recall from previous science classes that the temperature–time data from boiling water forms a curve. While the experimental data may reflect what happened in a trial, this is an opportunity for students to discuss that it is not appropriate to interpolate or extrapolate from this data and that a linear equation does not represent this situation.

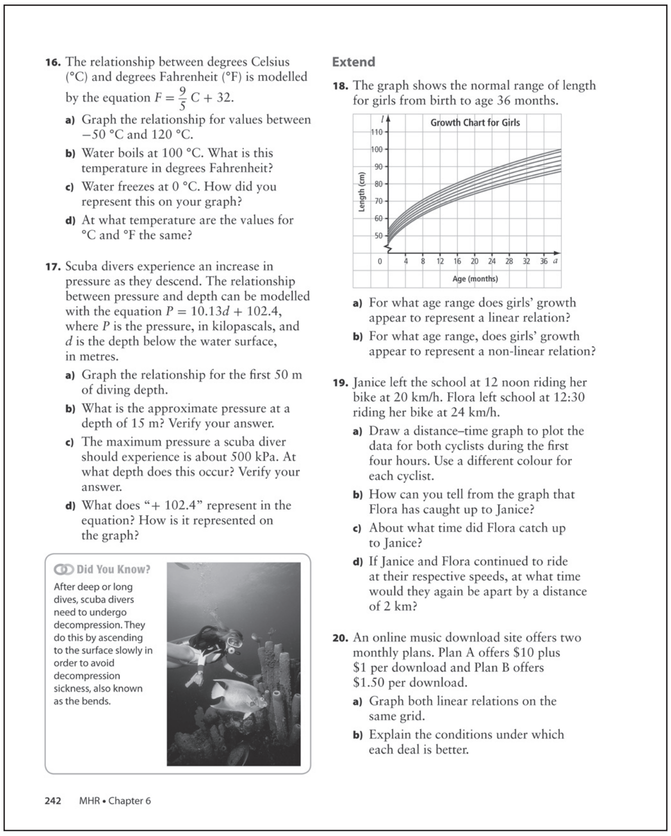
For #17, direct students' attention to the Did You Know? about decompression sickness.

Extend

In #18, which is more of a conceptual than a procedural question, point out that *non-linear* means not linear. Students could use a ruler to evaluate the graph visually for parts of the line that are straight.

For #19, students may choose to plot the data for each cyclist on separate graphs. If so, ensure they use the same scale on both graphs in order to make comparisons.

For #21, explain any unfamiliar concepts. *Principal* is the amount that is initially invested. *Payable at 3.5%* means a client is paid 3.5% of the original investment each year but the interest is kept separate from the principal until the end of the ten-year term.



16. The relationship between degrees Celsius (°C) and degrees Fahrenheit (°F) is modelled by the equation $F = \frac{9}{5}C + 32$.
   a) Graph the relationship for values between −50 °C and 120 °C.
   b) Water boils at 100 °C. What is this temperature in degrees Fahrenheit?
   c) Water freezes at 0 °C. How did you represent this on your graph?
   d) At what temperature are the values for °C and °F the same?

17. Scuba divers experience an increase in pressure as they descend. The relationship between pressure and depth can be modelled with the equation $P = 10.13d + 102.4$, where P is the pressure, in kilopascals, and d is the depth below the water surface, in metres.
   a) Graph the relationship for the first 50 m of diving depth.
   b) What is the approximate pressure at a depth of 15 m? Verify your answer.
   c) The maximum pressure a scuba diver should experience is about 500 kPa. At what depth does this occur? Verify your answer.
   d) What does "+ 102.4" represent in the equation? How is it represented on the graph?

Did You Know?

After deep or long dives, scuba divers need to undergo decompression. They do this by ascending to the surface slowly in order to avoid decompression sickness, also known as the bends.

Extend

18. The graph shows the normal range of length for girls from birth to age 36 months.

   a) For what age range does girls' growth appear to represent a linear relation?
   b) For what age range, does girls' growth appear to represent a non-linear relation?

19. Janice left the school at 12 noon riding her bike at 20 km/h. Flora left school at 12:30 riding her bike at 24 km/h.
   a) Draw a distance–time graph to plot the data for both cyclists during the first four hours. Use a different colour for each cyclist.
   b) How can you tell from the graph that Flora has caught up to Janice?
   c) About what time did Flora catch up to Janice?
   d) If Janice and Flora continued to ride at their respective speeds, at what time would they again be apart by a distance of 2 km?

20. An online music download site offers two monthly plans. Plan A offers $10 plus $1 per download and Plan B offers $1.50 per download.
   a) Graph both linear relations on the same grid.
   b) Explain the conditions under which each deal is better.

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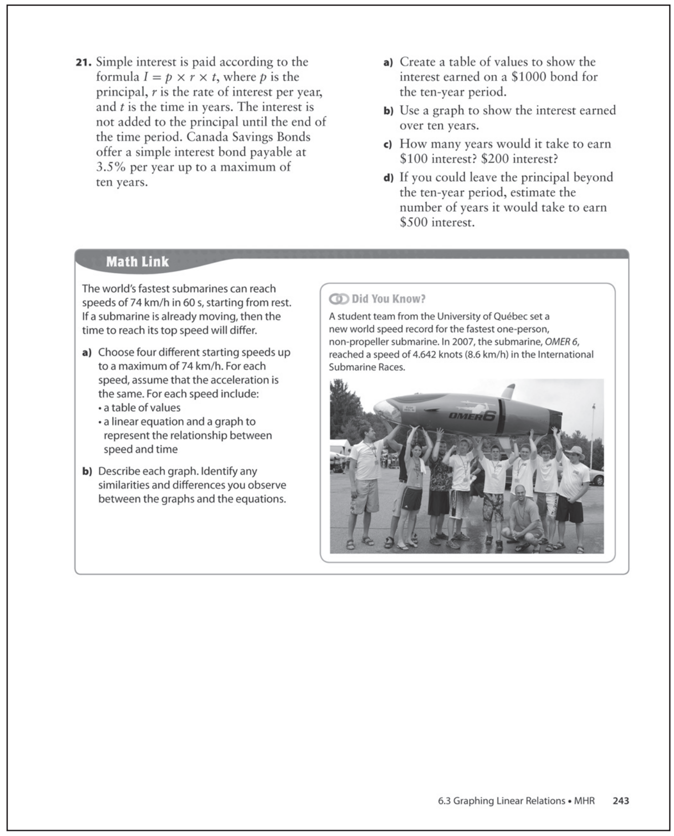


21. Simple interest is paid according to the formula $I = p \times r \times t$, where p is the principal, r is the rate of interest per year, and t is the time in years. The interest is not added to the principal until the end of the time period. Canada Savings Bonds offer a simple interest bond payable at 3.5% per year up to a maximum of ten years.
   a) Create a table of values to show the interest earned on a $1000 bond for the ten-year period.
   b) Use a graph to show the interest earned over ten years.
   c) How many years would it take to earn $100 interest? $200 interest?
   d) If you could leave the principal beyond the ten-year period, estimate the number of years it would take to earn $500 interest.

Math Link

The world's fastest submarines can reach speeds of 74 km/h in 60 s, starting from rest. If a submarine is already moving, then the time to reach its top speed will differ.

a) Choose four different starting speeds up to a maximum of 74 km/h. For each speed, assume that the acceleration is the same. For each speed include:
   - a table of values
   - a linear equation and a graph to represent the relationship between speed and time
b) Describe each graph. Identify any similarities and differences you observe between the graphs and the equations.

Did You Know?

A student team from the University of Québec set a new world speed record for the fastest one-person, non-propeller submarine. In 2007, the submarine, *OMER 6*, reached a speed of 4.642 knots (8.6 km/h) in the International Submarine Races.

6.3 Graphing Linear Relations • MHR 243

Literacy Link Using their sequence chart, have students complete the seventh box by describing the process for graphing a linear relation. Encourage students to develop a situation and a linear equation, and then graph the linear equation to help them explain the process.

Math Link

The Math Link provides students with an opportunity to practise creating tables of values, developing equations, and graphing equations.

For part a), students develop four tables of values and their corresponding linear equations and graphs.

For part b), consider having students discuss the similarities and differences between the graphs and equations with a classmate. Then, as a class, discuss students' findings. You might use the graphs and equations to generate discussion about how the numerical coefficients and constants in the equations affect the starting point and slope of the graphs.

Direct students to the Did You Know? about *Omer 6*, which is one of a series of human-powered submarines built by students in electrical, mechanical, software, and automation engineering from the École de technologies supérieure (School of Higher Technology) in Montreal, Québec. ETS is an engineering university member of the University of Québec. As of 2007, the *Omer 6* team held the human-powered submarine world speed record for the one-seater, non-propeller category.

Meeting Student Needs

- For #2, discuss multiple examples of situations that result in graphs with vertical or horizontal lines.
- Consider allowing students to use spreadsheet software or a graphing calculator to develop tables and graphs for selected questions.
- You may wish to assign only two parts of #12, as this should provide sufficient evidence of understanding.
- Provide **BLM 6–10 Section 6.3 Extra Practice** to students who would benefit from more practice.

ELL

- Ensure that English language learners understand the following terms: *hot-air balloon*, *degrees Celsius*, *degrees Fahrenheit*, *scuba diver*, *pressure*, *kilopascals*, *decompression*, *the bends*, *download site*, *principal*, *interest*, *Canada Savings Bonds*, *simple interest bond payable*, *submarines*, *acceleration*, and *non-propeller submarine*. Have them add any new terms to their personal dictionary.
- Assign a greater balance of Practise questions as they are less text dense.
- Ensure students understand what the word problems are asking.

Gifted and Enrichment

- For #17, to prevent decompression sickness, divers rise slowly to the surface. However, this ascent need not be steady or linear. Divers can rise in leaps, stopping to acclimatize along the way. A similar method is used by mountain climbers to avoid altitude sickness. Have students research the rule of thumb or equation that these adventurers use and create a distance–time graph of the recommended techniques.

WWW Web Link

For tutorials on using graphing calculators, go to www.mathlinks9.ca and follow the links.

Answers

Communicate the Ideas

1. Example: To graph a linear relation such as $y = -x + 3$, use the equation to create a table of values. Substitute values for x to find coordinate values of y, and then graph the pairs.

2. Look for at least one similarity and one difference. Examples:
Similarity: Horizontal and vertical lines and their equations have only one variable (e.g., $x = 5$ or $y = 3$).
Difference: Horizontal and vertical lines and their equations differ because the variables represent different axes on a graph.

3. a) Example: The graph represents fundraising revenue from the sale of $1.50 chocolate bars.

b) Example: Create a table of values from the graph, find the pattern in the data, and then determine the equation.

c) Example: Yes, as long as you consider only whole chocolate bars and neither the number of chocolate bars nor the cost is a negative number

Math Link

a) Examples: $s = 1.23t$

Graph A

Time, *t* (s)	Speed, *s* (km/h)
0	36.99
10	49.33
20	61.66
30	74

Graph B

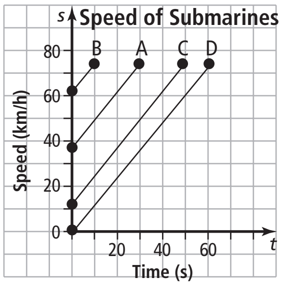
Time, *t* (s)	Speed, *s* (km/h)
0	61.66
10	74

Graph C

Time, *t* (s)	Speed, *s* (km/h)
0	12.33
10	24.66
20	36.99
30	49.33
40	61.66
50	74

Graph D

Time, *t* (s)	Speed, *s* (km/h)
0	0
10	12.33
20	24.66
30	36.99
40	49.33
50	61.66
60	74



b) Look for at least one similarity and one difference. Examples:
Similarities:
- All graphs end at 74 km/h.
- To move from one point to the next, you go 1 unit over and go 1.5 units up.

Differences:
- Each graph starts at a different y-coordinate.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #2 and 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Assign #1 only to students who have not done this earlier in their sequence chart or Foldable. • Direct students to make sure they title and label the axes on graphs to assist them in making connections between the different representations and interpreting the solutions.
Assessment *for* Learning	
Practise and Apply Have students do #4, 6 to 8, 10, 12a) and b), and 13. Students who have no problems with these questions can go on to the remaining Apply questions.	• Students who need help with #4 may benefit from additional coaching with Example 1. Help students work through #4, and then have them try #5 on their own. • Encourage students who find #6 challenging to develop a partial table of values for each equation, and then match coordinate pairs on the tables with those on the graph line. • Students may find #7 challenging because it contains variables that are neither *x* and *y* and they are required to develop a table of values. Prompt students to identify that the *y* value is the responding value, *r,* in part b) and *m,* in part c). • Students who need help with #8 may benefit from additional coaching with Example 2. Help students work through #8, and then have them try #9 on their own. • For #10, encourage students to create a table of values. Assign #11 to students who would benefit from additional practice. • Coach students who need assistance with #12a) and b). Then, have them try parts c) or d) on their own. • For #13, students may find it helpful to use the right angle from a geometry set to find the appropriate coordinates.
Math Link The Math Link on page 243 gives students practice with creating tables of values, developing equations, and graphing equations, and prepares them for the chapter problem titled Wrap It Up! on page 247.	• It is not necessary for students to do the Math Link as it is not required to complete the Wrap It Up! However, doing so may help students who would benefit from additional practice. • For struggling students, you may want to demonstrate that 74 km/h is the same as 12.3 km per 10 s. • For part b), consider having students compare the graphs and equations for two different starting speeds. • Students who need help getting started could use **BLM 6–11 Section 6.3 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Literacy Link By the end of section 6.3, have students complete the seventh box by describing how to graph a linear relation. As a class, consider having students discuss the process of graphing a linear equation.	• Encourage students to develop a situation, develop a linear equation, substitute values and create a table of values, and then graph the linear equation to help them explain the process. • Some students may benefit from being provided with a linear equation and coaching to generate a table of values, and then plot the points on a graph. • Consider allowing students to use technology for creating the graph. • Have students who need a challenge use interpolation and extrapolation to develop and solve problems related to their graph.
Math Learning Log Have students respond to the following prompts: • I know an equation and its graph are related because … • What I am most confident with about tables of values, equations, and graphs is … • What I find most confusing about tables of values, equations, and graphs is … • I know it is reasonable to use interpolation and extrapolation when …	• Encourage students to discuss their responses with a classmate and you. Based on the response, address any areas of concern before moving on. • Depending on students' learning styles, have them provide oral or written responses. • Encourage students to add notes to the definitions, examples, and Key Ideas for this section in their Foldable. • Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with.