Multiplying and Dividing Monomials

7.1

***MathLinks 9*, pages 254–263**

Suggested Timing
60–80 minutes

Materials
- algebra tiles
- ruler
- compass
- grid paper

Blackline Masters
Master 2 Communication Peer Evaluation
Master 6 Square Dot Paper
Master 7 Isometric Dot Paper
Master 8 Centimetre Grid Paper
Master 9 0.5 Centimetre Grid Paper
Master 11 Algebra Tiles (Positive Tiles)
Master 12 Algebra Tiles (Negative Tiles)
BLM 7–3 Chapter 7 Warm-Up
BLM 7–5 Section 7.1 Extra Practice
BLM 7–6 Section 7.1 Math Link

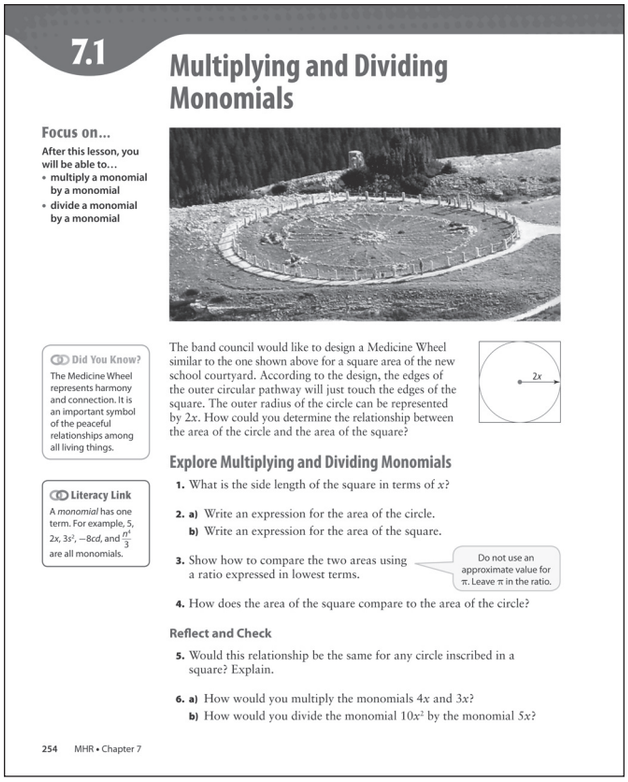
Mathematical Processes
- [x] Communication (C)
- [x] Connections (CN)
- [ ] Mental Mathematics and Estimation (ME)
- [ ] Problem Solving (PS)
- [x] Reasoning (R)
- [ ] Technology (T)
- [x] Visualization (V)

Specific Outcomes
PR7 Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 7, 9, 11, 13, 15, 18, Math Link
Typical	#1–3, 5, 7, 9, 11, 13, 15, 17–19, Math Link
Extension/Enrichment	#1–3, 7, 15, 17, 18, 21–25

Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections.



7.1

Multiplying and Dividing Monomials

Focus on...

After this lesson, you will be able to...
- multiply a monomial by a monomial
- divide a monomial by a monomial

The band council would like to design a Medicine Wheel similar to the one shown above for a square area of the new school courtyard. According to the design, the edges of the outer circular pathway will just touch the edges of the square. The outer radius of the circle can be represented by $2x$. How could you determine the relationship between the area of the circle and the area of the square?

2x

Did You Know?
The Medicine Wheel represents harmony and connection. It is an important symbol of the peaceful relationships among all living things.

Literacy Link
A *monomial* has one term. For example, 5, $2x$, $3s^2$, $-8cd$, and $\frac{n^2}{3}$ are all monomials.

Explore Multiplying and Dividing Monomials

1. What is the side length of the square in terms of x?
2. a) Write an expression for the area of the circle.
   b) Write an expression for the area of the square.
3. Show how to compare the two areas using a ratio expressed in lowest terms.

Do not use an approximate value for π. Leave π in the ratio.

4. How does the area of the square compare to the area of the circle?

Reflect and Check

5. Would this relationship be the same for any circle inscribed in a square? Explain.
6. a) How would you multiply the monomials $4x$ and $3x$?
   b) How would you divide the monomial $10x^2$ by the monomial $5x$?

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As a class, read and discuss the information concerning the dimensions of the square and circle. You may wish to introduce the term *inscribed circle*. Ask students:
- What are the diameter and radius of the circle?
- How do these dimensions relate to the side length of the square?

When students are working on the Explore, you might ask them:
- What are the exponent rules?
- How do you use the exponent rules to simplify algebraic expressions?

Explore Multiplying and Dividing Monomials

In this exploration, students compare the area formula for a square to that of a circle with a diameter that is the same measure as the side length of the square. Students can work individually or in pairs to develop the ratio and express it in lowest terms.

Students should be able to use the exponent rules to simplify the ratio.

Method 1 Have students work in pairs to complete #1 to 4 of the Explore. Then, have each pair discuss their ratio with another pair. As pairs of students work to identify the relationship between the area of the square and the area of the circle, you may wish to ask the following questions:

- What is the relationship between the diameter of the circle and the side length of the square?
- Why is it useful to leave π as an exact value rather than an approximation (e.g., 3.14)?
- Does it matter which area expression you use in the denominator of the ratio?

As a class, discuss group responses to #5 and 6. Use the discussion to arrive at a consensus concerning how to multiply and divide monomials.

Method 2 Have students work individually to complete #1 to 6 of the Explore. Then, have students discuss their solutions with a classmate.

Have each student contribute to a class discussion, and expand the discussion as outlined in Method 1.

Literacy Link Before beginning the Explore, refer students to the Literacy Link on page 254. Discuss the properties of a monomial and have students brainstorm examples. List these examples on the board. You may also suggest that students refer to the work they did on monomials in Chapter 5 of the student resource.

Meeting Student Needs

- It may be better for your class to work through the Explore as a whole-class activity.

ELL

- Teach the following terms in context: *edges*, *radius*, *determine*, *relationship*, *side length*, *lowest terms*, *compare*, and *inscribed*.
- Draw the image of the inscribed circle, and as you read the question, put a question mark to represent the side of the square to show that that is what #1 is asking.
- When defining *expression*, write another area question on the board and show what the expression for area would be. Then, have students do the work themselves for this Explore.
- For the Reflect and Check, students to work with a partner that speaks their own language.

Common Errors

- Some students may not apply the exponent rules appropriately.

R_x If necessary, review the applicable exponent rules using similar examples.

- Some students may multiply the monomial coefficients incorrectly (i.e., add them instead).

R_x Reinforce the multiplication concept using a rectangular area model. Remind students of the connection between area and multiplication vs. perimeter and addition.

Answers

Explore Multiplying and Dividing Monomials

1. $4x$

2. **a)** $4\pi x^2$ **b)** $16x^2$

3. $\frac{16x^2}{4\pi x^2} = \frac{4}{\pi}$ or $\frac{4\pi x^2}{16x^2} = \frac{\pi}{4}$

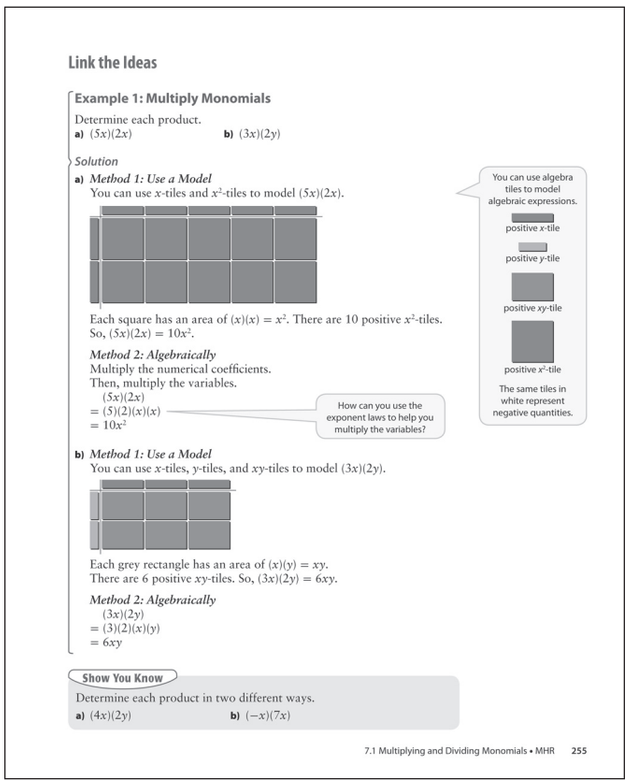
4. Example: The area of the square is slightly larger than the area of the circle.

5. Example: Yes, regardless of the dimensions of the circle and square, the ratio will always simplify to $\frac{4}{\pi}$ or $\frac{\pi}{4}$.

6. **a)** Example: To multiply monomials, you can multiply the numerical coefficients and use the exponent rules to multiply the variables. When you multiply variables with the same base, you add the exponents: $(4x)(3x) = 12x^2$.

b) Example: To divide monomials, you can divide the numerical coefficients and use the exponent rules to divide the variables. When you divide variables with the same base, you subtract the exponents: $10x^2 \div 5x = 2x$.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Try to have students explain how the different methods for representing monomial multiplication and division are similar and different.	• Have students share their responses to #5 as a class. Provide several different examples from #4 on the board for all students to see and then discuss generalizing to any inscribed circle in a square. • Make a list of the strategies used in #6 on the board and encourage students who are experiencing difficulty to include them in their Foldable.



Link the Ideas

Example 1: Multiply Monomials

Determine each product.

a) $(5x)(2x)$ b) $(3x)(2y)$

Solution

a) *Method 1: Use a Model*

You can use x-tiles and x^2-tiles to model $(5x)(2x)$.

You can use algebra tiles to model algebraic expressions.
positive x-tile
positive y-tile
positive xy-tile
positive x^2-tile
The same tiles in white represent negative quantities.

Each square has an area of $(x)(x) = x^2$. There are 10 positive x^2-tiles.
So, $(5x)(2x) = 10x^2$.

Method 2: Algebraically

Multiply the numerical coefficients.
Then, multiply the variables.

$(5x)(2x)$
$= (5)(2)(x)(x)$
$= 10x^2$

How can you use the exponent laws to help you multiply the variables?

b) *Method 1: Use a Model*

You can use x-tiles, y-tiles, and xy-tiles to model $(3x)(2y)$.

Each grey rectangle has an area of $(x)(y) = xy$.
There are 6 positive xy-tiles. So, $(3x)(2y) = 6xy$.

Method 2: Algebraically

$(3x)(2y)$
$= (3)(2)(x)(y)$
$= 6xy$

Show You Know

Determine each product in two different ways.

a) $(4x)(2y)$ b) $(-x)(7x)$

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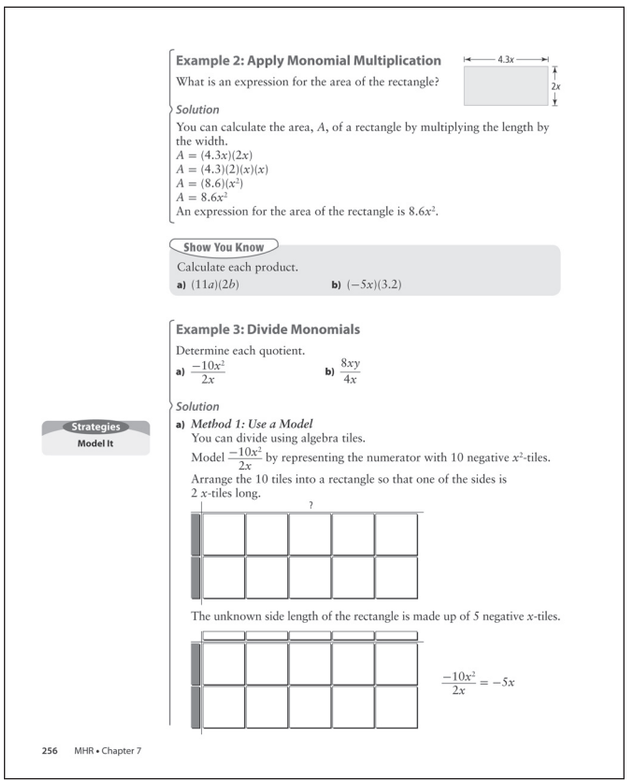

Example 2: Apply Monomial Multiplication

What is an expression for the area of the rectangle?

4.3x
2x

Solution

You can calculate the area, A, of a rectangle by multiplying the length by the width.

$A = (4.3x)(2x)$
$A = (4.3)(2)(x)(x)$
$A = (8.6)(x^2)$
$A = 8.6x^2$

An expression for the area of the rectangle is $8.6x^2$.

Show You Know

Calculate each product.

a) $(11a)(2b)$ b) $(-5x)(3.2)$

Example 3: Divide Monomials

Determine each quotient.

a) $\frac{-10x^2}{2x}$ b) $\frac{8xy}{4x}$

Strategies
Model It

Solution

a) *Method 1: Use a Model*

You can divide using algebra tiles.

Model $\frac{-10x^2}{2x}$ by representing the numerator with 10 negative x^2-tiles.

Arrange the 10 tiles into a rectangle so that one of the sides is 2 x-tiles long.

?

The unknown side length of the rectangle is made up of 5 negative x-tiles.

$\frac{-10x^2}{2x} = -5x$

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Link the Ideas

Example 1

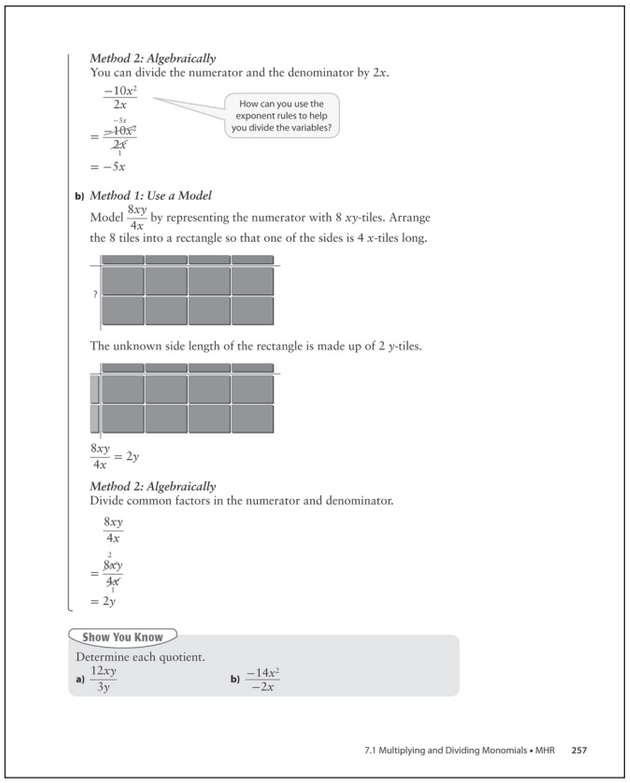
This example shows students how to multiply monomials using models as well as symbolically. Remind students that the tiles are used to represent positive and negative variables and constants. Reinforce with students that the first method uses a familiar problem-solving approach (Model It). Note that you may choose to use diagrams for modelling, instead of algebra tiles. Ask students if they can relate the two chosen methods to one another by identifying which parts of the model represent what is recorded in the symbolic method. You may wish to ask the following questions:

- Does it matter which way you arrange the tiles to model the multiplication of monomials?
- Is there a way to model a negative xy-tile? What might that be?
- How are the operations used to multiply monomials similar to those used for multiplying numbers?

Students should complete the Show You Know before continuing. Check to see which method each student prefers, and ask them to explain why.

Example 2

This example shows an application of monomial multiplication. It uses an area model to provide another way to relate the diagram to the symbolic method of solving the problem. Emphasize the importance of multiplying the coefficients and using the exponent rules correctly: adding exponents when multiplying expressions with the same variables.



Method 2: Algebraically
You can divide the numerator and the denominator by $2x$.

$$\frac{-10x^2}{2x}$$

How can you use the exponent rules to help you divide the variables?

$$= \frac{-5\cancel{x}\cdot x^{\cancel{2}}}{\cancel{2}\cdot\cancel{x}_{1}}$$

$$= -5x$$

b) *Method 1: Use a Model*
Model $\frac{8xy}{4x}$ by representing the numerator with 8 xy-tiles. Arrange the 8 tiles into a rectangle so that one of the sides is 4 x-tiles long.

The unknown side length of the rectangle is made up of 2 y-tiles.

$$\frac{8xy}{4x} = 2y$$

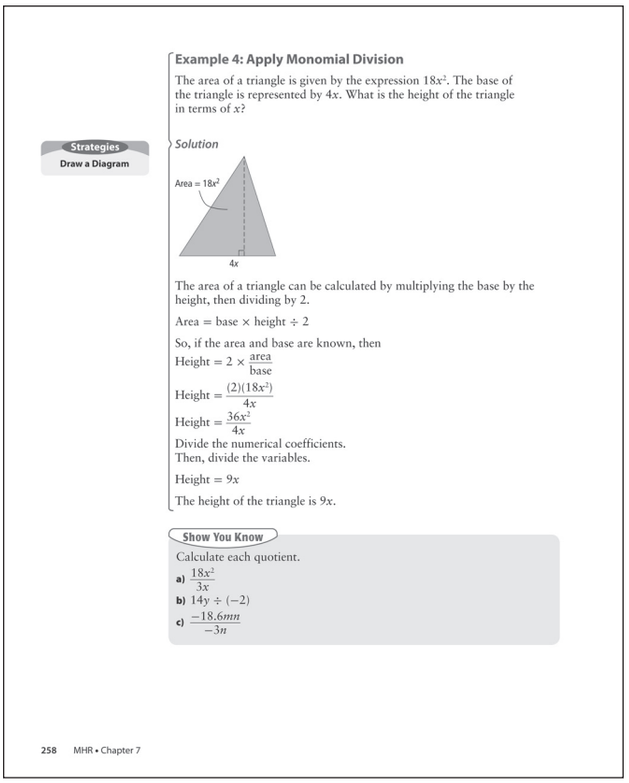
Method 2: Algebraically
Divide common factors in the numerator and denominator.

$$\frac{8xy}{4x}$$

$$= \frac{2\cdot\cancel{4}\cdot x\cdot y}{\cancel{4}\cdot\cancel{x}_{1}}$$

$$= 2y$$

Show You Know
Determine each quotient.
a) $\frac{12xy}{3y}$ **b)** $\frac{-14x^2}{-2x}$



Example 4: Apply Monomial Division
The area of a triangle is given by the expression $18x^2$. The base of the triangle is represented by $4x$. What is the height of the triangle in terms of x?

Strategies
Draw a Diagram

Solution

The area of a triangle can be calculated by multiplying the base by the height, then dividing by 2.

Area = base × height ÷ 2

So, if the area and base are known, then

$$\text{Height} = 2 \times \frac{\text{area}}{\text{base}}$$

$$\text{Height} = \frac{(2)(18x^2)}{4x}$$

$$\text{Height} = \frac{36x^2}{4x}$$

Divide the numerical coefficients.
Then, divide the variables.

$$\text{Height} = 9x$$

The height of the triangle is $9x$.

Show You Know
Calculate each quotient.
a) $\frac{18x^2}{3x}$
b) $14y \div (-2)$
c) $\frac{-18.6mn}{-3n}$

You may wish to ask the following questions:
- Why is it easier to represent this problem using an area model rather than algebra tiles?
- Does it matter in which order the numbers and variables are multiplied together?

Example 3

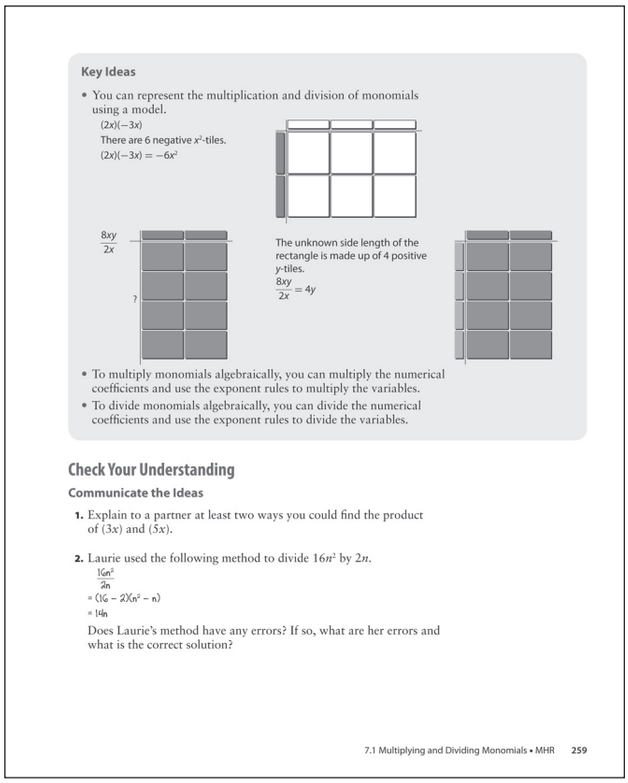
This example shows students how to divide monomials by using algebra tiles as well as by using symbols. Make sure that students understand the use of positive and negative integers to represent the expressions being divided. Similar to Example 1, you may choose to use diagrams for modelling, instead of algebra tiles. Again, ask students if they can relate the two methods to one another by identifying which parts of the model represent what is recorded in the symbolic method. You may wish to ask the following questions:
- Does it matter which way you arrange the tiles to model the division of monomials?
- Which method do you prefer when dividing monomials: using models or symbols? Explain why you prefer one method over another.
- How are the operations used for dividing monomials similar to those used for dividing numbers?

Example 4

This example shows an application of monomial division. It also uses an area model to provide another way to relate the diagram to the symbolic method of solving the problem. Emphasize the importance of dividing the coefficients and using the exponent rules correctly: subtracting exponents when dividing expressions with the same variables. It should also be noted that the area model can be used with negative expressions, but these may be more easily understood if modelled using algebra tiles or solved algebraically. You may wish to ask the following questions:
- Can algebra tiles be used to model the area of a triangle? How?
- What other ways can you model monomial division?

Key Ideas

Have students read and review the Key Ideas provided for section 7.1. They identify two ways that monomial multiplication and division can be performed: with models or algebraically. Stress the relationships between modelling multiplication and division and the symbols used for variables and coefficients. Encourage students to add the Key Ideas to their Foldable, writing them in their own words.



Key Ideas

- You can represent the multiplication and division of monomials using a model.

$(2x)(-3x)$
There are 6 negative x^2-tiles.
$(2x)(-3x) = -6x^2$

$\frac{8xy}{2x}$

?

The unknown side length of the rectangle is made up of 4 positive y-tiles.
$\frac{8xy}{2x} = 4y$

- To multiply monomials algebraically, you can multiply the numerical coefficients and use the exponent rules to multiply the variables.
- To divide monomials algebraically, you can divide the numerical coefficients and use the exponent rules to divide the variables.

Check Your Understanding

Communicate the Ideas

1. Explain to a partner at least two ways you could find the product of $(3x)$ and $(5x)$.

2. Laurie used the following method to divide $16n^2$ by $2n$.

$\frac{16n^2}{2n}$
$= (16 - 2)(n^2 - n)$
$= 14n$

Does Laurie's method have any errors? If so, what are her errors and what is the correct solution?

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Meeting Student Needs

- Help students explicitly connect the parts of the models (algebra tiles or area model) to the variables, coefficients, and operations being performed.
- Remind students that the rules that apply for multiplying and dividing integers also apply to coefficients of monomials.
- It may be better for some students to work only with positive coefficients for some time before moving to negative coefficients.
- When dividing monomials by monomials, students might be encouraged to write the numerator and denominator in expanded form before dividing; for example, $\frac{6x^2}{2x} = \frac{(6)(x)(x)}{(2)(x)}$. This step may help them in removing common factors.
- Some students may need an additional question, similar to the Show You Know, before proceeding to the next example.
- It may be better for your class to work through the examples as a whole-class activity. Assign one Show You Know question as a small-group or pair activity. Then, assign the other Show You Know question as individual student work.

ELL

- Teach the following terms in context: *product*, *numerator*, *denominator*, and *common factors*.

Common Errors

- Some students may have difficulty relating the models to the algebraic method.

R_x Ask students to represent monomial multiplication and division using both models and symbols. Have them work from one form to another in both directions (models to symbols and then symbols to models).

- Some students may have difficulty applying the exponent rules correctly.

R_x Remind students that when multiplying expressions with the same variable, the exponents are added, and when dividing, the exponents are subtracted. You may wish to refer them to the work they did on exponent rules in Chapter 5.

Answers

Example 1: Show You Know

a) $8xy$ **b)** $-7x^2$

Example 2: Show You Know

a) $22ab$ **b)** $-16x$

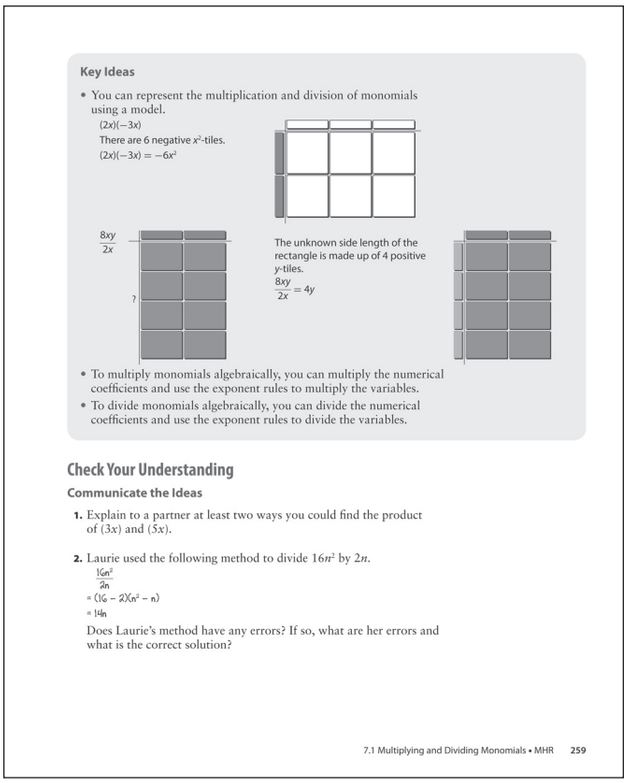
Example 3: Show You Know

a) $4x$ **b)** $7x$

Example 4: Show You Know

a) $6x$ **b)** $-7y$ **c)** $6.2m$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • If the algebra tiles you use in your classroom have different colours from the ones in the student resource, show students which ones are equivalent. You may wish to post visuals of these equivalents for easy reference. • Encourage students to model the questions using actual algebra tiles. If algebra tiles are not available, provide students with **Master 11 Algebra Tiles (Positive Tiles)** and **Master 12 Algebra Tiles (Negative Tiles)**. • Explain to students that the different methods of representing monomial multiplication and division are equivalent, but that one approach may be more useful depending upon what is being asked. For example, working with negative dimensions in an area model is not realistic, but algebra tiles can be used to represent both positive and negative monomials. • Ensure that all students understand the relationship between the algebra-tile model and the symbolic approach. You may find it beneficial to model several examples for the class using algebra tiles on an overhead. This allows you to watch students and observe who may be experiencing difficulty. • Students need to understand that when two different variables are involved in the monomials, the exponent rules cannot be applied. • Remind students that the sign rules for multiplying monomials with positive and negative signs are the same as those used for multiplying integers. • You may wish to use the term *distribute* at this early stage and carry through with it throughout the chapter. Tell students that they distribute by multiplying each term in the first polynomial (which can be a monomial) by each term in the second polynomial. They can then apply this method throughout the chapter. • Reinforce the concept that when opposite-signed tiles are used, the inside of the rectangle will always be negative.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Ensure that students are able to relate the area model to the symbolic representation. • Ask students to explain the difficulty of using algebra tiles for multiplying or dividing when there are decimal coefficients.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to model the questions using actual algebra tiles. If algebra tiles are not available, provide students with **Master 11 Algebra Tiles (Positive Tiles)** and **Master 12 Algebra Tiles (Negative Tiles)**. Ensure that all students understand the relationship between the algebra tile model and the symbolic approach. • All students should understand that when two different variables are involved in the monomials, the exponent rules can not be applied. • Reinforce that the sign rules for multiplying monomials with positive and negative signs are the same as those used for multiplying integers.
Example 4 Have students do the Show You Know related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students should be able to relate the area model to the symbolic representation.



Key Ideas

- You can represent the multiplication and division of monomials using a model.

(2x)(−3x)
There are 6 negative x^2-tiles.
$(2x)(-3x) = -6x^2$

$\frac{8xy}{2x}$

?

The unknown side length of the rectangle is made up of 4 positive y-tiles.
$\frac{8xy}{2x} = 4y$

- To multiply monomials algebraically, you can multiply the numerical coefficients and use the exponent rules to multiply the variables.
- To divide monomials algebraically, you can divide the numerical coefficients and use the exponent rules to divide the variables.

Check Your Understanding

Communicate the Ideas

1. Explain to a partner at least two ways you could find the product of $(3x)$ and $(5x)$.

2. Laurie used the following method to divide $16n^2$ by $2n$.

$\frac{16n^2}{2n}$
$= (16 - 2)(n^2 - n)$
$= 14n$

Does Laurie's method have any errors? If so, what are her errors and what is the correct solution?

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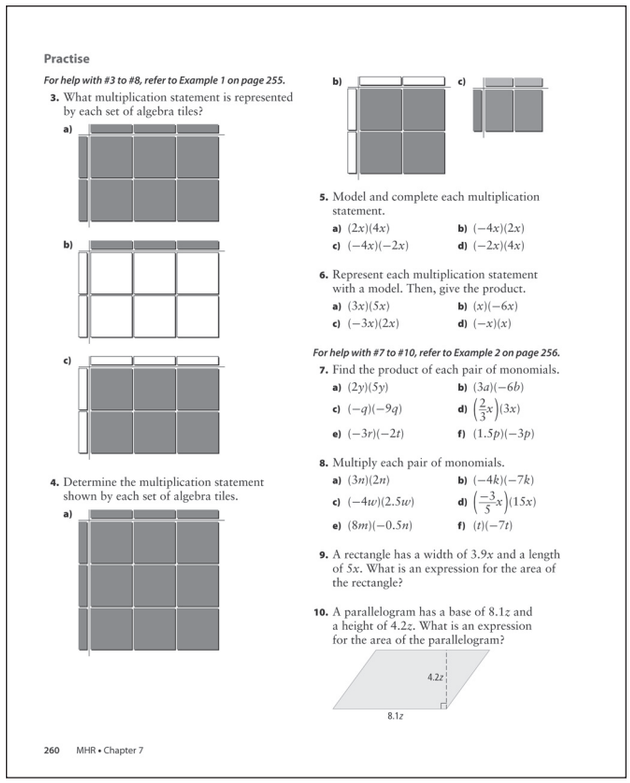

Practise

For help with #3 to #8, refer to Example 1 on page 255.

3. What multiplication statement is represented by each set of algebra tiles?

a)

b)

c)

4. Determine the multiplication statement shown by each set of algebra tiles.

a)

b)

c)

5. Model and complete each multiplication statement.
   a) $(2x)(4x)$ b) $(-4x)(2x)$
   c) $(-4x)(-2x)$ d) $(-2x)(4x)$

6. Represent each multiplication statement with a model. Then, give the product.
   a) $(3x)(5x)$ b) $(x)(-6x)$
   c) $(-3x)(2x)$ d) $(-x)(x)$

For help with #7 to #10, refer to Example 2 on page 256.

7. Find the product of each pair of monomials.
   a) $(2y)(5y)$ b) $(3a)(-6b)$
   c) $(-q)(-9q)$ d) $\left(\frac{2}{3}x\right)(3x)$
   e) $(-3r)(-2t)$ f) $(1.5p)(-3p)$

8. Multiply each pair of monomials.
   a) $(3n)(2n)$ b) $(-4k)(-7k)$
   c) $(-4w)(2.5w)$ d) $\left(-\frac{3}{5}x\right)(15x)$
   e) $(8m)(-0.5n)$ f) $(t)(-7t)$

9. A rectangle has a width of $3.9x$ and a length of $5x$. What is an expression for the area of the rectangle?

10. A parallelogram has a base of $8.1z$ and a height of $4.2z$. What is an expression for the area of the parallelogram?

$4.2z$

$8.1z$

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Check Your Understanding

Communicate the Ideas

You may wish to have students complete the questions in groups and then discuss their answers. When students have considered #1, reinforce the relationship between the physical attributes of the model(s) and the symbolic representation of the variables, exponents, and coefficients. For #2, make sure students understand that the exponent rule for division allows them to subtract exponents when the exponents are the same, but that they cannot subtract coefficients; the coefficients must be divided.

Practise

Students should complete at least one of each pair of questions (#3 and 4, #5 and 6, #7 and 8, #11 and 12, #13 and 14, and #15 and 16) in order to reinforce the connection between the models and the symbolic representations. Students might also complete either #9 or 10 so that they have practice with an application question similar to Example 2.

For #5, 6, 13, and 14, students use models. You may wish to provide students with **Master 8 Centimetre Grid Paper** or **Master 9 0.5 Centimetre Grid Paper** on which to draw diagrams.

Apply

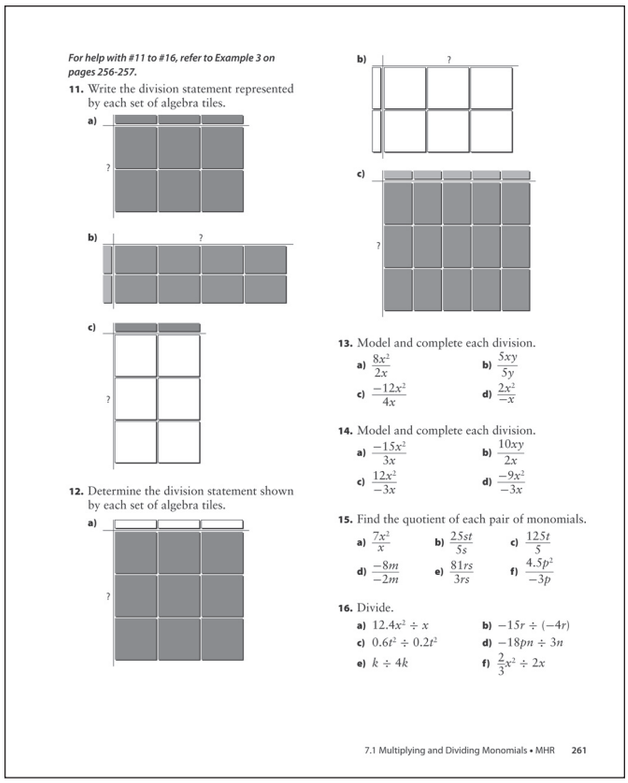
The Apply questions consist primarily of area-model problems. Assign questions based on student interest and/or familiarity with the contexts.

For both #17 and 18, students need an understanding of area, but in #17 they are required to perform multiplication, while in #18 they must perform division. In #19, students are completing the same type of question as in #18, except that they are not given a visual representation. It may be helpful for students to draw a diagram before they begin.

Some students may have difficulty visualizing what is described in #20. Direct their attention to the photo. Also, it may be beneficial for students to draw an overhead diagram before they begin this question.

For #21 and 22, students will need to recall how to calculate the area of a circle. For #22, they will also need to remember how to determine the circumference of a circle.

You may wish to use #23 to give students practice in factoring by having them identify some possible dimensions for a dogsled that has a rectangular base area of 3.2 m^2 and an unknown length and width.



For help with #11 to #16, refer to Example 3 on pages 256-257.

11. Write the division statement represented by each set of algebra tiles.

a)

b)

c)

12. Determine the division statement shown by each set of algebra tiles.

a)

b)

c)

13. Model and complete each division.

a) $\frac{8x^2}{2x}$ b) $\frac{5xy}{5y}$

c) $\frac{-12x^2}{4x}$ d) $\frac{2x^2}{-x}$

14. Model and complete each division.

a) $\frac{-15x^2}{3x}$ b) $\frac{10xy}{2x}$

c) $\frac{12x^2}{-3x}$ d) $\frac{-9x^2}{-3x}$

15. Find the quotient of each pair of monomials.

a) $\frac{7x^2}{x}$ b) $\frac{25st}{5s}$ c) $\frac{125t}{5}$

d) $\frac{-8m}{-2m}$ e) $\frac{81rs}{3rs}$ f) $\frac{4.5p^2}{-3p}$

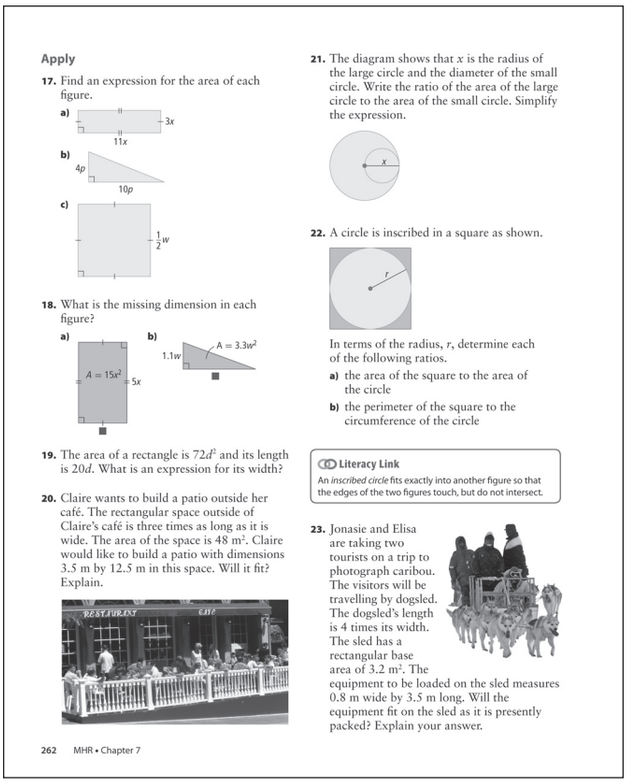
16. Divide.

a) $12.4x^2 \div x$ b) $-15r \div (-4r)$

c) $0.6t^2 \div 0.2t^2$ d) $-18pn \div 3n$

e) $k \div 4k$ f) $\frac{2}{3}x^2 \div 2x$

7.1 Multiplying and Dividing Monomials • MHR 261



Apply

17. Find an expression for the area of each figure.

a)

b)

c)

18. What is the missing dimension in each figure?

a)

b)

19. The area of a rectangle is $72d^2$ and its length is $20d$. What is an expression for its width?

20. Claire wants to build a patio outside her café. The rectangular space outside of Claire's café is three times as long as it is wide. The area of the space is 48 m². Claire would like to build a patio with dimensions 3.5 m by 12.5 m in this space. Will it fit? Explain.

21. The diagram shows that x is the radius of the large circle and the diameter of the small circle. Write the ratio of the area of the large circle to the area of the small circle. Simplify the expression.

22. A circle is inscribed in a square as shown.

In terms of the radius, r, determine each of the following ratios.

a) the area of the square to the area of the circle

b) the perimeter of the square to the circumference of the circle

Literacy Link

An *inscribed circle* fits exactly into another figure so that the edges of the two figures touch, but do not intersect.

23. Jonasie and Elisa are taking two tourists on a trip to photograph caribou. The visitors will be travelling by dogsled. The dogsled's length is 4 times its width. The sled has a rectangular base area of 3.2 m². The equipment to be loaded on the sled measures 0.8 m wide by 3.5 m long. Will the equipment fit on the sled as it is presently packed? Explain your answer.

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Literacy Link Before students attempt #22, draw their attention to the Literacy Link on page 262. You may wish to refer them to the beginning of section 7.1 and ask whether the diagram in the introduction represents an inscribed circle. Have them justify their answer.

Extend

The Extend questions range from abstract mathematical questions to additional area-model problems. Students could be asked to select one question based on their interest.

For #24, students will need to recall how to determine the volume and surface area of a rectangular prism.

Note that for #25, grade 9 students are not expected to be able to perform division when the exponent in the divisor is of a higher degree than the exponent in the dividend. Have students focus on the differences between the two expressions rather than determining the division of $9n$ by $3n^2$.

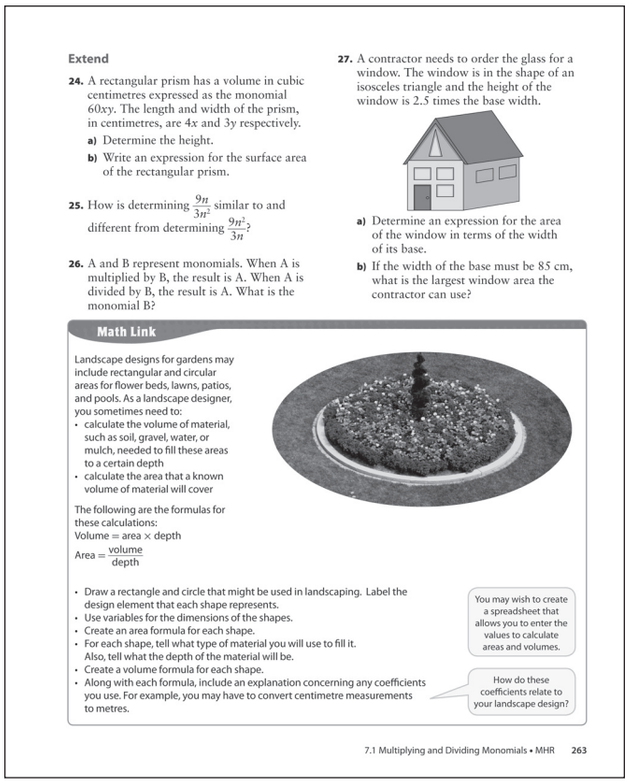
Encourage students to use a method of their choice to solve #26. Most students will likely use Guess and Check.

For #27, students will need to recall the properties of an isosceles triangle and how to determine the area of a triangle.

Literacy Link After completing section 7.1, have students fill in the upper left leg of their spider map. You may wish to have them provide their own examples of monomials and to have them illustrate how to multiply monomials. In the upper right leg, consider having them provide their own examples of monomials and then have them illustrate how to divide monomials.

Math Link

The Math Link involves creating an area formula and a volume formula for a rectangular feature and a circular feature of a landscape design. Students are given a set of parameters and asked to develop the formula that could be used to permit repeated calculations for different area dimensions. Students might create a spreadsheet that allows them to enter the values (including unit cost) to calculate the areas, volumes, and costs.



Extend

24. A rectangular prism has a volume in cubic centimetres expressed as the monomial $60xy$. The length and width of the prism, in centimetres, are $4x$ and $3y$ respectively.

a) Determine the height.

b) Write an expression for the surface area of the rectangular prism.

25. How is determining $\frac{9n}{3n^2}$ similar to and different from determining $\frac{9n^2}{3n}$?

26. A and B represent monomials. When A is multiplied by B, the result is A. When A is divided by B, the result is A. What is the monomial B?

27. A contractor needs to order the glass for a window. The window is in the shape of an isosceles triangle and the height of the window is 2.5 times the base width.

a) Determine an expression for the area of the window in terms of the width of its base.

b) If the width of the base must be 85 cm, what is the largest window area the contractor can use?

Math Link

Landscape designs for gardens may include rectangular and circular areas for flower beds, lawns, patios, and pools. As a landscape designer, you sometimes need to:

- calculate the volume of material, such as soil, gravel, water, or mulch, needed to fill these areas to a certain depth
- calculate the area that a known volume of material will cover

The following are the formulas for these calculations:

Volume = area × depth

Area = $\frac{\text{volume}}{\text{depth}}$

- Draw a rectangle and circle that might be used in landscaping. Label the design element that each shape represents.
- Use variables for the dimensions of the shapes.
- Create an area formula for each shape.
- For each shape, tell what type of material you will use to fill it. Also, tell what the depth of the material will be.
- Create a volume formula for each shape.
- Along with each formula, include an explanation concerning any coefficients you use. For example, you may have to convert centimetre measurements to metres.

You may wish to create a spreadsheet that allows you to enter the values to calculate areas and volumes.

How do these coefficients relate to your landscape design?

7.1 Multiplying and Dividing Monomials • MHR 263

Meeting Student Needs

- Provide **BLM 7–5 Section 7.1 Extra Practice** to students who would benefit from more practice.
- You may wish to change the Math Link activity to make it more relevant to your students. For example, Northern communities have tank farms where heating, vehicle, and sometimes aviation fuel is stored. The farm consists of a number of huge cylindrical storage tanks set inside a rectangular gravel area to contain possible fuel spills. Students might complete the Math Link by planning a tank farm instead of a landscape design. Encourage students to contribute their own ideas for alternative designs.

ELL

- Teach the following terms in context: *parallelogram*, *base*, *errors*, *statement*, *patio*, *inscribed*, *circumference*, *contractor*, *glass*, *isosceles*, and *convert*.
- Have students write their ideas down in their first language and then partner them up with another student in that class to discuss their answers. They can then do their best to translate their answers to English.
- For the questions in the Practise, model one part of each type of question. For example, model number #3a) on the board and explain that the rest of #3 and 4 are asking for the same thing. Model #5a) and explain that the rest of #5 and 6 are the same.

Gifted and Enrichment

- Direct students to focus on the Apply and Extend questions.

Common Errors

- If using spreadsheet software for the Math Link, some students may not be familiar with entering formulas into the cells.

R_x Provide multiple examples of spreadsheets with simple formulas. Demonstrate for students the procedures for multiplying a cell value by a number or another cell value.

Answers

Communicate the Ideas

1. Example: To multiply $3x$ and $5x$, you can use a model or you can use symbols.

2. Example: Yes, Laurie's method has errors. Laurie should have divided the numerical coefficients instead of subtracting them. Also, she should have shown her work as:

$$\frac{16n^2}{2n} = (16 \div 2)(n^{2-1})$$
$$= 8n$$

Math Link

Example:
Rectangle:
$A = l \times w$; depth $= 0.1$ m; $V = (l \times w)(0.1w) = l \times 0.1w$.
The coefficient 0.1 represents the depth of dirt, which is 0.1 m.
Circle:
$A = \pi r^2$; depth $= 0.2$ m; $V = \pi r^2 \times 0.2 = 0.2\pi r^2$.
The coefficient 0.2 represents the depth of water, which is 0.2 m.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students experiencing difficulty start with models before trying the algebraic approach. This way they can compare their algebraic response to their model to verify their answer. • Some students may require coaching in the exponent rules. Having these students return to the exponent rules in their Foldable may be beneficial. • Have students assess each other's answers to #1 using **Master 2 Communication Peer Evaluation**.
Assessment *for* Learning	
Practise Have students do #3, 5, 7, 9, 11, 13, and 15. Students who have no problems with these questions can go on to the Apply questions.	• Students who have difficulty with any of the questions will need additional coaching on the corresponding example. • Students having difficulty with #9 should be encouraged to draw and label a diagram. • Have students complete #13 with a partner so that they can share and compare how they set up the models. • Having students verbalize their models will assist them in transferring the learning to an algebraic approach. • Reinforce the concept that when opposite-signed tiles are used, the inside of the rectangle will always be negative.
Math Link The Math Link on page 263 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 281.	• You may wish to have students complete the Math Link in order to apply their understanding of monomial multiplication and division. Listen to any discussion about how students solve the problems and assist with any terminology that may cause confusion. • You may wish to use this as a starter for the Wrap It Up!, especially for students who find it challenging. Have students use what they completed in this Math Link for the Wrap It Up! • Some students may wish to use square dot paper, isometric dot paper, or grid paper to draw their design features. Make available **Master 6 Square Dot Paper, Master 7 Isometric Dot Paper**, **Master 8 Centimetre Grid Paper**, and **Master 9 0.5 Centimetre Grid Paper**. • Encourage students to use a ruler to draw the rectangular design and a compass to draw the circular design. • To help them before they begin, you might want to discuss with students how the thought bubble relates to the landscape-design activity. • Provide students with formative feedback for the Math Link as they work towards the Wrap It Up! • Students who need help getting started could use **BLM 7–6 Section 7.1 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Literacy Link Help students recall the concepts introduced in this section, and then have them add examples and methods that they learned in this section to their spider map.	• Work with students to model the use of spider maps if they are unfamiliar with them. Model how to start the upper left leg of the spider map. • You may wish to have students print the following subheadings beside the heading in the upper left leg of the spider map: Example of Monomial, Example of Monomial, and Multiplying Monomials. Consider having students record two examples of a monomial. Then, have them multiply those two monomials. • You may wish to have students print the following subheadings beside the heading in the upper right leg of the spider map: Example of Monomial, Example of Monomial, and Dividing Monomials. Have them record two examples of a monomial. Then, you might have them show how to divide these two monomials. Note that the dividend they choose must be evenly divisible by the divisor they choose. • Encourage students to fill in the spider map with examples and methods of their choice. Have students write the explanations in their owns words, as opposed to copying what appears in the student resource.
Math Learning Log Have students respond to the following questions: • What relationships are there between the models and symbolic representations? • What method do you like best? Why?	• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. • Encourage students who struggle with writing to incorporate diagrams into their writing.