Dividing Polynomials by Monomials

MathLinks 9, pages 272–277

Suggested Timing

60–80 minutes

Materials

- algebra tiles
- centimetre cubes
- grid paper
- ruler

Blackline Masters

Master 2 Communication Peer Evaluation Master 6 Square Dot Paper Master 7 Isometric Dot Paper Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper Master 11 Algebra Tiles (Positive Tiles) Master 12 Algebra Tiles (Negative Tiles) BLM 7–3 Chapter 7 Warm-Up BLM 7–9 Section 7.3 Extra Practice BLM 7–10 Section 7.3 Math Link

Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Mathematics and Estimation (ME)
- Problem Solving (PS)
- 🖌 Reasoning (R)
- Technology (T)
- Visualization (V)

Specific Outcomes

PR7 Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2, 4, 6, 8, 10, Math Link
Typical	#1, 2, 4, 6, 8, 10, 11, 14, Math Link
Extension/Enrichment	#1-3, 8, 10-15, 17



Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections.

As a class, read and discuss the information about fish tanks and cichlids. Have students discuss possible answers to the question. Emphasize that the equation and solution can be used to make predictions for the unknown dimension of the tank. Ask students questions such as the following:

- What does each dimension of the tank represent?
- What does the volume of the tank represent?
- What polynomial multiplication statement can be represented by the volume of the tank?

Explore Dividing a Polynomial by a Monomial

In this exploration, students work with a volume model of a polynomial that represents the dimensions of a three-dimensional object similar to the fish tank. This model assists students in understanding how dividing a polynomial by a monomial results in an expression that represents an unknown quantity. You might wish to discuss with students the relationship between tank size and the size and habits of a fish. Students may be interested in this information about the jaguar cichlid: An adult male jaguar cichlid, or *Parachromis managuensis*, typically has a mass of 1.5 kg and a length of 50 cm. Since it is not a strong swimmer, its tank does not have to be as large as it would be for a better swimmer.

Method 1 Have students work in pairs to complete #1 to 5 of the Explore. Have each pair of students use algebra tiles or, alternatively, diagrams to model the problem situation. Invite each group to report their conclusions to the class. Then, have a class discussion to ensure that all students have made the connection between the model and the abstract symbolic representation of polynomial division. You may wish to ask the following questions:

- Is it practical to use a volume model for all polynomial division? Explain.
- Is there a way to model a negative value using a volume model? What might that be?

No part of the exploration involves the division of a polynomial by a negative monomial. The reason is that the volume model as presented does not provide an intuitive solution involving negative dimensions. This is better demonstrated with an algebra-tiles model or diagram related to the algebraic expression.

Method 2 Have students work individually to complete #1 to 5 of the Explore. Then, have students discuss their conclusions with a classmate. Students could be provided with other modelling materials (e.g., centimetre cubes) to help them make the connection between the resulting polynomial and the three-dimensional representations.

It is important to make sure that students are able to relate the attributes of the volume model (e.g., length, width, missing dimension, total volume) to the algebraic components (e.g., monomial/polynomial, solution, final quotient). This is critical information that helps to bridge the conceptual and procedural aspects of polynomial division.

Meeting Student Needs

- It may benefit your class to work through the Explore as a whole-class activity.
- You may wish to replace the fish tank scenario with a context that is more relevant to students' lives and culture. For example, in some Aboriginal communities, traditional women dancers carry a feathered fan to complete their outfit. Some dancers will have a special box made to carry the fan. Consider inviting to the classroom a member of the Aboriginal community who has crafted fan boxes. Have this guest discuss with students how this box is made from a solid piece of wood. Then, have a discussion about the dimensions of the fan box, and perhaps have someone bring in a fan for viewing. As a class, create a context for the Explore based on the fan box.

ELL

- Teach the following terms in context: *fish tank*, *habits*, *dimensions*, *volume*, and *base*.
- Read the introduction to the class. Have other students add any information they might have regarding the size of tanks for different types of fish. Use the illustration to teach the name *jaguar cichlid*.
- Pair up English language learners with other students in the class to do the steps in the Explore.
- For #4 and 5 of the Reflect and Check, model what it means to *show* that your expression is correct and *describe* the steps taken. Some students from other educational systems may not be used to having to use words to explain how something works in math.

Common Errors

- Students may have difficulty relating the volume of the rectangular solid to the numerator in the polynomial division statement.
- $\mathbf{R}_{\mathbf{x}}$ Show students how the dimensions of the solid are multiplied to create a product (volume). Start with numerical values only and introduce first one dimension as a variable and then a second dimension as a variable. Then, have students work backward from a volume based on one singlevariable dimension.

Answers**Explore Dividing a Polynomial by a Monomial**1. volume = area × height; area = $2x^2 + 4x$; height = 3
 $volume = (2x^2 + 4x)(3)$
 $= 6x^2 + 12x$ 2.3.4.4.(x + 2)(2x) = $2x^2 + 4x$, which is the given area in the question.5.5.5.6.6.7.

Assessment	Supporting Learning
Assessment as Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Observe whether students are able to identify which symbolic representation corresponds to the model.	 Encourage students to model the Explore using actual concrete materials, such as algebra tiles. If algebra tiles are not available, provide students with Master 11 Algebra Tiles (Positive Tiles) and Master 12 Algebra Tiles (Negative Tiles). Some students may need assistance generalizing the algebra tile representation to the symbolic representation. Have them verbalize the similarities and differences between the two methods. All students would benefit from a class discussion of #5. Write down the various approaches that students used. It will benefit students to see and hear the thinking behind each solution.





Link the Ideas

Example 1

This example reinforces how algebra tiles can be used to represent the symbolic division of polynomials. If needed, students should be reminded of how the dimensions of the rectangle represent the monomial and polynomial and that the unknown is the quotient of the polynomial and monomial. You may wish to ask the following questions:

- Can algebra tiles be used to model the division of a trinomial by a monomial? Explain.
- Is this method easier to use than a volume model when modelling negative values in a polynomial? Explain why or why not.
- Can algebra tiles be used to model polynomial division involving rational number coefficients? Explain.
- Refer to the thought bubble that shows this division using symbols. Which method do you prefer? Why?

Note that, as an alternative to algebra tiles, students could use diagrams to model the division.

When completing the Show You Know, monitor if students are able to model using the negative tiles and if they are able to interpret the product correctly in part b).

Example 2

This example provides students with procedural information related to the algebraic approach to polynomial division. Point out that, although this is a three-dimensional problem, only the variable r is considered the unknown as h is eliminated in the division process. You may wish to ask the following questions:

- Is it simpler to keep the ratio as a single expression or as two expressions? Why do you think so?
- What would happen if an approximate value for π were substituted?
- What is the common factor in part a)? Could that value be factored out of the expression first? Would that change the final answer?

Key Ideas

Have students read and review the Key Ideas. Have students explain how the same algebra-tile model represents two different division statements. You may wish to provide additional examples of polynomial division. Ask students if there is anything in the Key Ideas that they would like to add to their Foldable.

Meeting Student Needs

- It may be better for your class to work through each example as a whole-class activity. Assign one part of the Show You Know as a small-group or pair activity. Then, assign the other part as individual student work.
- Some students may benefit from continuing to use algebra tiles for the Example 1 Show You Know.

Gifted and Enrichment

ELL

• Challenge students to examine the detail in the thinking for Example 2. Have them use that thinking to develop their own similar ratio problem and worked example.

• Teach the word *quotient* in context.

Common Errors

- Some students may have difficulty using the algebra tiles to correctly model the polynomial division.
- $\mathbf{R}_{\mathbf{x}}$ Emphasize to students that the result of the polynomial division is the unknown dimension of the rectangle.

Answers

Example 1: Show You Know a) x + 2 **b)** 4x - 1

Example 2: Show You Know

a) 5x - 4 **b)** -t + 2

Assessment	Supporting Learning
Assessment for Learning	
Example 1 Have students do the Show You Know related to Example 1.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Encourage students to model the questions using actual algebra tiles. If algebra tiles are not available, provide students with Master 11 Algebra Tiles (Positive Tiles) and Master 12 Algebra Tiles (Negative Tiles). Ensure that all students understand the relationship between the algebra-tile components and the symbolic representation of the polynomial division.
Example 2 Have students do the Show You Know related to Example 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Ensure that all students are able to substitute <i>h</i> for <i>r</i> and are able to simplify the resulting expression. Some students will benefit from extra coaching and a review of exponent laws to assist them with Example 2.



Check Your Understanding Communicate the Ideas

You may wish to have students complete these

questions in groups and discuss their answers.

In #1, students are asked to demonstrate their understanding of polynomial division using either a concrete or abstract model. You may wish to ask students why they selected to explain the particular method that they did.

All students should answer #2 as it requires them to use higher-order thinking skills by identifying the error in a solution and explaining how to correct it.

In #3, students should be able to use algebra tiles or another model to represent polynomial division.

Practise

For #4 and 5, encourage students to think about how algebra tiles are used to represent polynomial division and how the model relates to the solutions obtained in #6 and 7. Note that students may use models of their choice, such as diagrams or algebra tiles, to complete #6 and 7.

For #8 and 9, encourage students to use an algebraic approach to solve the polynomial division. It may

be beneficial for students to verify their solutions. They might use algebra tiles to verify their solutions for problems involving integer coefficients, and they might use volume/area models for problems involving decimal-number coefficients. You could ask students to explain why they selected a particular method to solve a problem or to confirm their answers.

Apply

Question #13 provides a good entry-point problem involving polynomial division and it is an appropriate question for struggling students to begin with. Students must model the descriptions mathematically for the real-world applications in #10, 11, 12, and 14. Remind students to choose the algebra tiles that represent the product and divisor in order to determine the final solution.

Extend

Each part of #15 involves rational coefficients and requires students to apply fraction operations or operations involving decimal numbers when determining their solutions.

For #16 to 18, students solve abstract problems involving more complex polynomial division, including expressions with two variables.



Literacy Link At the end of section 7.3, have students fill out the lower left leg of their spider map. Brainstorm and discuss as a class the information needed to complete this leg. Have students provide their own examples of a polynomial and a monomial and have them illustrate how to divide a polynomial by a monomial.

Math Link

The Math Link involves designing two parking lots. Although the truck is three-dimensional, the emphasis is on the two-dimensional shape. Students are given known values for three different truck widths and use them to explore the different volumes that each truck can carry. The use of computers with spreadsheet software will enhance this activity.

Meeting Student Needs

- Provide **BLM 7–9 Section 7.3 Extra Practice** to students who would benefit from more practice.
- You may wish to alter the Math Link so that it relates more closely to the community where your students live. For example, students might design a gravel road, a house pad, or an airstrip.

ELL

• Teach the following terms in context: *decorate*, *end wall*, *gym*, *red poster paper*, *dump truck*, *yard*, *distance*, *object*, *common sides*, *parking lot*, *covered*, *gravel*, and *efficient*.

Common Errors

- Some students may find it challenging to use different area or volume formulas in their polynomials.
- R_x Refer students to Example 4 in section 7.1 where the area formula for a triangle is used. Emphasize that the same principles apply to using any area or volume formula (rectangle or circle).

Answers

Communicate the Ideas

- 1. Example: To divide a polynomial by a monomial, divide the numerical coefficients and apply the exponent laws to the variables. When you divide variables with the same base, you subtract the exponents: 1.5x + 3.
- **2.** a) $\frac{3k}{3} \neq 1$ b) The second term in her solution should be k, not 1.





Math Link

- a) Example: Rectangular lot 24 m \times 15 m; circular lot with 24 m diameter b) $20x^2 + 80x$
- c) Example: Rectangular lot with $A = 360 \text{ m}^2$, truck with width of 1.5 m: 2.2 truckloads; truck with width of 2 m: 1.5 truckloads; truck with width of 3 m: 0.9 truckloads. Example: Circular lot with $A = 452 \text{ m}^2$, truck with width of 1.5 m: 2.8 truckloads; truck with width of 2 m: 1.9 truckloads; truck with width of 3 m: 1.1 truckloads
- d) Example: The truck with a 3-m width would be most efficient for the rectangular lot and the truck with a 2-m width would be most efficient for the circular lot because it would be more efficient to take a full load than a fraction of a load.

Assessment	Supporting Learning	
Assessment <i>as</i> Learning		
Communicate the Ideas Have all students complete #1 and 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Some students will benefit from setting up an answer for #1 before explaining it. This question would be suitable to use for oral assessment. Have students provide feedback regarding each other's response to #2, using Master 2 Communication Peer Evaluation. 	
Assessment <i>for</i> Learning		
Practise Have students do #4, 6, 8, and 10. Students who have no problems with these questions can go on to the Apply questions.	 Students who have difficulty with #4 and/or 6 will need additional coaching on Example 1. Help students work through #4, and then have them attempt 5. Similarly, after students work through #6, assign 7. Encourage the use of tiles for any visual or struggling learner. 	
Math Link The Math Link on page 277 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 281.	 You may wish to have students complete the Math Link in order to apply their understanding of polynomial division. Listen to any discussion about how students solve the problem. Consider having students use what they completed in this Math Link for the Wrap It Up! You may wish to provide students with Master 6 Square Dot Paper, Master 7 Isometric Dot Paper, Master 8 Centimetre Grid Paper, or Master 9 0.5 Centimetre Grid Paper for drawing their diagrams. Students who need help getting started could use BLM 7–10 Section 7.3 Math Link, which provides scaffolding. Struggling learners might be told to design only one parking lot as it will still allow them to demonstrate their learning. 	
Assessment <i>as</i> Learning		
Literacy Link By the end of section 7.3, have students complete the part of the spider map related to dividing polynomials by monomials.	• You may wish to have students print the following subheadings beside the heading in the lower left leg of the spider map: Example of Monomial, Example of Polynomial, and Dividing Polynomials by Monomials. Have them write an example of a polynomial and an example of a monomial. Then, have them show how to divide the polynomial by the monomial. Note that the polynomial they choose as the dividend will have to be evenly divisible by the monomial they choose as they divisor.	
 Math Learning Log Have students respond to the following questions: What methods can you use to divide polynomials? What method do you prefer? Explain. 	 Some students may benefit from the use of tiles to answer the questions. You may wish to refer students back to the examples and encourage them to create their own examples in their Foldable. Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. 	