Solving Equations: $ax + b = c$, $\frac{x}{a} + b = c$

8.2

***MathLinks 9*, pages 304–313**

Suggested Timing

80–100 minutes

Materials

- coins or items to represent coins of different denominations
- cups or small containers
- paper clips

Blackline Masters

Master 2 Communication Peer Evaluation
Master 4 Number Lines
Master 8 Centimetre Grid Paper
Master 9 0.5 Centimetre Grid Paper
Master 14 Coin Models
BLM 8–3 Chapter 8 Warm-Up
BLM 8–5 Canadian Coins and Their Values
BLM 8–8 Section 8.2 Extra Practice
BLM 8–9 Section 8.2 Math Link

Mathematical Processes

- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes

PR3 Model and solve problems using linear equations of the form:

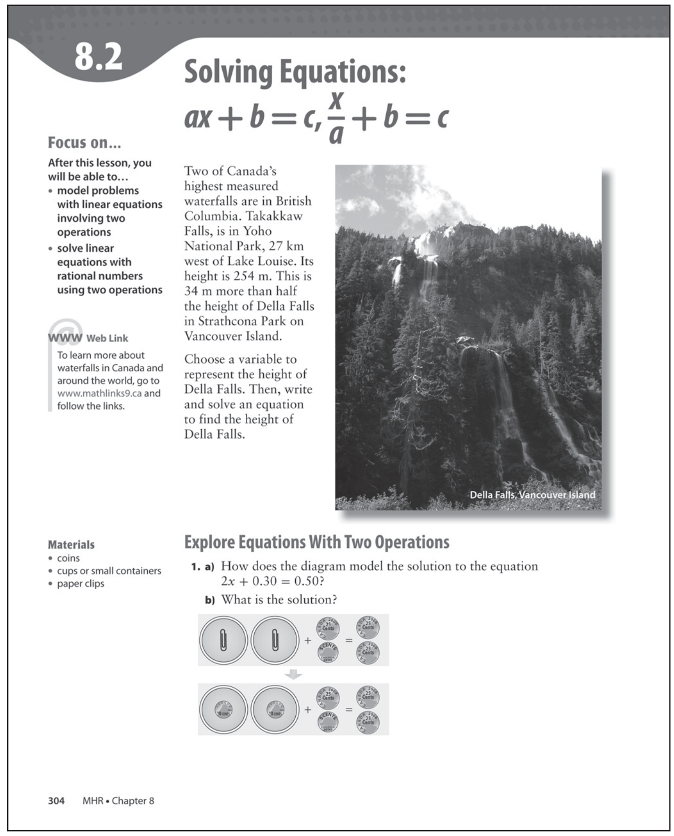
- $ax + b = c$
- $\frac{x}{a} + b = c, a \neq 0$

where a, b, c, d, e and f are rational numbers.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5–7, 9, 11, 13, 15, Math Link
Typical	#1–3, 5–7, 9, 11, 13, 15 , 16, 19, Math Link
Extension/Enrichment	#1–3, 19, 20, 23, 26–32

Planning Notes

Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce material learned in previous sections.



8.2 Solving Equations: $ax + b = c, \frac{x}{a} + b = c$

Focus on...

After this lesson, you will be able to...

- model problems with linear equations involving two operations
- solve linear equations with rational numbers using two operations

Web Link

To learn more about waterfalls in Canada and around the world, go to www.mathlinks9.ca and follow the links.

Two of Canada's highest measured waterfalls are in British Columbia. Takakkaw Falls, is in Yoho National Park, 27 km west of Lake Louise. Its height is 254 m. This is 34 m more than half the height of Della Falls in Strathcona Park on Vancouver Island.

Choose a variable to represent the height of Della Falls. Then, write and solve an equation to find the height of Della Falls.

Della Falls, Vancouver Island

Explore Equations With Two Operations

Materials

- coins
- cups or small containers
- paper clips

1. a) How does the diagram model the solution to the equation $2x + 0.30 = 0.50$?

   b) What is the solution?

304 MHR • Chapter 8

The first paragraph on page 304 is designed to activate students' knowledge and skills with equations of the form $\frac{x}{a} + b = c$, where a, b, and c are integers.

Point out the photograph of Della Falls, one of Canada's highest measured waterfalls. (Note that we have specified *measured* because, although the Canadian government lists Della Falls as the highest falls in Canada, the World Waterfall Database suggests that there are other falls—perhaps many others—in rugged parts of BC that are higher, but which have not been officially mapped and measured.)

Have students read the paragraph and individually choose a variable and write the required equation. Then, have students discuss the equations they wrote so that the class can reach a consensus about a correct equation. Have students solve the equation individually, compare their solutions, and make any necessary corrections.

2. a) Explain how the second part of the diagram in #1 can model the equation $0.10y + 0.30 = 0.50$. What is the solution? Explain.
   b) How does the second part of the diagram in #1 also model the solution to the equation $\frac{z}{10} + \frac{3}{10} = \frac{1}{2}$? What is the solution?

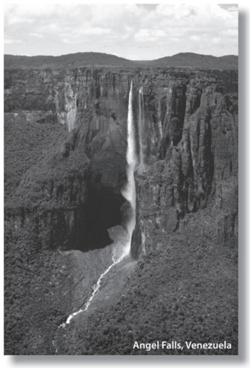
3. Describe how you would use manipulatives or diagrams to model the solution to each of the following.
   a) $3x + 0.05 = 0.26$
   b) $0.01x + 0.05 = 0.08$
   c) $\frac{x}{4} + \frac{1}{5} = \frac{7}{10}$

4. Work with a partner to explore how to model the solution to the equation $2x - 0.11 = 0.15$. Share your models with other classmates.

Reflect and Check

5. a) How can you model solutions to equations of the form $ax + b = c$ and $\frac{x}{a} + b = c$ using manipulatives or diagrams?
   b) Think of other ways to model the solutions. Explain how you would use them.

6. The tallest waterfall in the world is Angel Falls in Venezuela, with a height of about 0.8 km. This height is 0.08 km less than twice the height of Della Falls. Write and solve an equation to determine the height of Della Falls in kilometres. Check that your answer agrees with the height in metres you determined at the beginning of this section.


Angel Falls, Venezuela

Did You Know?
Canada's tallest free-standing structure is the CN Tower, with a height of about 550 m. This is about 250 m less than the height of Angel Falls.

8.2 Solving Equations: $ax + b = c, \frac{x}{a} + b = c$ • MHR 305

While students are solving the equation, you might ask some coaching questions:

- Which operation will you perform first? Why? (If students are unsure of why they subtract first, you might remind them of the reverse order of operations. Some students may benefit from the acronym SAMDEB when working with the reverse order of operations.)
- Which operation will you perform second? Why?
- Why do you need to perform the same operations on both sides?
- How can you check that your calculated answer is correct?
- What are the units of the answer?

Explore Equations With Two Operations

In this exploration, students model and solve linear equations of the form $ax + b = c$ and $\frac{x}{a} + b = c$ where a, b, and c are positive rational numbers.

Method 1 Have students work on the exploration with a partner or in small groups, using manipulatives. Encourage students to compare their models and discuss their answers. You may wish to provide **Master 14 Coin Models**, if actual coins are not available.In #1, students observe a model of a solution to an equation of the form $ax + b = c$ involving decimals. You might ask:

- How does the first part of the diagram in #1 model the equation $2x + 0.30 = 0.50$?
- How do you know that the solution modelled by the second part of the diagram is correct? (Use this question to check students' understanding that each paper clip represents the same value and that the total value on each side of the equation must be the same.)

In #2a), students see a decimal value for a numerical coefficient, with the variable y representing the number of dimes on the left side of the model. In #2b), students see an equation of the form $\frac{x}{a} + b = c$ with fractional constants. You might ask:

- How are the equations in #2a) and b) the same? How are they different?
- If you write the equation in #2b) in the form, $ax + b = c$, what is the value of a?
- How can you check that the solution to #2b) is correct?

In #3, encourage students to use manipulatives to model solutions in whatever way they prefer. The strategies they use may include inspection, Guess and Check, and methods that make use of their growing knowledge of algebra. Encourage students to see connections between such uses of the model and algebraic techniques.

In #4, students construct a model to represent and solve an equation that involves subtraction. Encourage students to share their solution methods and to explain their preferences. If students would benefit from modelling and solving more equations of this type, you might have them try $3x - 0.05 = 0.25$ and $4x - 0.20 = 0.28$.

Encourage students to engage in open discussion of #5a) and b).

In #6, students apply their learning to solve a problem that involves the heights of waterfalls, the context introduced before the Explore. You may wish to point out the photograph of Angel Falls, which follows #6. Have students compare their equations and solutions for #6.

You may wish to read with students the Did You Know? next to #6. It compares the height of Angel Falls to the height of the CN Tower. The highest building in the world is Burj Dubai, in Dubai, United Arab Emirates. The uncompleted structure was 707 m high on September 21, 2008. Its finished height has been kept secret, but some drawings suggest a final height of 818 m.

Method 2 Have students complete #1 to 4 in the exploration by sketching diagrams.

Meeting Student Needs

- Ask students to research the word *takakkaw* (TA-kuh-koh). What does it mean? What is its origin? (It means "it is wonderful, magnificent" in Cree.)
- When discussing the equation in the section opener, encourage students to solve the equation in the form $\frac{d}{2} + 34 = 254$, where d is the height of Della Falls. Some students may write $\frac{1}{2}d + 34 = 254$ or $0.5d + 34 = 254$. However, they have not previously worked with equations of the form $ax + b = c$, when a, b, or c is a fraction or decimal, so they may be unsure of how to solve it. This is an opportunity to show that multiplying both sides of an equation by 2 is equivalent to dividing both sides by $\frac{1}{2}$ or 0.5.
- Some students may have difficulty conceiving of the heights of the falls. To give a clearer sense of this, you might ask students to compare the height with more familiar lengths, such as the length of a football field, or to identify a location that is about 0.8 km from the school.
- When the constant is a fraction, students will need to recall how to add and subtract, and multiply and divide, fractions.
- It may be better for your class to work through the Explore as a whole-class activity.
- Some students may have difficulty understanding how to model fractions with coins; for example, in #3c) of the Explore. If so, encourage them to rewrite the equation in decimal form before modelling it with manipulatives.
- If students would benefit from modelling and solving more equations such as the ones in #3, they might try $4x + 0.02 = 0.14$, $0.35x + 0.50 = 1.20$, and $\frac{x}{10} + \frac{1}{2} = \frac{4}{5}$.
- Consider holding a class discussion around questions #5a) and b) of the Explore so that students have the opportunity to share any creative models they developed.
- In #5b), stress that students are not limited to models that involve cups, coins, and paper clips.
- Encourage algebraic reasoning by asking students to describe the operations they would perform in solving each equation in the Explore symbolically.
- In #6 of the Explore, the units may cause some students difficulty. If students see their answer as being different from the height of Della Falls in metres, ask them to specify the units of their answer. If they solve the equation $2d - 0.08 = 0.8$ in #6, their answer should be expressed in kilometres, and a unit conversion is needed to confirm that the two height values agree.
- Remind students that the first step when solving equations of this form is always to "remove" the constant by using the reverse operation.

ELL

- Teach the word *highest* using pictures of objects of varying heights, such as buildings.
- Very new Canadians will not know the name of the coins yet. Clarify the value of each type of coin. Provide **BLM 8–5 Canadian Coins and Their Values**.
- Have English language learners work with English-speaking partners to complete the Explore.

Answers

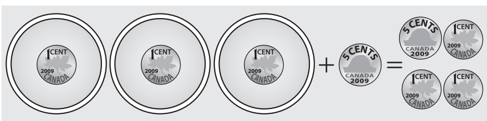
Explore Equations With Two Operations

1. a) The value of two paper clips plus 30¢ equals 50¢.
b) $x = 0.10$

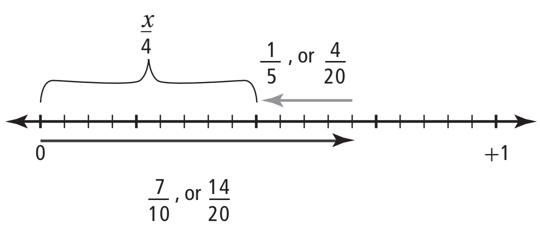
2. a) y represents the number of cups in this equation. So, $y = 2$.
b) z represents the number of dimes in both cups. So, $z = 2$.

3. a)

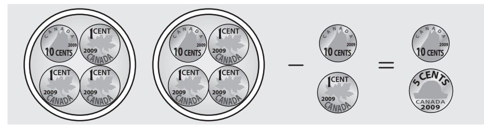
b)



c)



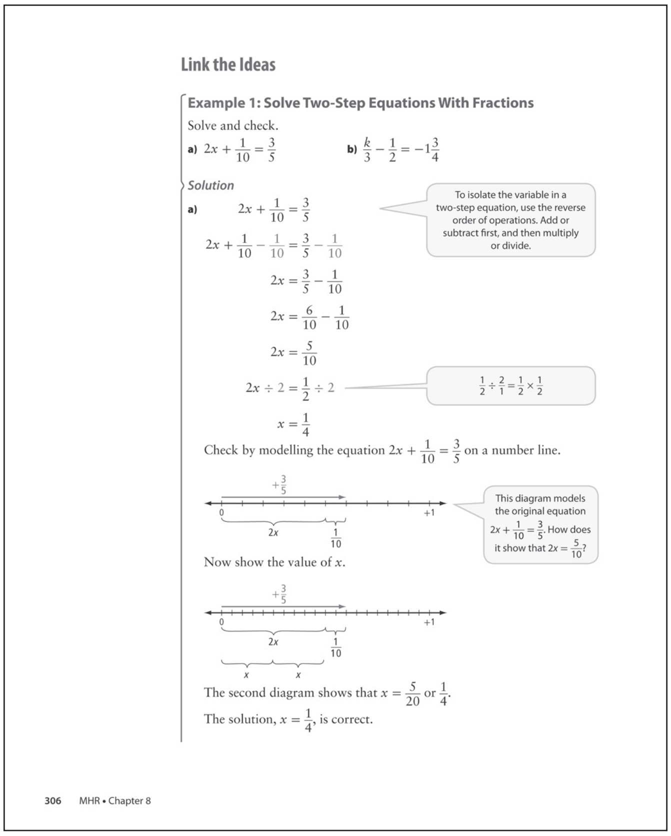
4.



5. a) Using coins, paper clips, and cups
b) Number lines, algebra tiles

6. $2x - 0.08 = 0.8$, so, the height of Della Falls is 0.44 km.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Try to have students generalize how they can use manipulatives of diagrams to model the solution to equations.	• Students may discuss their ideas with a classmate to help articulate a method for #5a). • Have a class discussion to assist students with #5b). •Make a list of the suggested methods, on the board, that allow students to check their answer. • Some students may need to be directed to find key words in word problems to assist them in identifying the relevant parts to be used to design an equation. For example, key words in #6 include *height, 0.8 km, 0.08 less than, twice the height*.



Link the Ideas

Example 1: Solve Two-Step Equations With Fractions

Solve and check.

a) $2x + \frac{1}{10} = \frac{3}{5}$ **b)** $\frac{k}{3} - \frac{1}{2} = -1\frac{3}{4}$

Solution

a)

$$2x + \frac{1}{10} = \frac{3}{5}$$
$$2x + \frac{1}{10} - \frac{1}{10} = \frac{3}{5} - \frac{1}{10}$$
$$2x = \frac{3}{5} - \frac{1}{10}$$
$$2x = \frac{6}{10} - \frac{1}{10}$$
$$2x = \frac{5}{10}$$
$$2x \div 2 = \frac{1}{2} \div 2$$
$$x = \frac{1}{4}$$

To isolate the variable in a two-step equation, use the reverse order of operations. Add or subtract first, and then multiply or divide.

$\frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2}$

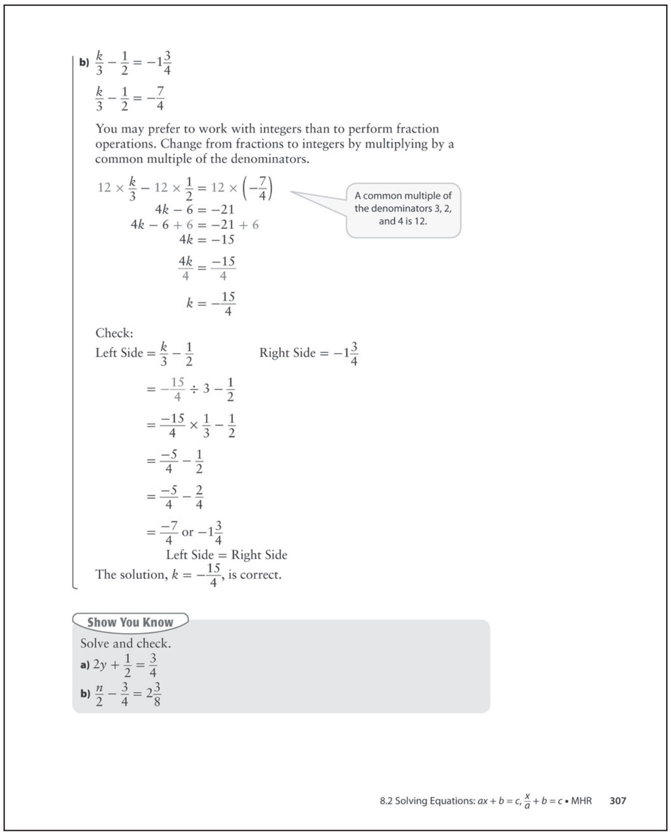
Check by modelling the equation $2x + \frac{1}{10} = \frac{3}{5}$ on a number line.

This diagram models the original equation $2x + \frac{1}{10} = \frac{3}{5}$. How does it show that $2x = \frac{5}{10}$?

Now show the value of x.

The second diagram shows that $x = \frac{5}{20}$ or $\frac{1}{4}$.

The solution, $x = \frac{1}{4}$, is correct.



b) $\frac{k}{3} - \frac{1}{2} = -1\frac{3}{4}$

$\frac{k}{3} - \frac{1}{2} = -\frac{7}{4}$

You may prefer to work with integers than to perform fraction operations. Change from fractions to integers by multiplying by a common multiple of the denominators.

$$12 \times \frac{k}{3} - 12 \times \frac{1}{2} = 12 \times \left(-\frac{7}{4}\right)$$
$$4k - 6 = -21$$
$$4k - 6 + 6 = -21 + 6$$
$$4k = -15$$
$$\frac{4k}{4} = \frac{-15}{4}$$
$$k = -\frac{15}{4}$$

A common multiple of the denominators 3, 2, and 4 is 12.

Check:

Left Side $= \frac{k}{3} - \frac{1}{2}$ Right Side $= -1\frac{3}{4}$

$= -\frac{15}{4} \div 3 - \frac{1}{2}$

$= \frac{-15}{4} \times \frac{1}{3} - \frac{1}{2}$

$= \frac{-5}{4} - \frac{1}{2}$

$= \frac{-5}{4} - \frac{2}{4}$

$= \frac{-7}{4}$ or $-1\frac{3}{4}$

Left Side = Right Side

The solution, $k = -\frac{15}{4}$, is correct.

Show You Know

Solve and check.

a) $2y + \frac{1}{2} = \frac{3}{4}$

b) $\frac{n}{2} - \frac{3}{4} = 2\frac{3}{8}$

Link the Ideas

Example 1

This example presents algebraic solutions for equations of the forms $ax + b = c$ and $\frac{x}{a} + b = c$, involving positive and negative fractions and mixed numbers.

Part a) shows an algebraic solution for an equation of the form $ax + b = c$. Point out the first thought bubble to remind students of the reverse order of operations. To check for understanding, you might ask:

- Why is $\frac{3}{5} - \frac{1}{10}$ expressed as $\frac{6}{10} - \frac{1}{10}$?
- Could you use a different common denominator? If so, would the subtraction be any harder or easier?
- Why does $\frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2}$?

The check in Example 1 makes use of a number-line model, which also serves to help students visualize the original equation and its solution. Have students answer the question in the thought bubble beside the first number-line diagram. You might also ask:

- To show the value of x, why are the divisions on the number line changed from tenths to twentieths?
- How could you check the solution by substitution?
- Which method of checking do you prefer? Why?

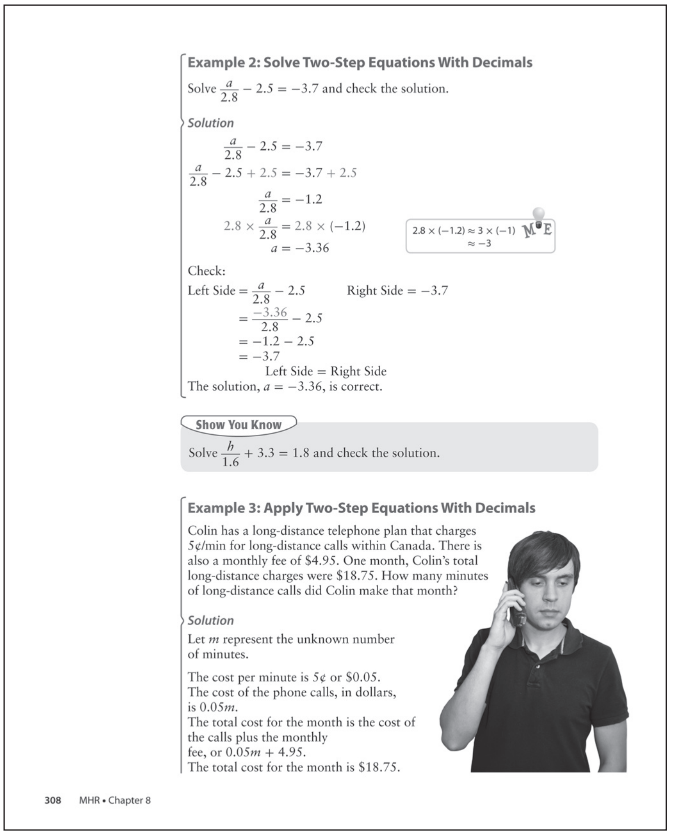
Part b) shows an algebraic solution for an equation of the form $\frac{x}{a} + b = c$. To support understanding you might ask:

- Why does $12 \times \left(-\frac{7}{4}\right) = -21$?
- Could you use a different common multiple of the denominators? If so, would the solution be any easier or harder?
- Solve the equation $\frac{k}{3} - \frac{1}{2} = -\frac{7}{4}$ by performing fraction operations. Do you find this method easier or harder than changing from fractions to integers? Why?
- How is the order of operations used in the check?

Example 2

This example presents an algebraic solution for an equation of the form $\frac{x}{a} + b =$ c involving rational numbers in decimal form. To promote understanding, you might ask:

- Why does the addition step come before the multiplication step?
- How would you estimate $-3.7 + 2.5$? Is the calculated sum close to your estimate?
- How could you predict the sign of the product $2.8 \times (-1.2)$ and the sign of the quotient $\frac{-3.36}{2.8}$?



Example 2: Solve Two-Step Equations With Decimals

Solve $\frac{a}{2.8} - 2.5 = -3.7$ and check the solution.

Solution

$$\frac{a}{2.8} - 2.5 = -3.7$$
$$\frac{a}{2.8} - 2.5 + 2.5 = -3.7 + 2.5$$
$$\frac{a}{2.8} = -1.2$$
$$2.8 \times \frac{a}{2.8} = 2.8 \times (-1.2)$$
$$a = -3.36$$

$2.8 \times (-1.2) \approx 3 \times (-1)$ ≈ -3 ME

Check:

Left Side $= \frac{a}{2.8} - 2.5$ Right Side $= -3.7$

$= \frac{-3.36}{2.8} - 2.5$

$= -1.2 - 2.5$

$= -3.7$

Left Side = Right Side

The solution, $a = -3.36$, is correct.

Show You Know

Solve $\frac{h}{1.6} + 3.3 = 1.8$ and check the solution.

Example 3: Apply Two-Step Equations With Decimals

Colin has a long-distance telephone plan that charges 5¢/min for long-distance calls within Canada. There is also a monthly fee of \$4.95. One month, Colin's total long-distance charges were \$18.75. How many minutes of long-distance calls did Colin make that month?

Solution

Let m represent the unknown number of minutes.

The cost per minute is 5¢ or \$0.05.
The cost of the phone calls, in dollars, is $0.05m$.
The total cost for the month is the cost of the calls plus the monthly fee, or $0.05m + 4.95$.
The total cost for the month is \$18.75.

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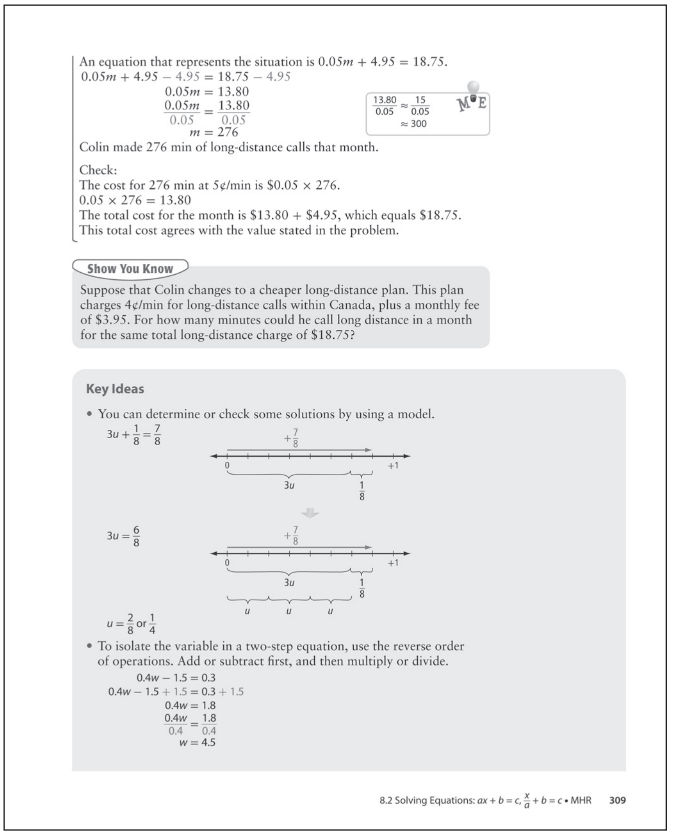


An equation that represents the situation is $0.05m + 4.95 = 18.75$.

$$0.05m + 4.95 - 4.95 = 18.75 - 4.95$$
$$0.05m = 13.80$$
$$\frac{0.05m}{0.05} = \frac{13.80}{0.05}$$
$$m = 276$$

$\frac{13.80}{0.05} \approx \frac{15}{0.05}$ ≈ 300 ME

Colin made 276 min of long-distance calls that month.

Check:
The cost for 276 min at 5¢/min is \$0.05 × 276.
$0.05 \times 276 = 13.80$
The total cost for the month is \$13.80 + \$4.95, which equals \$18.75.
This total cost agrees with the value stated in the problem.

Show You Know

Suppose that Colin changes to a cheaper long-distance plan. This plan charges 4¢/min for long-distance calls within Canada, plus a monthly fee of \$3.95. For how many minutes could he call long distance in a month for the same total long-distance charge of \$18.75?

Key Ideas

- You can determine or check some solutions by using a model.

$3u + \frac{1}{8} = \frac{7}{8}$

$3u = \frac{6}{8}$

$u = \frac{2}{8}$ or $\frac{1}{4}$

- To isolate the variable in a two-step equation, use the reverse order of operations. Add or subtract first, and then multiply or divide.

$$0.4w - 1.5 = 0.3$$
$$0.4w - 1.5 + 1.5 = 0.3 + 1.5$$
$$0.4w = 1.8$$
$$\frac{0.4w}{0.4} = \frac{1.8}{0.4}$$
$$w = 4.5$$

8.2 Solving Equations: $ax + b = c, \frac{x}{a} + b = c$ • MHR 309

- What steps would you use to calculate $\frac{-3.36}{2.8} - 2.5$ on your calculator?

Example 3

This example shows the writing and solving of an equation of the form $ax + b = c$ in an applied setting. To promote understanding, you might ask:

- What would the equation be if you worked in cents, instead of dollars?
- Would the solution be any easier or harder if you worked in cents, rather than dollars? Why?
- How would you estimate $18.75 - 4.95$? Is the calculated difference close to your estimate?

Key Ideas

The Key Ideas summarize solution methods for two-step equations with forms $ax + b = c$ and $\frac{x}{a} + b = c$, involving rational numbers. To check for understanding, you might ask students to describe how the diagrams model the solution to $3u + \frac{1}{8} = \frac{7}{8}$. You might then have students describe, in their own words, the steps used in the algebraic solutions shown and the reasoning behind each step.

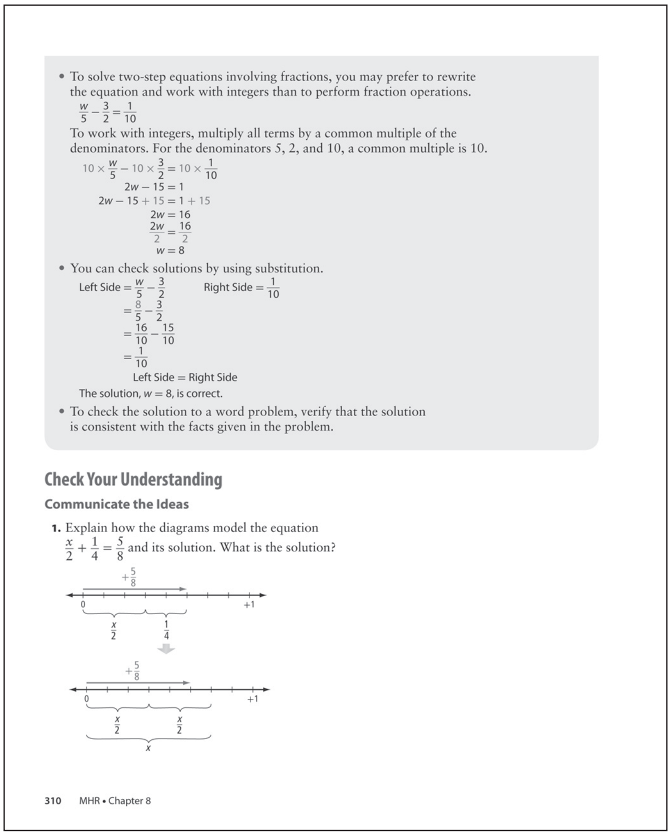
Have students review the Key Ideas for any additional material they would like to add to their Foldable.

Meeting Student Needs

- You might have students work in pairs or small groups to complete the Show You Knows. Encourage them to correct errors in each other's solutions and to suggest ways of completing them more efficiently. Alternatively, have students work individually and then have them compare and comment on each other's solutions.
- You may wish to supply **Master 4 Number Lines** for students to use as they work on the Show You Know questions.
- In Example 3, you might have students discuss how else they could solve the question. Tell them to see the Communicate the Ideas for another method.
- In Example 3, emphasize the importance in word problems of making a summary statement and checking that the calculated answer is consistent with the information given in the problem.

ELL

- Teach the following terms in context (use pictures where possible): *long distance*, *monthly fee*, *two-step equation*, *reverse order of operations*, *rewrite*, *common multiple*, *denominator*, and *substitution*.
- For the Show You Know following Example 3, read through the question with any English language learners to ensure they understand what it is asking.



- To solve two-step equations involving fractions, you may prefer to rewrite the equation and work with integers than to perform fraction operations.

  $\frac{w}{5} - \frac{3}{2} = \frac{1}{10}$

  To work with integers, multiply all terms by a common multiple of the denominators. For the denominators 5, 2, and 10, a common multiple is 10.

  $$\begin{aligned} 10 \times \frac{w}{5} - 10 \times \frac{3}{2} &= 10 \times \frac{1}{10} \\ 2w - 15 &= 1 \\ 2w - 15 + 15 &= 1 + 15 \\ 2w &= 16 \\ \frac{2w}{2} &= \frac{16}{2} \\ w &= 8 \end{aligned}$$

- You can check solutions by using substitution.

  $$\begin{aligned} \text{Left Side} &= \frac{w}{5} - \frac{3}{2} \\ &= \frac{8}{5} - \frac{3}{2} \\ &= \frac{16}{10} - \frac{15}{10} \\ &= \frac{1}{10} \end{aligned} \qquad \text{Right Side} = \frac{1}{10}$$

  Left Side = Right Side

  The solution, $w = 8$, is correct.

- To check the solution to a word problem, verify that the solution is consistent with the facts given in the problem.

Check Your Understanding

Communicate the Ideas

1. Explain how the diagrams model the equation $\frac{x}{2} + \frac{1}{4} = \frac{5}{8}$ and its solution. What is the solution?

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Gifted and Enrichment

- To challenge students, you might have them solve $0.4w - 1.5 = 0.3$ in the Key Ideas by first multiplying both sides by 10 and then working with integers. Then, have them solve $\frac{w}{5} - \frac{3}{2} = \frac{1}{10}$ by using fraction operations. Have students decide whether they prefer this method, or the one presented in the student resource.

Common Errors

- Some students may obtain incorrect solutions to equations of the forms $ax + b = c$ and $\frac{x}{a} + b = c$ by performing mental computations without recording the two separate steps.

Rx Encourage students to write full solutions until they develop their skills in solving equations. Also, encourage them to check their solutions by substitution.

Answers

Example 1: Show You Know

a) $y = \frac{1}{8}$ **b)** $n = \frac{25}{4}$

Example 2: Show You Know

$h = -2.4$; $\frac{-2.4}{1.6} + 3.3 = 1.8$

Example 3: Show You Know

He could call for 370 min.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to look back at the Explore to look for a comparison of the two methods shown there. Ask students which of the two they prefer and why. • Some students may benefit from a review of finding lowest common multiples. • Providing struggling learners with grid paper will assist them in drawing the number line. You may wish to hand out **Master 8 Centimetre Grid Paper** and/or **Master 9 0.5 Centimetre Grid Paper**. • Encourage students to solve each of the questions a different way. • Remind students of the importance of Left Side = Right Side.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students who are having difficulty to verbalize what they plan to do for the Show You Know. Clarify any misunderstandings. • Remind students to move the constant term first and then multiply by the denominator.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some learners may find it difficult to create the equation. • Encourage them to review the format of the solution in Example 3, and to build the equation by listing known values and what they represent.

- To solve two-step equations involving fractions, you may prefer to rewrite the equation and work with integers than to perform fraction operations.

$\frac{w}{5} - \frac{3}{2} = \frac{1}{10}$

To work with integers, multiply all terms by a common multiple of the denominators. For the denominators 5, 2, and 10, a common multiple is 10.

$$10 \times \frac{w}{5} - 10 \times \frac{3}{2} = 10 \times \frac{1}{10}$$
$$2w - 15 = 1$$
$$2w - 15 + 15 = 1 + 15$$
$$2w = 16$$
$$\frac{2w}{2} = \frac{16}{2}$$
$$w = 8$$

- You can check solutions by using substitution.

Left Side $= \frac{w}{5} - \frac{3}{2}$ Right Side $= \frac{1}{10}$
$= \frac{8}{5} - \frac{3}{2}$
$= \frac{16}{10} - \frac{15}{10}$
$= \frac{1}{10}$
Left Side = Right Side

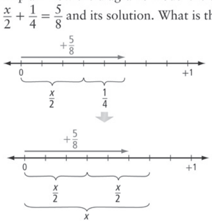
The solution, $w = 8$, is correct.

- To check the solution to a word problem, verify that the solution is consistent with the facts given in the problem.

Check Your Understanding

Communicate the Ideas

1. Explain how the diagrams model the equation $\frac{x}{2} + \frac{1}{4} = \frac{5}{8}$ and its solution. What is the solution?



2. Ryan solved $2r + 0.3 = 0.7$ as follows. Do you agree with his solution? Explain.

$$\frac{2r}{2} + 0.3 = \frac{0.7}{2}$$
$$r + 0.3 = 0.35$$
$$r + 0.3 - 0.3 = 0.35 - 0.3$$
$$r = 0.05$$

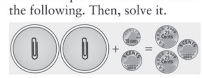
3. Jenna did not want to perform fraction operations to solve the equation $\frac{x}{2} - \frac{1}{9} = \frac{5}{6}$, so she first multiplied both sides by 54. Is this the common multiple you would have chosen? Explain.

4. a) Milos solved $0.05x - 0.12 = 0.08$ by multiplying both sides by 100 and then solving $5x - 12 = 8$. Show how he used this method to determine the correct solution.

b) When Milos was asked to solve $\frac{x}{0.05} - 0.12 = 0.08$, he reasoned that he could determine the correct solution by solving $\frac{x}{5} - 12 = 8$. Do you agree with his reasoning? Explain.

Practise

5. Write an equation that is modelled by the following. Then, solve it.



6. Model the equation $3x + 0.14 = 0.50$ using concrete materials. Solve using your model.

For help with #7 and #8, refer to Example 1 on pages 306–307.

7. Solve.

a) $4y - \frac{2}{5} = \frac{3}{5}$ **b)** $2d - \frac{1}{2} = \frac{5}{4}$

c) $\frac{n}{2} + 1\frac{2}{3} = \frac{1}{6}$ **d)** $\frac{4}{5} - 2\frac{1}{2}r = \frac{3}{10}$

8. Solve.

a) $1\frac{1}{2} = 4h + \frac{2}{3}$ **b)** $\frac{4}{3}x + \frac{3}{4} = \frac{1}{2}$

c) $\frac{3}{4} - \frac{d}{3} = \frac{3}{8}$ **d)** $-4\frac{2}{5} = -3\frac{1}{5} + \frac{7}{10}g$

For help with #9 and #10, refer to Example 2 on page 308.

9. Solve and check.

a) $\frac{x}{0.6} + 2.5 = -1$

b) $0.38 = 6.2 - \frac{r}{1.2}$

10. Solve.

a) $-0.02 - \frac{n}{3.7} = -0.01$

b) $\frac{k}{-0.54} + 0.67 = 3.47$

For help with #11 and #12, refer to Example 3 on pages 308–309.

11. Solve and check.

a) $2 + 12.5v = 0.55$

b) $-0.77 = -0.1x - 0.45$

12. Solve.

a) $0.074d - 3.4 = 0.707$

b) $67 = 5.51 + 4.3a$

Check Your Understanding

Communicate the Ideas

These questions allow students to explain aspects of solving equations of the forms $ax + b = c$ and $\frac{x}{a} + b = c$ using rational numbers.

In #2, ask students to provide a correct algebraic solution.

In #3, encourage students to solve the equation using Jenna's common multiple and to try a different common multiple as well. You might extend the question by asking students to solve it using fraction operations and to decide which approach they prefer.

In #4a), you might have students discuss whether they prefer to solve the question using integers or decimals. Encourage students to discuss the reasoning error in b). You might ask them to write and solve the correct equation that results from multiplying both sides by 100 (i.e., $2000x - 12 = 8$). As in #4a), you might have students discuss whether they prefer to solve b) using integers or decimals.

Practise

Encourage students to compare their solution methods, check each other's answers, and to suggest corrections or other improvements.

Apply

Have students suggest and discuss alternative solutions for the same problem. For example, #13 readily lends itself to a Guess and Check approach as well as to an algebraic solution. Students may solve #23 by making a table.

If necessary, in #16, remind students of the definition of a regular polygon.

In #20, you might have the following class discussion after students have finished the question:

- Did you solve this problem by working in centimetres and using integers, or by working in metres and using decimal numbers? Which of these possibilities do you prefer? Why?
- Which of you let the shorter piece of the post be represented by the variable? Which of you let the longer piece of the post be represented by the variable? How do the operations in the solutions compare for the resulting equations? Is one of these equations easier to solve than the other?

Apply

13. The cost of a pizza is \$8.50 plus \$1.35 per topping. How many toppings are on a pizza that costs \$13.90?

14. Hiroshi paid \$34.95 to rent a car for a day, plus 12¢ for each kilometre he drove. The total rental cost, before taxes, was \$55.11. How far did Hiroshi drive that day?

15. On Saturday morning, Marc had a quarter of his weekly allowance left. He spent a total of \$6.50 on bus fares and a freshly squeezed orange juice on Saturday afternoon. He then had \$2.25 left. How much is his weekly allowance?

16. Nadia has a summer job in an electronics store. She is paid \$400 per week, plus 5% commission on the total value of her sales.

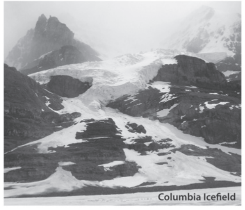
a) One week, when the store was not busy, Nadia earned only \$510.30. What was the total value of her sales that week?

b) Nadia's average earnings are \$780 per week. What is the average value of her weekly sales?

17. Benoit was helping his family build a new fence along one side of their yard. The total length of the fence is 28 m. They worked for two days and completed an equal length of fence on each day. On the third day, they completed the remaining 4.8 m of fence. What length of fence did they build on each of the first two days?

18. The perimeter of a regular hexagon is 3.04 cm less than the perimeter of a regular pentagon. The perimeter of the regular hexagon is 21.06 cm. What is the side length of the regular pentagon?

19. The greatest average annual snowfall in Alberta is on the Columbia Icefield. The greatest average annual snowfall in Manitoba is at Island Lake. An average of 642.9 mm of snow falls on the Columbia Icefield in a year. This amount of snow is 22.5 mm less than twice the annual average at Island Lake. What is the average annual snowfall at Island Lake?



Columbia Icefield

20. During a camping trip, Nina was making a lean-to for sleeping. She cut a 2.5-m long post into two pieces, so that one piece was 26 cm longer than the other. What was the length of each piece?

21. The average monthly rainfall in Victoria in July is 2.6 mm less than one fifth of the amount of rain that falls in Edmonton in the same period. Victoria averages 17.6 mm of rainfall in July. What is the average monthly rainfall in Edmonton in July?

22. The temperature in Winnipeg was 7 °C and was falling by 2.5 °C/h. How many hours did it take for the temperature to reach −5.5 °C?

23. Max and Sharifa are both saving to buy the same model of DVD player, which costs \$99, including tax. Max already has \$31.00 and decides to save \$8.50 per week from now on. Sharifa already has \$25.50 and decides to save \$10.50 per week from now on. Who can pay for the DVD player first? Explain.

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24. A cylindrical storage tank that holds 375 L of water is completely full. A pump removes water at a rate of 0.6 L/s. For how many minutes must the pump work until 240 L of water remain in the tank?

25. The average distance of Mercury from the sun is 57.9 million kilometres. This distance is 3.8 million kilometres more than half the average distance of Venus from the sun. What is the average distance of Venus from the sun?

26. Create an equation of the form $\frac{x}{a} + b = c$ with each given solution. Compare your equations with your classmates' equations.

a) $\frac{2}{3}$

b) -0.8

27. Write a word problem that can be solved using an equation of the form $ax + b = c$. Include at least one decimal or fraction. Have a classmate solve your problem.

Extend

28. Solve.

a) $\frac{3}{2} + \frac{w}{4} = \frac{5}{6} - \frac{1}{2}$

b) $\frac{3}{4}\left(-\frac{2}{9}\right) = 4\frac{1}{2}x + \frac{1}{3}$

29. Solve. Express each solution to the nearest hundredth.

a) $0.75 + 0.16y + 0.2y = 0.34$

b) $\frac{-1.85}{0.74} = 2.22 - 0.57s$

30. Solve.

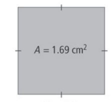
a) $\frac{0.2}{x} + 0.8 = 1.2$

b) $\frac{1}{2} - \frac{4}{n} = -\frac{1}{4}$

c) $-\frac{3.52}{h} - 1.31 = 1.19$

d) $4\frac{5}{6} = 3\frac{1}{3} - \frac{3}{y}$

31. Determine the value of x.



32. A freight train passes through a 750-m long tunnel at 50 km/h. The back of the train exits the tunnel 1.5 min after the front of the train enters it. What is the length of the train, in metres?

Revelstoke, British Columbia

Math Link

A slice of canned corned beef contains about 0.21 g of sodium. This much sodium is 0.01 g more than the mass of sodium in four slices of roast beef. What is the mass of sodium in a slice of roast beef?

a) Write an equation that models the situation.

b) Solve the equation in two different ways.

c) Which of your solution methods do you prefer? Explain.

8.2 Solving Equations: $ax + b = c$, $\frac{x}{a} + b = c$ • MHR 313

In #24, students consider the units of measurement. After students have solved the problem, you might check whether they worked in minutes using integers (i.e., solving $375 - 36t = 240$) or in seconds using decimals (i.e., solving $375 - 0.6t = 240$, and then converting the answer to minutes).

In #25, you might ask students if it is easier to let the average distance of Venus from the Sun be measured in kilometres, or in millions of kilometres. (Using the latter option, students can avoid including all the zeros in the distances.) Emphasize that they cannot mix the two options. That is, they could not let v be the average distance of Venus from the Sun in kilometres, and then write the equation as $\frac{v}{2} + 3.8 = 57.9$. In this equation, v must be measured in millions of kilometres. You might also poll the class to see how many students wrote an equation in the form $\frac{x}{a} + b = c$, and how many chose the form $ax + b = c$ by writing $0.5v + 3.8 = 57.9$ or $\frac{1}{2}v + 3.8 = 57.9$. You might ask students if one form of the equation is any easier to solve than the other.

Have students discuss their strategies in #26, which involves working backward from the solution to an equation. One possible approach is to use the given value of x, assume values of a and b, and calculate the corresponding value of c. An infinite number of equations of the form $\frac{x}{a} + b = c$ have each of the given solutions.

In #27, the easiest way to write this word problem is to model it on a previous problem, such as #13. Emphasize that students must check that they can solve their own problem before giving it to a classmate to solve. Encourage students to use original contexts. You may wish to share particularly creative problems with the whole class.

Extend

In #28 and 29, students need to perform additional steps in solving equations. You might have them discuss ways of solving the equations most easily. Note that #30 introduces the form $\frac{a}{x} + b = c$ as an extension of the work students did with the form $\frac{a}{x} = b$ in section 8.1. Before students solve the equations in this question, you might ask them to predict how many operations will be needed in each solution.

In #31, students must recall and apply their knowledge of square roots from Chapter 2 to write the equation $3 + 2x = 1.3$. Have students verify that their calculated value of x, which is a negative number, is consistent with the area given in the problem.

In #32, the reasoning involved in writing an equation will be challenging for some students. You might encourage students to try different strategies for solving the problem and to discuss their solutions. If students have difficulty, you might suggest that they start by drawing a diagram.

Literacy Link Remind students to complete the second oval in their concept map, which is labelled *with two operations*. They should provide an example of the equation form, and outline the steps required to solve the equation, using a strategy of their choice.

Math Link

In this Math Link, students can apply their skills in writing and solving equations of the form $ax + b = c$ to the nutrition field. After students have answered part b), you might challenge them to think of and discuss more ways of solving the equation $4r + 0.01 = 0.21$. A few possible solution methods include modelling with cups, coins, and paper clips, solving by Guess and Check, and multiplying both sides of the equation by 100 and solving $400r + 1 = 21$.

After students have completed the questions, you might have them research the maximum amount of sodium they should eat in a day, the amounts of sodium in some of their favourite foods, and the physical effects of consuming too much sodium.

Meeting Student Needs

- Provide **BLM 8–8 Section 8.2 Extra Practice** to students who would benefit from more practice.
- Give students as much variety and choice as possible to encourage them to take responsibility for their learning. Variety can provide differentiated learning opportunities.
- For #4, if students have difficulty in writing the correct equation, you might first suggest that they rewrite the given equation in the form $ax + b = c$, which results in $20x - 0.12 = 0.08$.
- You may wish to have students complete the Practise questions individually, in pairs, or in small groups. Stress the importance of estimation in algebraic solutions, the usefulness of the sign rules for multiplication and division, and the need to check answers.
- For all word problems, emphasize that students should make sure that answers are reasonable within the context of the problem.

ELL

- Teach the following terms in context (use pictures where possible): *quarter* (referring to a quarter of an amount), *rainfall*, *allowance*, *storage tank*, *snowfall*, and *freight train*.
- Clarify any questions with English language learners as they complete the Check Your Understanding section. Remind them what *least common multiple* means by showing them another set of numbers and circling the least common multiple as you say the word.
- In the Apply questions, read through the questions with the English language learners whenever possible, and use pictures in their book to help clarify meaning. For example, for #13, draw a pizza. Then, add a picture of a mushroom that is labelled with \$1.35, and then add a picture of a piece of cheese that is also labelled \$1.35. Point to the cheese and mushroom and say, "These are toppings." Ask the English language learner if they have tried pizza, and if so, what toppings they like. Then, calculate with them how much their own personal pizza might cost.

Gifted and Enrichment

- If you wish to extend #1, you might ask students how the number-line models for the solutions to $\frac{x}{2} + \frac{1}{4} = \frac{5}{8}$ and $2x + \frac{1}{4} = \frac{5}{8}$ are the same and how they are different.
- It is often easier to work with integers than with fractions when solving equations. Challenge students to develop methods of solving equations without removing the fractions. Have them show their thinking.

Common Errors

- Some students may have more difficulty in solving equations correctly when the variable is on the right side, such as in #9b), for example.

R_x Show that the operations involved in isolating the variable are the same whether the equation is written as $0.38 = 6.2 - \frac{r}{1.2}$, or as $-\frac{r}{1.2} + 6.2 = 0.38$. Have students check the solution by substituting their answer into both forms of the equation.

Answers

Communicate the Ideas

1. Example $x = \frac{3}{4}$

2. No, he should have subtracted 0.3 from both sides of the equation before he divided by 2.

3. 18 is the lowest common multiple she could have chosen, because 2, 9, and 6 are all factors of 18.

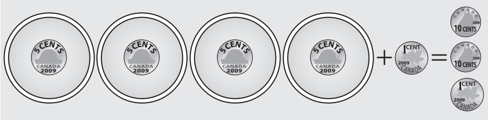
4. a)
$$\begin{aligned} 100\,(0.05x - 0.12) &= 100(0.08) \\ 5x - 12 &= 8 \\ 5x - 12 + 12 &= 8 + 12 \\ 5x &= 20 \\ \frac{5x}{5} &= \frac{20}{5} \\ x &= 4 \end{aligned}$$

b) No, he multiplied 0.12 and 0.08 by 100, but he multiplied $\frac{x}{0.05}$ by $\frac{1}{100}$ instead of 100.

Math Link

a) Example: $0.21 = 4x + 0.01$

b) Example: $x = 0.05$



c) Example: The algebraic method is faster than modelling the equation.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1–3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • For students who are having difficulty setting up the solution using the number line, review with them that below the number line reflects the left side of the equation and above the number line shows the right side of the equation. Remind students that they are seeking the value of x and not $\frac{x}{2}$; hence, they need two groups of $\frac{x}{2}$ to create a single x. • Provide additional questions for students to set up a number line with, such as: $\frac{x}{2} + \frac{1}{3} = \frac{5}{6}$. • For #2, review the order in which you approach solving an equation. Remind students it is based on opposites including the opposite of the order of operations: add/subtract, then multiply/divide. • For #3, students should review finding the lowest common denominator. Although the student is not wrong in choosing 54, it is not the most efficient way of solving. Have students place solutions on the board and have them compare answers. Some students may use 36 rather than 18. • You may wish to have students use **Master 2 Communication Peer Evaluation** to assess one or more of each other's responses to the questions.
Assessment *for* Learning	
Practise and Apply Have students do #5–7, 9, 11, 13, 15, 16, and 19. Students who have no problems with these questions can go on to the remaining Apply questions.	• For students having difficulty with #5, they may find it helpful to review the Explore at the beginning of the chapter section. If cups confuse students, have them model the equation with another manipulative that they feel more comfortable with. Have them verbalize what they see and what it represents before writing. • Note that #6 is similar to #5 in that students must still have a good understanding of modelling. Again, provide an alternate manipulative or have students use number lines, whatever they are more familiar with. Provide grid paper for students needing visual support. • For #7, 9, and 11, provide students with appropriate manipulative support if this assists them in solving. Reviewing the opposite operations may also assist students in getting started. Have students model a solution for you once they have been coached in the question. Use questions from #8, 10, and 12, respectively, as additional questions to check for understanding. • For #13 and 15, encourage students to review the steps used in identifying the variables and values in the word problems. Students could review the format used in Example 3.
Math Link The Math Link on page 313 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 333.	• Students who need help getting started could use **BLM 8–9 Section 8.2 Math Link**, which provides scaffolding. • As they did in #13 and 15, have students use identify their variables and values prior to setting up the equation. • The Math Link allows students to chose the methods they feel most comfortable with to solve the equations. You could challenge them to solve the problems in as many ways as they can.
Assessment *as* Learning	
Literacy Link At the end of this section, have students work in pairs to complete the next oval of the concept map, entitled *with two operations*.	• Brainstorm and discuss as a class the information needed to complete this oval. • Some learners may benefit from listing the steps, sequentially, to follow when solving two-step equations. This could be completed with an additional sub-topic oval.
Math Learning Log Have students respond to the following prompt: • The steps I would use to write and equation from a word problem are …	• Encourage students to use an example and solve the equation that results from the problem. It is important that students have a good understanding of the process of developing an equation.