Solving Equations: $ax = b, \frac{x}{a} = b, \frac{a}{x} = b$

MathLinks 9, pages 292–303

Suggested Timing

80–100 minutes

Materials

- coins or items to represent coins of different denominations
- paper cups or small containers
- paper clips

Blackline Masters

Master 2 Communication Peer Evaluation Master 4 Number Lines Master 8 Centimetre Grid Paper Master 9 0.5 Centimetre Grid Paper Master 14 Coin Models BLM 8–3 Chapter 8 Warm-Up BLM 8–5 Canadian Coins and Their Values BLM 8–6 Section 8.1 Extra Practice BLM 8–7 Section 8.1 Math Link

Mathematical Processes

🖌 Communication (C)

- Connections (CN)
- Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- 🖌 Reasoning (R)
- 🖌 Technology (T)
- 🖌 Visualization (V)

Specific Outcomes

PR3 Model and solve problems using linear equations of the form:

- ax = b
- $\frac{x}{a} = b, a \neq 0$
- $\frac{a}{x} = b, x \neq 0$

where a, b, c, d, e and f are rational numbers.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 4–6, 8, 11, 12, Math Link
Typical	#1, 4–6, 8, 11, 12, 14, 17, 19, 23, Math Link
Extension/Enrichment	#1, 20, 21, 23–29

Planning Notes

Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce material learned in previous sections.



Have students examine the photo of Steve Nash and the information in the first paragraph on page 292. You might ask students if they follow basketball and if they have ever watched Steve Nash play. Ask those students who do follow basketball to name their favourite player.

Point out the Did You Know? beside the opening paragraph to highlight the fact that basketball was invented by a Canadian.

Next, direct students' attention to the second paragraph on page 292, and have students consider the two questions. If students have difficulty with the first question, you might encourage understanding and provide assistance by asking coaching questions, such as:

- What does 20 represent in the equation?
- What does 407 represent in the equation?
- Did Steve score the same number of points in every game? (This question is intended to prompt students to consider that *x* must represent an average value.)



When answering the second question, most students can be expected to choose division on the basis of their previous experiences in solving equations. However, you might ask if there is an alternative. Multiplication is a possible answer, if it is used in conjunction with a Guess and Check strategy. After students have solved some equations that involve rational numbers, you may wish to return to this situation to point out the difficulty of using Guess and Check, as opposed to an algebraic method.

After students have answered the questions, you might point out the Web Link on page 292. Students who admire Steve Nash's playing skills may be interested in learning more about him as a player and as a person.

Explore Equations Involving Multiplication and Division

In this exploration, students model, for the first time, solutions to linear equations that involve decimals.

Before students begin to answer the questions, tell them that the diagrams show one way to use cups, coins, and paper clips to model solutions. However, reinforce that they are free to explore other ways of using the model. Emphasize that whatever solution **Literacy Link** Use the Literacy Link on page 293 to activate students' knowledge of the meaning of equation and the appropriate terminology for describing equations. To check for understanding, you might have students identify each variable, numerical coefficient, and constant in the following equations. 0.2g = -1.3

3.2 = -1.5w = 7.7

When students are completing the exploration, encourage them to use the appropriate terminology throughout.

method is used, the two sides of an equation must always be equal. In the present model, the two sides have equal monetary values expressed in dollars.

If there is no convenient supply of coins or you do not want the responsibility of ensuring that students get their contributed coins back, other manipulatives can be used. For example, students might use paper cut-outs of coins. (You may wish to provide copies of **Master 14 Coin Models**.) Another possibility might be to use counters of different colours, labelled penny, nickel, dime, etc., or with their decimal values. Other small objects can be used in place of the paper clips (e.g., small strips of coloured paper). Small boxes of any shape can be used in place of the cups.

Method 1 Have students work on the exploration with a partner or in small groups by using manipulatives. Encourage students to compare their models and discuss their answers.

In #1 and 2, students model and solve equations of the form ax = b. When students examine #1a), you might ask:

- How many paper clips are there?
- How do you know that the coins represent 0.12?

Direct students' attention to the thought bubble. Note that there are four combinations of coins that could be used to model 0.12: a dime and two pennies; two nickels and two pennies; one nickel and seven pennies; and twelve pennies.

To promote understanding in #1b), you might ask:

- Could you model the solution by having 5¢ in one cup and 7¢ in the other? Why or why not?
- Could you model the solution by having 5¢ in one cup and 5¢ in the other? Why or why not?
- When you solve a linear equation, how many values can the variable have?

• In an equation, how are the left side and the right side related?

In #1c), students encounter a decimal value of the numerical coefficient for the first time. You might ask:

- Where can you see a value of 0.06 represented in the diagram?
- How many times is 0.06 represented in the diagram?
- What part of the diagram represents the variable *y*? (Some students may indicate that *y* is the number of groups on the left side that each represent 6¢, or 0.06. Other students may indicate that *y* is the number of cups, since each cup holds a value of 0.06. Either response is reasonable.)

In #2a), to assist students in modelling the equation, you might ask:

- How many cups and paper clips will you use in your model of 3x = 0.6?
- What coins will you use in your model of 3x = 0.6?

In #2a), the way that students choose to use the model to solve the equation 3x = 0.6 is deliberately left open. Encourage students to work in whatever way is most comfortable for them. For some students, the solution may be obvious by inspection of the model of the equation. For other students, the diagram in #1b), and the discussion around it, may suggest a Guess and Check approach to #2a). Other students may organize the manipulatives into three identical groups, with each group representing x = 0.2. To do this, these students may need to replace the coins on the right with other coins of equal value. If students would benefit from modelling and solving more equations such as the one in #2a), you might have them try 3d = 0.54 and 4f = 0.76.

In #2b), you might ask:

- What does the variable *y* represent?
- What is the value of the coins in each cup?
- How many cups do you need? Why?

Answering the last of these questions provides the solution to the equation. Some students may organize the manipulatives into groups, in which 5ϕ in a cup on the left equals 5ϕ in a cup on the right, and then count the number of cups they need to reach a total of 25ϕ on each side. If students would benefit from modelling and solving more equations such as the one in #2b), you might have them try 0.03n = 0.12 and 0.13m = 0.39.

In #3 and 4, students model and solve equations of the form $\frac{x}{a} = b$ and $\frac{a}{x} = b$. In #3, students see how

separating 12¢ into two equal groups of coins can model the solution to $\frac{n}{2} = 0.06$, or $\frac{0.12}{k} = 0.06$.

After students have completed #3, you might ask:

- Suppose you changed the right side of the model to show four cups with 3¢ in each. What equations could you solve with this model? ($\frac{x}{4} = 0.03$ and $\frac{0.12}{a} = 0.03$)
- What are the solutions to the equations? (0.12 and 4, respectively)

In #4, encourage students to apply their own reasoning. For example, in #4a), some students may draw a diagram that shows a third of a paper clip in a cup as being equal to 5ϕ , and then creating two more identical diagrams and reasoning that a whole paper clip must represent 15ϕ , or 0.15. If students would benefit from modelling and solving more equations

such as the ones in #4, you might have them try $\frac{x}{2} = 0.12$ and $\frac{1.25}{2} = 0.25$

$$\frac{x}{4} = 0.13$$
 and $\frac{1.25}{x} = 0.25$.

Encourage students to engage in an open discussion of #5a) and b). Consider holding a class discussion around these questions so that students have the opportunity to share any creative models they developed. In discussing #5b), stress that students are not limited to models that involve cups, coins, and paper clips. For example, students might solve #3a) using a number line. Some students may also suggest algebraic solutions. Encourage students to describe the operations they would perform in solving each equation in the Explore symbolically and to identify any cases in which they are unsure.

In #6, students apply their learning to solve a problem in the context introduced in the section opener. In #6a), have students compare their equations and explain any differences between them. There may, of course, be variations in the letter chosen to represent the variable, but the equations may also be of different forms. The equations 0.8g = 4 and $\frac{4}{g} = 0.8$ both model the problem. Encourage students to draw on their knowledge from other parts of the math curriculum to answer #6b). They may use their previous reasoning with rates, or they may solve a proportion, such as $\frac{0.8}{1} = \frac{4}{g}$.

Method 2 Have students complete #1 to 4 in the exploration by using diagrams. Emphasize that laboriously exact drawings are not necessary; clear sketches are sufficient. You can modify the instructions to make sketching easier. For example, you may want

students to use rectangular boxes, rather than trying to sketch large circles to represent cups. Instead of trying to sketch a coin, students might just record the appropriate decimal value and circle it.

Meeting Student Needs

- Students may be interested in other sports invented in Canada. You might also suggest that interested students do some research to identify some of these sports. Possible answers include lacrosse, five-pin bowling, ice hockey, and synchronized swimming, although there may be disagreements over the origin of a particular sport. Students could also research the games played by the First Nations and Inuit peoples of Canada.
- Invite a member of a First Nations community to talk about the contributions that Aboriginal people have made in Canadian sports. Have students research Aboriginal athletes and collect data about the performance of some of the many famous Canadian Aboriginal athletes.
- Tell students that whenever they are solving an equation such as the one presented in the section opener, 20x = 407, they may wish to estimate the answer first, using whole numbers. They might then compare their calculated answer with their estimate.
- Students might check that their calculated answer makes sense in the context of the given data. (For example, if students were to make the mistake of solving by multiplying both sides by 20, their answer of 8140 points per game would make no sense.)
- Some students may, at this time, be uncomfortable with solving the equation in the opener, 20x = 407. If so, you may wish to come back to this equation after students have worked on Example 2.
- Some students may benefit from first working through an equation of the forms presented in the section, using whole numbers. They can then transition to the material in the chapter dealing with decimals and fractions.
- Less visual and tactile learners may find it easier to work through an algebraic example before working with the manipulatives in the Explore. The cup-andcoins method will then reinforce their learning.
- You may decide that it would be better for the class to work through the Explore as a whole-class activity.
- Students who experience difficulty solving by multiplication may need to be reminded that, when multiplying two fractions with the same number in the numerator and denominator, the two values can be cancelled out. Also, remind students of the

alternative strategy of multiplying the numerators and denominators, and then reducing the fraction to its lowest terms.

ELL

- Teach the following terms in context: *arguably*, *produced*, *situation*, *represents*, *paper clip*, *explore*, *reversing*, and *at least*.
- Very new Canadians may not know the names of the coins yet. Clarify the value of each type of coin.
 Provide BLM 8–5: Canadian Coins and Their Values, which outlines the different coins and their values.

Gifted and Enrichment

- You could challenge students by including fractions in the exploration. You might ask them to model the solution to the equation 0.25x = 0.75 and then ask them to explain how their model also shows the solution to the equation $\frac{1}{4}x = \frac{3}{4}$. You then might ask them to model and solve $\frac{3}{10}y = \frac{3}{5}$.
- To tie into the idea of mathematics and statistics in sports, you could tell students that a poorly performing major league baseball team once hired a mathematician to pick their new team, despite the fact that he knew very little about baseball. He made his picks by evaluating statistics. He set up an equation to identify players who were not highly valued, but contributed a great deal statistically. (See the book Moneyball: The Art of Winning an Unfair Game by Michael Lewis (Norton, 2003)). This is an interesting example of how mathematical modelling and equations can be used to predict results. Another good example is the +/- (i.e., plus/minus) rating system used in hockey. Ask students to pick a sport and evaluate the value of players from a mathematical perspective. They could develop an equation that could be used to evaluate players. They could then predict and track their performance.



There are many sites on the Internet that provide information on equations. Some offer interactive tools that help students to practise their skills, and to visualize the concept of balancing expressions on each side. For examples of these sites, students can go to www.mathlinks9.ca and follow the links.

Answers

Explore Equations Involving Multiplication and Division

- a) Example: The two cups represents the 2 in the equations, the variable *x* represents the value of each paper clip, and the 12¢ represents the total value of the paper clips. b) Example: Each cup has 6¢ worth of paper clips inside and because there is one paper clip per cup, each paper clip is worth 6¢.
 - c) In the second part of the diagram, the number of cups is y. So, y = 2.



3. a) The variable *n* represents the total amount of money, and this total divided by *t*2 represents the money in each cup, 6¢.

b) *n* = 0.12

c) k = 2. This represents the number of cups into which the total amount of money is divided.

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- **5.** a) Using coins, paper clips, and cupsb) Number lines, algebra tiles
- **6.** a) Five games
- **b)** Example: 0.8g = 4,



Assessment	Supporting Learning
Assessment as Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Try to have students generalize the conclusion about their findings.	 Guide students back to the class discussion to assist with #5b). It may be beneficial to write the suggested methods on the board. Students could enter these methods into their Foldable as possible alternative strategies for future use. Have students compare their answers to #6a), and then look at each other's strategies. Encourage them to see if they can solve using more than one additional method.





Link the Ideas

Example 1

This example presents strategies for solving equations of the forms ax = b and $\frac{x}{a} = b$, involving positive and negative fractions and mixed numbers.

When discussing Method 1 for part a), draw students' attention to the thought bubble beside the second diagram. To promote student understanding, you might ask:

- If the divisions on the number line were not changed from quarters to eighths, could you read the value of *x* from the number line?
- Could you read the value of *x* if the divisions were changed to twelfths? What about sixteenths?
- Is there a disadvantage to using sixteenths?
- Why do you think that eighths were chosen?

When discussing Method 2 for solving part a), you may want to first refer students to the Literacy Link beside the algebraic solution.

After discussing the Literacy Link, ask students to answer the question in the first thought bubble in the solution. Then, direct their attention to the second thought bubble to reactivate their knowledge of division and multiplication of rational numbers. Stress the importance of the check. **Literacy Link** To check for understanding, ask students to give examples with whole numbers to show that addition and subtraction, and multiplication and division, are pairs of opposite operations. You might also ask if students can name another pair of opposite operations. (They worked with squares and square roots in Chapter 2.)

In the check, you may want to remind students that the expression $2\left(\frac{3}{8}\right)$ is the same as $\frac{2}{1} \times \frac{3}{8}$. This may make it easier for some students to understand the multiplication step.

For Method 1 for solving part b), you might ask:

- Why does the total length of three of the smaller curly brackets represent *m*?
- How do you know that $-\frac{6}{5} = -1\frac{1}{5}$?
- How are Method 1 in part a) and Method 1 in part b) the same? How are they different?

For Method 2 for part b), draw students' attention to the thought bubbles. Stress the importance of the comparison with the estimate. You might ask:

• Why is $-1\frac{1}{2}$ a reasonable estimate? $\left(-\frac{2}{5}\right)$ is closest to $-\frac{1}{2}$.)





- In the check, how do you know that
 - $-\frac{6}{5} \div 3 = \frac{-6}{5} \times \frac{1}{3}$? How do you know that $\frac{-6}{15} = \frac{-2}{5}$?

For part c) of Example 1, draw students' attention to the two thought bubbles in the solution. The first will reactivate students' knowledge of the sign rule for division, and the second their understanding of the division of rational numbers in fraction form. In Chapter 2, they multiplied by the reciprocal.

In discussing part c), you might ask:

- How do you know that $-3\frac{1}{2} = -\frac{7}{2}$ and that $-2\frac{1}{2} = -\frac{5}{2}$? • If you estimated the quotient $-3\frac{1}{2} \div \left(-2\frac{1}{2}\right)$ using
- If you estimated the quotient $-3\frac{1}{2} \div \left(-2\frac{1}{2}\right)$ using integers, what would your estimate be? Why? (Students may estimate using $-4 \div (-3) = \frac{4}{3}$ or $-3 \div (-2) = \frac{3}{2}$. You might ask if either $-4 \div (-2) = 2$ or $-3 \div (-3) = 1$ would be a reasonable estimate.)
- In the check, would you prefer to remove a common factor from $-\frac{5}{2} \times \frac{7}{5}$ before multiplying? Why or why not?

For the Show You Know, you may wish to provide students with **Master 4 Number Lines**.

Example 2

This example presents strategies for solving equations of the forms ax = b and $\frac{x}{a} = b$, involving positive and negative decimal numbers. Stress the importance of the estimates and make sure that students are able to obtain the correct answers on their calculators. Use the thought bubble in part b) to reactivate students' knowledge of the sign rule for multiplication.

Example 3

This example shows an algebraic method of solving an equation of the form $\frac{a}{x} = b$ in an applied context. Begin by pointing out the Did You Know? beside the example and the photograph of Quick Six and Karyn Drake. Have students read the problem. Then, to emphasize the importance of units of measurement in applied problems, you might ask:

- Are the units of measurement included in the problem consistent with each other?
- What would the units of speed be if distance was measured in kilometres and time was measured in hours?

C Did You Know? At Calgary Stampeders football games, a white horse manded Quick Six changes the length of the field each time the Stampeders score a touchdown. The ridge, Karyn Dake, carries the team flag.	<section-header><section-header><text><text><text><text><equation-block><equation-block><equation-block></equation-block></equation-block></equation-block></text></text></text></text></section-header></section-header>	<section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><text><text><text><text><text><text><text><text></text></text></text></text></text></text></text></text></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header>
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Draw students' attention to the question in the thought bubble in the solution and have them discuss it. Students have previously solved equations of the form $\frac{x}{a} = b$ by multiplying both sides by a number, but the present example is the first time they have seen multiplication of both sides by a variable.

When discussing the solution, you might ask:

- How many values can *t* have in this problem? (The fact that it can have only one value means that they are multiplying both sides of the equation by the same quantity, but they do not know what its value is at this point.)
- How many operations does it take to solve the equation $13.4 = \frac{137.2}{t}$? Why?
- Why do you think that the expression 130 ÷ 13 is used to make the estimate? (This question could be used to reactivate students' knowledge of compatible numbers.)
- Would $140 \div 13$ be a good way to make the estimate?
- Are there other ways you might make the estimate? (Some students may suggest 134 ÷ 13.4, in which case you might mention that, when estimating with a decimal, it is not always easiest to use the nearest integer.)
- Does the estimate suggest that the calculated answer is reasonable?

Reinforce the importance of the summary statement in explaining the numerical solution, and point out how the summary statement directly addresses the wording of the original problem.

The method shown in the check refers back to the information in the original problem. This checking method is more comprehensive than checking by substituting the calculated value of *t* into the equation $13.4 = \frac{137.2}{t}$. The reason is that the substitution method would not detect any errors made in writing this equation; for example, if a student inadvertently wrote the equation as $13.4 = \frac{173.2}{t}$.

For the check using a calculator, have students discuss why this answer is reasonable. They might also discuss how rounding up affects the estimate.

Example 4

This example shows the writing and solving of an equation of the form ax = b in an applied setting.

Have students answer the question in the thought bubble in the solution. You might check students' understanding of the solution by asking:

- Does the equation 0.25p = 176.25 represent the situation? Why or why not?
- Why are both sides of the equation divided by 0.75?



Have students check the answer in different ways. For example, they might reason:

$$\frac{\frac{3}{4}}{\frac{1}{4}} = \frac{176.25}{\frac{1}{4}} = \frac{176.25}{\frac{3}{2}} = 58.75$$

Therefore, $\frac{4}{4} = 176.25 + 58.75 = 235$.

Encourage them to solve the Show You Know using more than one method and compare their answers. They may share their approach with their classmates. Ask

- What method do you prefer? Why?
- What method is more efficient?

Key Ideas

This section summarizes solution methods for equations with forms ax = b, $\frac{x}{a} = b$ and $\frac{a}{x} = b$, involving rational numbers. To check for understanding, you might ask students to describe how the diagrams model the solution to $\frac{g}{2} = \frac{2}{3}$, and how the concrete materials model the solution to 2x = 0.10. You might have students describe in their own words the steps used in the algebraic solution shown and the reasoning behind each step. You might ask students to suggest a reasonable estimate for $\frac{-1.4}{-0.8}$ and to compare their estimate with the calculated value of p. Have students review the Key Ideas for any additional material they would like to add to their Foldable.

Meeting Student Needs

- Possibly work through the Examples as a wholeclass activity.
- You may consider teaching this section over a couple of classes.
- Ensure that students are comfortable with the concepts in Examples 1 and 2 before moving on to the word problems in Examples 3 and 4.
- In Example 3, a good way for students to remember the formula for speed is to think of the speed of a car. The speed of a car is usually referred to in kilometres per hour, which is distance over time.
- You might have students work in pairs or small groups to complete the Show You Know. Alternatively, you could have them work individually and then compare their solutions. Or, assign one question as a small-group or pairs activity, and a second as individual work.
- Encourage students to choose their preferred solution method(s), explain their choices, and suggest improvements to each other's solutions.
- In this section, it is important that students be given the time to work through the manipulatives and the number lines.

ELL

• Teach the following terms in context: *curly bracket*, *isolate*, *speed*, *distance*, *gallops*, *end zones*, *formula*, *consistent*, *verifying musher*, *dog team*, *sled*, *express your answer*, *ski jackets*, *regular price*, and *ski mitts*.

Gifted and Enrichment

- For Example 3, have students research the difference between a Canadian football field and an American football field. Have them calculate how long it would take this horse and rider to gallop the length of an American field.
- To challenge students in Example 4, you might ask:
 Does the equation ^{176.25}/_p = 0.75 represent the situation? Why or why not?
- How would you solve $\frac{176.25}{p} = 0.75$?
- How does the calculated answer from this equation compare with the answer shown in Example 4?
- Is it easier to solve 0.75p = 176.25 or $\frac{176.25}{p} = 0.75$? Why?

Common Errors

- Some students may not know which operation to apply when solving an equation of the form ax = b or $\frac{x}{a} = b$.
- $\mathbf{R}_{\mathbf{x}}$ Emphasize that the aim is to isolate the variable. Give examples to show that the way to do this is to apply the opposite operation.

Answers

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Example 1: Show You Know
a) x = -\frac{2}{9} b) x = \frac{5}{3} c) x = -\frac{7}{5}
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Example 2: Show You Know a) u = 1.04 **b)** k = -0.62

Example 3: Show You Know t = 2.1 h

Example 4: Show You Know

p = \$49.99

Assessment	Supporting Learning		
Assessment for Learning			
Example 1 Have students do the Show You Know related to Example 1.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Some students may benefit from using grid paper with each second square marked off as ¹/₄. When the need to change it to eighths arises, they can make the visual link to solving for 2<i>x</i>. It may be necessary to provide a few more examples of questions following the format of a), b), and/or c), depending on students' need; for example: 3<i>x</i> = ³/₄, 2<i>x</i> = ⁵/₆. You may wish to provide Master 8 Centimetre Grid Paper and/or Master 9 0.5 Centimetre Grid Paper. Some students may benefit from a quick oral review of the Literacy Link beside Method 2. Writing a list of opposites into their Foldables for quick reference may assist some students. Reviewing the opposite of multiplying with fractions may also be helpful. For part c), it may benefit students for you to show them why you require 3 of ^m/₃ to make a whole. Show the process with common denominators and adding. 		
Example 2 Have students do the Show You Know related to Example 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. As students will not be using number lines for these questions, they may benefit from a review of opposites. If may be necessary to review why, when a decimal number is in the denominator, we multiply by that decimal number. As an example, show them that m/3 = 10 is the same as 1/3 m = 10. You multiply by the reciprocal. Link this to the fact that m/2.4 = 25 is the same as 1/2.4 m = 25. To clear out the denominator, you multiply by 2.4 / 1. 		
Example 3 Have students do the Show You Know related to Example 3.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Encourage students to look back at the example and make sure they are able to answer the question in the thought bubble, "Why do you multiply by <i>t</i>?" Review the concept of opposites and perhaps have students write in one extra step when copying their equations to show that 13.4 = 137.2 / t is the same as 13.4 = (1/t)137.2. It makes it visually easier to see the opposite of 1/t. Some students may benefit from identifying each variable's value before attempting to solve. For example, in the Show You Know, s = 23.5, d = 50, and t = ? This may assist students in linking the equation to the values, thereby facilitating student success. 		
Example 4 Have students do the Show You Know related to Example 4.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Some students will need a refresher on percents, how to change a percent to a decimal, and/or how to find a percent of a number. It is important in Example 4 that students recognize where the 75% came from (100 - 25). It may be necessary to go over the check with students so they are clear why 25% was used here. Have students verbalize how they will treat the 30% values in the Show You Know. Have them draw parallels to the example. 		



Check Your Understanding

Communicate the Ideas

In #1a), students can discuss a solution step that is not "wrong," but is also not helpful in isolating the variable. In part b), they can suggest a more effective approach.

In #2, students consider the difficulty of checking approximate answers by substitution. To challenge students, you might ask whether Ming would have encountered the same difficulty if she had multiplied both sides of the equation by 10 and expressed the solution to the resulting equation as a fraction.

In #3, students consider how equations of the forms ax = b and $\frac{a}{x} = b$ can be used to represent the same situation.

Practise

You may wish to have students work in pairs or small groups when completing the Practise questions. Stress the importance of estimation in algebraic solutions and the need to check answers even when the wording of the question does not include the word "check." Encourage students to compare their solution methods, check each other's answers, and suggest corrections or other improvements.

Apply

When students work on the Apply problems individually, in pairs, or in small groups—encourage them to compare solutions, discuss strategies, identify errors, and suggest improvements to each other's solutions.

Literacy Link Before students complete #17, refer them to the Literacy Link box that follows it. To check for understanding, you might ask if each of the following qualifies as a regular polygon:

- a square
- a rectangle that is not a square
- a rhombus that is not a square

Extend

In #26 and 27, students need to perform additional steps in solving equations. You might have them discuss ways of solving the equations most easily. For example, in #26b), they might consider whether it is easier to evaluate the left side before multiplying both sides by z, or to begin by multiplying both sides by 1.8 and z.



In #29 and 30, you might have students suggest alternative strategies. Both parts of #29 lend themselves to the use of concrete models and/or the use the Guess and Check, as well as algebraic methods. In #30, students can also use Guess and Check, as well as writing and solving an equation.

Meeting Student Needs

- Provide **BLM 8–6 Section 8.1 Extra Practice** to students who would benefit from more practice.
- If students need help with #12, refer them to Example 3, which closely resembles part b).
- For #18, most students are expected to write and solve $\frac{4}{5}s = 856$ and then subtract 856 from the value of *s*. Encourage students to think of another solution, not necessarily involving an equation.

Literacy Link Remind students to complete the first oval in their concept map, which is labelled *with multiplication and division*. They should provide an example of the equation form, and outline the steps required to solve the equation, using a strategy of their choice.

Math Link

In this Math Link, students apply their skills in writing and solving equations to the field of nutrition.

After students have shared and commented on each other's solutions to parts a) and b), you might ask which solution strategies they prefer and why. Methods other than writing and solving an equation may vary, even within the same general strategy. For example, if students decide to use a proportion for part b), they may reason from the original data (e.g., by writing and solving $\frac{12}{f} = \frac{1.2}{3}$) or from the answer to part a) (e.g., by writing and solving $\frac{12}{f} = \frac{0.4}{1}$). Because of the wording in part c), the most likely form for the formula is m = 0.4f. After students have completed parts c) and d), you might ask whether the formula $\frac{m}{f} = 0.4$ could also be used, and whether parts c) and d) would be easier or more

difficult if the formula was written in this form. You may wish to ask students if it is a good idea for them to try to get all of the iron they need from dried figs. Then, have students research why they need iron, and some other common sources for getting it. You may consider partnering with the Health teacher to present a unit on nutrition.

One possible approach involves reasoning that, since $\frac{4}{5}$ of the students bought a yearbook, $\frac{1}{5}$ of the students did not. Thus, the number of students who bought a yearbook was four times the number who did not. Therefore, dividing 856 by 4 gives the answer.

• For #19, encourage students to suggest alternative solutions. One strategy is to write and solve

 $\frac{17}{s} = 0.68$. A possible alternative is to write the proportion $\frac{17}{s} = \frac{68}{100}$, and then solve it by identifying the multiple of 4, which relates the equivalent ratios.

- In #21, you may need to explain the concept of net income to students.
- For #25, if students are having difficulty, encourage them to model the problem concretely by folding a square piece of paper, or semi-concretely by drawing a diagram.

ELL

Teach the following terms in context: *checked by substitution, roll of nickels, regular polygon, perimeter, yearbook, Nunavut,* and *Yukon Territory.*

- The Communicate the Ideas questions include a lot of text. Excuse very new students from completing this section. Have the other English language learners partner up with English-speaking students to complete this section.
- Reduce the number of word problems in the Apply section given to English language learners. Clarify with students what, exactly, the questions are asking.
- Read #24 with English language learners. Rephrase and teach in context the difficult words. Students will need to understand the difference between experienced and volunteers and first-time volunteers.
- Before attempting #29, ensure that very new Canadians understand the value of a nickel and a dime.

Common Errors

- Some students may forget to apply the same operation to both sides when solving an equation.
- R_x Encourage students to check their solutions, even when the wording of a problem does not explicitly include the word "check," so that they can detect the presence of any errors and can take steps to correct them.
- Students may ignore negative signs when solving equations of the form ax = b, $\frac{x}{a} = b$, or $\frac{a}{x} = b$.
- R_x Have students apply the sign rule for multiplication or division to check that the sign of their answer is correct.

Answers

Communicate the Ideas

- **1.** a) No, John's first step does not isolate the variable.b) To isolate *y*, multiply both sides of the equation by 2.
- **2.** a) Ming could use the fraction equivalent in the check.
- **b)** Her solution was correct; the check is off slightly due to her approximating the answers.
- **3.** a) Amalia thinks that 0.75 m, times the number of strides, equals the distance.
 - **b)** Gustav thinks that the distance, divided by the number of paces, equals the length of each stride.

Math Link





c) 0.4f = I; 0.4(8) = 3.2 mg

d)
$$f = \frac{1.8}{0.4}$$
, or 4.5 figs

Assessment	Supporting Learning		
Assessment <i>as</i> Learning			
Communicate the Ideas Have all students complete #1.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. For #1, students my benefit from having a class discussion about the fact that the process used is not wrong, but it is not the most efficient. For students who say this is the way they would choose, have them verbalize the rest of the solution, ensuring that they understand that they will multiply by the inverse of ³/₂ to isolate the variable. You may wish to have students use Master 2 Communication Peer Evaluation to assess each other's responses to #1. 		
Assessment <i>for</i> Learning			
Practise and Apply Have students do #4–6, 8, 9, 11, and 12. Students who have no problems with these questions can go on to the Apply questions.	 For students having difficulty with #4, they may find it helpful to review the Explore or to receive additional coaching. If cups confuse students, have them model with another manipulative that they feel more comfortable with. Have them verbalize what they see and what it represents before writing their solution. Note that #5 is similar to #4 in that students must have a good understanding of modelling. Again, provide alternativee manipulatives that students may be more familiar with if the number line confuses them. Provide grid paper for students needing visual support. 		
Math Link The Math Link on page 303 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 333.	 Students who need help getting started could use BLM 8–7 Section 8.1 Math Link, which provides scaffolding. The Math Link allows students to apply multiple methods in demonstrating their understanding of solving equations. It may help students to work in pairs to generate the formulas and then individually to use the methods of their own choosing. 		
Assessment <i>as</i> Learning			
Literacy Link (page 289) After completing this section, work with students to work on the first oval. Have them list the steps that lead to solving an equation using multiplication and division.	• You may want to model how the organizer works by discussing with the class how to complete this first oval.		
 Math Learning Log Have students respond to the following prompts: The method I find easiest to model solving equations is The opposite operations are so to solve 2.2x = 7, the first step is 	 It may benefit some students to explain their responses orally first, then write them down. If students are having difficulties solving the 2.2x = 7, have them choose a question from the student resource and use this as their example. 		