Multiplying Polynomials by Monomials



	Suggested Timing
6	0–80 minutes
ľ	Materials
	grid paper
•	algebra tiles
E	Blackline Masters
N	Naster 2 Communication Peer Evaluation
	Naster 8 Centimetre Grid Paper
	Aaster 9 0.5 Centimetre Grid Paper
	Aaster 11 Algebra Tiles (Positive Tiles) Aaster 12 Algebra Tiles (Negative Tiles)
	BLM 7–3 Chapter 7 Warm-Up
E	3LM 7–7 Section 7.2 Extra Practice
E	BLM 7–8 Section 7.2 Math Link
ľ	Nathematical Processes
•	Communication (C)
•	Connections (CN)
	Mental Mathematics and Estimation (ME)
	Problem Solving (PS)
•	Reasoning (R)
	Technology (T)
	Visualization (V)

Specific Outcomes

PR7 Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.

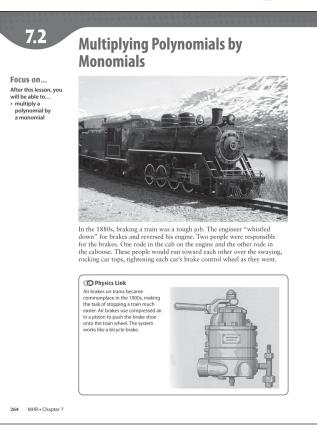
Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1-4, 6, 8, 10, 12, 15, Math Link
Typical	#1–4, 6, 8, 10, 12, 15, 16, Math Link
Extension/Enrichment	#1-3, 6, 10, 12, 15, 16, 19-21

Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections.

Discuss with students the section opener. Talk about how long it might take to stop the train using the described procedure. Ask:

• Will the train stop the same instant that the last set of brakes is applied? Why or why not?



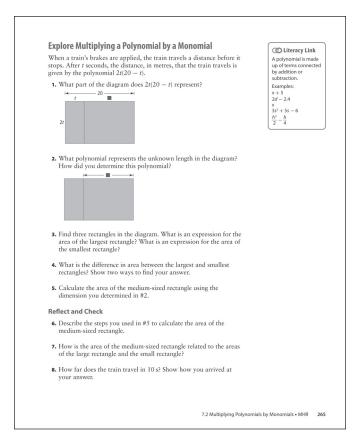
• What is a possible equation that can be used to represent the distance the train travels in a certain amount of time?

This discussion will lead the class into the information provided in the opening paragraph of the Explore.

Explore Multiplying a Polynomial by a Monomial

In this exploration, students are given an area model of a polynomial that represents the stopping distance of a train. With this model, students are better able to understand how multiplying a polynomial by a monomial results in terms that include t^2 and terms that include *t*.

As students study the area model, remind them that the variable, *t*, represents time. Emphasize that the equation and solution can be used to make predictions for stopping distance, speed, and acceleration of the train. Have students describe and discuss how they could use polynomial multiplication to determine the time it takes the train to travel a specified distance.



Method 1 Have students work individually to complete #1 to 7 of the Explore. Then, have each student discuss their conclusions with a classmate. Students could be provided with other modelling materials (e.g., algebra tiles) to help them make the connection between the resulting polynomial and the two-dimensional representations.

It is important to make sure that students are able to relate the attributes of the area model (e.g., length, width, composite areas, total area) to the algebraic components (e.g., monomial/polynomial, distributed products, the final product). This is critical information that helps bridge between the conceptual and procedural aspects of polynomial multiplication. Ask:

- What does each dimension of the rectangle represent?
- Which part of the diagram represents the result of using the distributive property?

Method 2 Have students work in pairs to complete #1 to 7 of the Explore. Give each pair of students algebra tiles to model the problem situation (rather than using an area model). Have each group report their conclusions to the class, and then have a class discussion to ensure that all students have made the connection between the concrete model and the abstract, symbolic representation of polynomial multiplication.

Literacy Link Direct students' attention to the Literacy Link on page 265. Discuss the difference between a *monomial*, *binomial*, *trinomial*, and the general term, *polynomial*. You may wish to refer students back to what they learned in Chapter 5 about these terms.

Meeting Student Needs

- Talk about why it might be important to know how much distance a train will take to stop. Relate the discussion to contexts that are more familiar to students. For example, ask students to describe how to stop a snowmobile or komatik (dogsled). When they drive a snowmobile or *komatik*, do they have to allow more distance for braking? What is the difference between braking on ice, snow, or gravel?
- You may wish to change the area model so that instead of representing the time it takes for a train to stop, it represents something concrete, perhaps related directly to students' lives and interests. For example, you may want to have students draw a star quilt with dimensions that are represented by polynomials, as in the Explore. You might have students work in small groups to create an expression for the area of the quilt.

ELL

- Teach the following terms in context: *braking*, *tough* (meaning hard), *reversed*, *cab*, *caboose*, *swaying*, *rocking*, *car tops*, *tightening*, *applied*, *largest*, *smallest*, *dimension*, and *describe*.
- Read through the Explore with the class and either use the board to illustrate the concepts or have students model how they would have to stop the trains. English language learners may not understand from the words that it is the train that continues to move a distance before it stops. Clarify this concept.
- For the Reflect and Check, allow students to answer the questions in their first language and then have them share their ideas in English.

Common Errors

- Students may have difficulty relating the unknown dimension with the expression 20 t.
- R_x Provide students with a similar model based on algebra tiles and have them physically create the unknown dimensions. Ensure that students do not count the resulting tiles, as this does not represent the final solution.

Answers

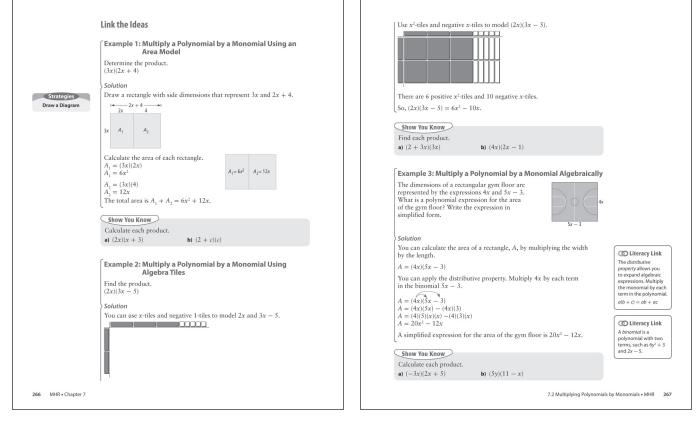
Explore Multiplying a Polynomial by a Monomial

- **1.** The area of the middle-sized rectangle.
- **2.** 20 − *t*. It can be determined from the diagram or from the expression in #1.
- **3.** The expression for the largest is 40t, and the expression for the smallest is $2t^2$.
- **4.** $40t 2t^2$

5. $40t - 2t^2$

- **6.** Example: I used the distributive property on the expression 2t(20 t).
- **7.** Example: The area of the middle-sized rectangle is the difference between the area of the largest rectangle and the area of the smallest rectangle.
- **8.** 2t(20 t); 2(10)(20 10) = 200. The train travels 200 m in 10 s.

Assessment	Supporting Learning	
Assessment <i>as</i> Learning		
Reflect and Check Listen as students discuss what they discovered during the Explore. See if students are able to identify which symbolic representation corresponds to the graphical or concrete representation.	 Some students may not be able to visualize the three rectangles. They may benefit from dividing them into three separate diagrams and labelling them individually. Some students may need assistance generalizing the area model to the symbolic representation. Have them verbalize the similarities and differences between the two methods. 	

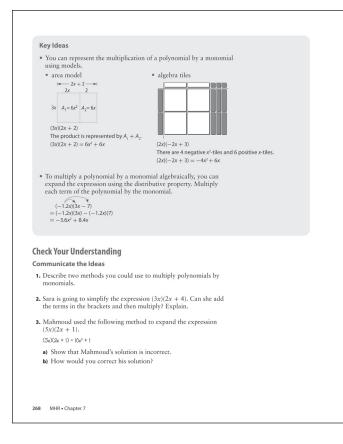


Link the Ideas

Example 1

This example reinforces how an area model can be used to represent the symbolic multiplication of polynomials. If needed, students should be reminded that the dimensions of the rectangle represent the monomial and binomial and that the total area is the product of the monomial and binomial. You may wish to ask the following questions:

- Can the rectangles be drawn with the length and width dimensions reversed (i.e., 3*x* as the width and 2*x* + 4 as the height)?
- How are negative values represented in the diagram?
- Will this method of multiplying polynomials work with polynomials that have rational numbers as coefficients?



Some students may find it beneficial to use algebra tiles to model the products, in addition to the area model.

Example 2

This example is similar to Example 1 except that it focuses on students using algebra tiles to model the polynomial multiplication. The main difference to point out is that algebra tiles make it easier to work with negative coefficients and constants. You may wish to ask the following questions:

- What is a limitation to using algebra tiles to model polynomial multiplication?
- Is it easier or more difficult to represent negative values using algebra tiles compared to using an area model? Which method do you prefer, and why?

While students are completing the Show You Know, monitor if they are able to model using the negative tiles and if they are able to interpret the product correctly in part b).

Example 3

This example provides students with an application of polynomial multiplication. Point out how the distributive property can help them create the component parts in an area model. You may wish to ask the following questions:

- What is the relationship between using an area model and using algebraic symbols to perform polynomial multiplication?
- How is using algebra tiles similar to or different from using algebra to perform polynomial multiplication?

Literacy Link Direction students to the first Literacy Link on page 267. Discuss the distributive property and have students provide examples. Relate the coefficients, variables, and constants in the examples to *a*, *b*, and *c* in the Literacy Link to assist students in generalizing this property.

Discuss with students the second Literacy Link on page 267. Have students share other ideas for examples of binomials.

Key Ideas

Have students read and review the Key Ideas. Emphasize the importance of knowing how to model the multiplication of a polynomial by a monomial using algebra tiles when at least one of the polynomials is negative. Have students provide their own summary of the Key Ideas in their Foldable.

Meeting Student Needs

- Encourage students to use the arrows as reminders of which term in the polynomial is multiplied first when using the distributive property.
- Consider posting the integer sign rules on a poster or on the board.

Gifted and Enrichment

• One of the difficulties of multiplying and expanding is keeping track of each of the parts. Challenge students to develop their own method of tracking and accounting for each of the parts of a complex polynomial.

Common Errors

- Some students may have difficulty using the algebra tiles to correctly model a monomial and/or polynomial.
- **R**_x Help students recall how they used integer chips in grade 8 and that the convention is similar for algebra tiles.

Answers

Example 1: Show You Know

a) $2x^2 + 6x$ **b)** $2c + c^2$

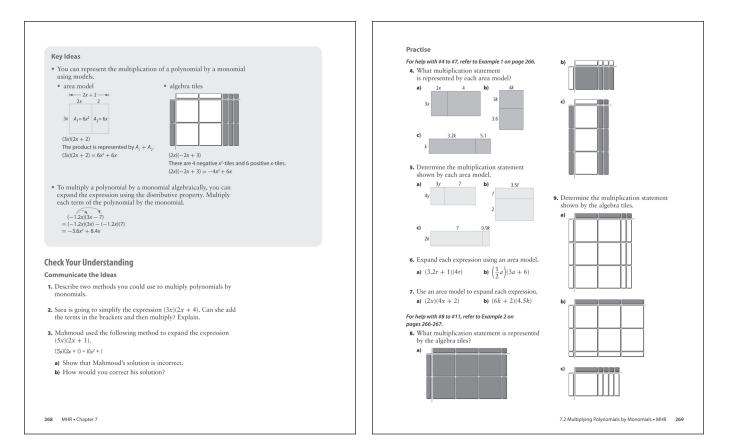
Example 3: Show You Know

a) $-6x^2 - 15x$ **b)** 55y - 5xy

Example 2: Show You Know

a) $6x + 9x^2$ **b)** $8x^2 - 4x$

Assessment	Supporting Learning			
Assessment for Learning	Assessment <i>for</i> Learning			
Example 1 Have students do the Show You Know related to Example 1.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. You may wish to provide students with Master 8 Centimetre Grid Paper or Master 9 0.5 Centimetre Grid Paper for drawing their area models. Ensure that all students understand the relationship between the rectangle dimensions and the polynomials, as well as the rectangle area and the polynomial product. Some students may benefit from separating the two rectangles and labelling their dimensions. Adding them together may help students to see the relationship between the algebraic expression and the tiles. 			
Example 2 Have students do the Show You Know related to Example 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Encourage students to model the questions using actual algebra tiles. If algebra tiles are not available, provide students with Master 11 Algebra Tiles (Positive Tiles) and Master 12 Algebra Tiles (Negative Tiles). Ensure that all students are able to use the negative tiles correctly when modelling the polynomials and interpreting the product. 			
Example 3 Have students do the Show You Know related to Example 3.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. You may wish to provide students with Master 8 Centimetre Grid Paper or Master 9 0.5 Centimetre Grid Paper for drawing their area models. It is important that students are able to use the distributive property correctly. Encourage students to use a systematic approach to the multiplication of the terms. 			



Check Your Understanding

Communicate the Ideas

You may wish to have students complete these questions in groups and discuss their answers. In #1, students should be able to describe both a method involving symbols and a method involving models (area model, algebra tiles, or diagram) for multiplying polynomials. This question also provides students with an opportunity to show that they understand the relationship between the concrete and algebraic models.

In #2, students are asked to demonstrate their understanding of the distributive property and order of operations as they apply to polynomial multiplication.

All students should answer question #3 as it requires them to use higher-order thinking skills by identifying the error in a solution and how to correct it.

Practise

Students will differ in the extent to which they continue to rely on models to multiply polynomials. Some students may already be comfortable with symbolic representations and the use of the distributive property. Students may be given a choice of completing any three questions from #4 to 7 and #8 to 13; however, suggest that they select at least one question that uses algebra tiles and one that does not. Ask students which questions they selected and why to determine whether they are having difficulty with certain types of questions.

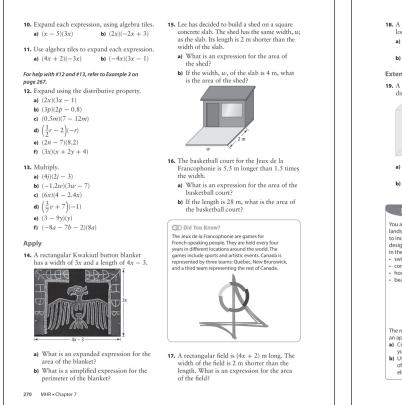
While #4 and 5 present area models representing polynomial multiplication, #6 and 7 require students to develop the area model from the polynomial multiplication statement.

While #8 and 9 present algebra tiles representing polynomial multiplication, #10 and 11 require students to use algebra tiles to perform polynomial multiplication.

For #12 and 13, students are required to use algebra (including the distributive property) to perform polynomial multiplication. Students may be encouraged to use either of the manipulative approaches to confirm their solutions, if needed.

Apply

For #14 and 15, diagrams are included that help students visualize the relationship between the area dimensions and the polynomial product. Assign additional questions based on student interest and/or



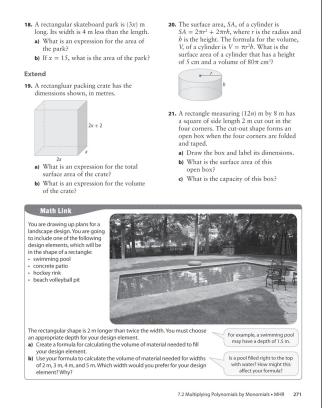
familiarity with the contexts. At least one of #16 to 18 should be assigned as these questions require students to create their own diagram in the given context or to work algebraically.

In #16 and its accompanying Did You Know?, students read about the International Jeux de la Francophonie. They may wish to research the Jeux de la Francophonie held in Canada. Refer to the Web Link that follows.

Extend

The three Extend questions involve three-dimensional objects that result in students working with polynomials of degree 3. Note that the curriculum does not require students to work with polynomials of a degree higher than 2. These questions serve to provide additional opportunities for students to use higher-order thinking to extend their learning.

Literacy Link At the end of section 7.2, have students fill out the lower right leg of their spider map. Brainstorm and discuss as a class the information needed to complete this leg. Have them provide their own examples and then illustrate how to multiply a polynomial by a monomial.



Math Link

The Math Link involves selecting a park feature as part of a landscape design. Although the features are three-dimensional, the emphasis is on the two-dimensional shape with known depths being used to create volume options. The use of computers with spreadsheet software will enhance this activity.

Web Link

For information about the Canadian Jeux de la Francophonie, go to www.mathlinks9.ca and follow the links.

Meeting Student Needs

- Consider expanding the Math Link to allow students to work on a design element that is more relevant to their own lives. For example, you may wish to add the following to the list of possible design elements: holding pond, watershed pool, curling rink, concrete foundation for a stable or a basketball court, or gravel driveway. You might also encourage students to come up with their own ideas.
- Provide **BLM 7–7 Section 7.2 Extra Practice** to students who would benefit from more practice.

ELL

- Teach the following terms in context: *brackets*, *expand*, *button blanket*, *shed*, *concrete slab*, *basketball court*, *skateboard park*, *packing crate*, *cylinder*, *taped* (meaning used masking tape), *hockey rink*, *beach volleyball pit*, and *prefer*.
- For #15, use the picture to teach *shed* and *concrete slab* in context.
- For #16, English language learners may have the skills to do the questions but not understand the language. For example, students need to understand that *5.5 m longer* means addition.

Common Errors

• Students may have difficulty converting the written description into a formula.

 R_x Provide students with similar expressions and assist them in developing the appropriate algebraic statements. Students could also work in small groups to develop the statements.

Answers

Communicate the Ideas

- **1.** Example: He could use a model or symbols.
- **2.** No, she cannot add the terms inside the brackets because 2*x* and 4 are not like terms.
- **3.** a) $(5x)(1) \neq 1$
 - **b)** Multiply the 5x by 1 to give 5x. The solution should be $10x^2 + 5x$.

Math Link

- a) Example: Beach volleyball court with a depth of 0.5 m. Therefore, a formula is (w)(2w + 2)(0.5), and the simplified formula is $w^2 + w$.
- b) Example: w = 2 m, V = 6 m³; w = 3 m, V = 12 m³; w = 4 m, V = 20 m³; w = 5 m, V = 30 m³. For the beach volleyball court, the width of 5 m would be best to ensure there is enough room to play.

Assessment	Supporting Learning			
Assessment as Learning				
Communicate the Ideas Have all students complete #1 to 3.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Some students may benefit from using tiles to obtain a visual model for #2 and 3 before answering the questions. Consider providing students with Master 2 Communication Peer Evaluation to assess each other's answers on one or more of these questions. 			
Assessment for Learning				
Practise Have students do #4, 6, 8, and 10. Students who have no problems with these questions can go on to the Apply questions.	 Students who have difficulty with #4 and/or 6 will need additional coaching on Example 1. Help students work through #4, and then have them try some parts of #5. Similarly, after students work through #6, assign #7. Some students may benefit form working with a partner for #8 and 10. Provide tiles for as long as students require them for support. For #15, some students may benefit from drawing the shed and slab separately and labelling their dimensions. 			
Math Link The Math Link on page 271 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 281.	 You may wish to have students complete the Math Link in order to apply their understanding of polynomial multiplication. Listen to any discussion about how students solve the problem. You may wish to have students save the park feature they worked on and use it for the Wrap It Up! they will do at the end of the chapter. Students who need help getting started could use BLM 7–8 Section 7.2 Math Link, which provides scaffolding. Provide students with formative feedback for the Math Link as they work towards the Wrap It Up! 			
Assessment <i>as</i> Learning				
Literacy Link By the end of section 7.2, have students complete the part of the spider map related to multiplying polynomials by monomials.	 You may wish to have students print the following subheadings beside the heading in the lower right leg of the spider map: Example of Monomial, Example of Polynomial, and Multiplying Polynomials by Monomials. Have them provide their own examples of a polynomial and a monomial and then have them illustrate how to multiply them. Encourage students to fill in the spider map with examples and methods of their choice. 			
 Math Learning Log Have students respond to the following questions: What methods can you use to multiply polynomials? What method do you prefer? Explain. 	 Some students may benefit from the use of tiles to answer the questions. You may wish to refer students back to the examples and encourage students to create their own examples in their Foldable. Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. 			