Solving Equations: $a(x + b) = c$

8.3

***MathLinks 9*, pages 314–321**

Suggested Timing

80–100 minutes

Materials

- coins or items to represent coins of different denominations
- paper cups or small containers
- paper clips

Blackline Masters

Master 2 Peer Evaluation
Master 4 Number Lines
Master 14 Coin Models
BLM 8–3 Chapter 8 Warm-Up
BLM 8–5 Canadian Coins and Their Values
BLM 8–10 Section 8.3 Extra Practice
BLM 8–11 Section 8.3 Math Link

Mathematical Processes

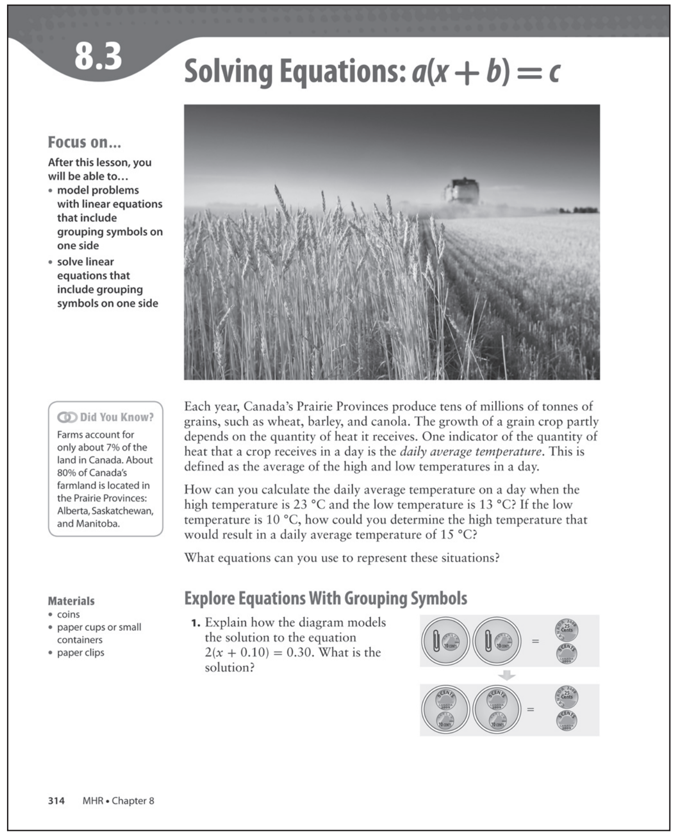
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes

PR3 Model and solve problems using linear equations of the form:
- $a(x + b) = c$

where a, b, c, d, e and f are rational numbers.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 4, 5, 6, 8, 10, 12, 14, 15, 17, Math Link
Typical	#1, 4, 5, 6, 8, 10, 12, 14, 15, 17, 20, 22, Math Link
Extension/Enrichment	#1, 4, 10, 20, 23–28


8.3

Solving Equations: $a(x + b) = c$

Focus on...

After this lesson, you will be able to...
- model problems with linear equations that include grouping symbols on one side
- solve linear equations that include grouping symbols on one side

Did You Know?

Farms account for only about 7% of the land in Canada. About 80% of Canada's farmland is located in the Prairie Provinces: Alberta, Saskatchewan, and Manitoba.

Each year, Canada's Prairie Provinces produce tens of millions of tonnes of grains, such as wheat, barley, and canola. The growth of a grain crop partly depends on the quantity of heat it receives. One indicator of the quantity of heat that a crop receives in a day is the *daily average temperature*. This is defined as the average of the high and low temperatures in a day.

How can you calculate the daily average temperature on a day when the high temperature is 23 °C and the low temperature is 13 °C? If the low temperature is 10 °C, how could you determine the high temperature that would result in a daily average temperature of 15 °C?

What equations can you use to represent these situations?

Materials
- coins
- paper cups or small containers
- paper clips

Explore Equations With Grouping Symbols

1. Explain how the diagram models the solution to the equation $2(x + 0.10) = 0.30$. What is the solution?

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Planning Notes

Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce material learned in previous sections.

Since the context used at the beginning of section 8.3 is farming, you may first wish to point out the Did You Know? on page 314. You might have students research the total area of land in Canada (which is less than the total area of the country because the total area includes bodies of water), and to use the given percents to answer the following questions:
- What is a reasonable estimate of the land area used for farming in Canada?
- What is a reasonable estimate of the land area used for farming in the Prairie provinces?

Have students discuss and justify their estimates.

The opening paragraph on page 314 introduces the term *daily average temperature* in the context of grain farming on the Prairies. The first question in the second paragraph will activate students' skills in working with averages. Because of their past experiences, students are likely to add the temperatures and then divide by 2 without thinking in terms of a formula. Solutions to the second question in the second paragraph may involve the use of Guess and Check, again without a written formula. The concluding question in the opener will prompt students to consider their solutions in terms of equations. You might ask:
- What variables will you choose to represent the high temperature, the low temperature, and the daily average temperature?

2. Work with a partner to explore how to model the solutions to the following equations using manipulatives or diagrams. Share your models with other classmates.

a) $2(x + 0.25) = 0.54$

b) $3\left(x + \frac{1}{20}\right) = \frac{9}{10}$

Reflect and Check

3. a) How can you model solutions to equations of the form $a(x + b) = c$ using manipulatives or diagrams?

b) Think of other ways to model the solutions. Explain how you would use them.

4. Stefan works on a farm in the Fraser Valley of British Columbia. Two of the fields are square. The perimeter of the larger field is 2.4 km. The side length of the larger field is 0.1 km more than the side length of the smaller field. Create a labelled drawing of this situation. Suggest ways of determining the side length of the smaller field. Include a suggestion for solving an equation of the form $a(x + b) = c$. Share your ideas with your classmates.

Link the Ideas

Example 1: Solve Equations With Grouping Symbols

Solve and check.

a) $3(d + 0.4) = -3.9$

b) $\frac{t - 1}{5} = \frac{3}{2}$

Solution

a) ***Method 1: Use the Distributive Property First***
Use the distributive property to remove the brackets.

$$\begin{aligned} 3(d + 0.4) &= -3.9 \\ (3 \times d) + (3 \times 0.4) &= -3.9 \\ 3d + 1.2 &= -3.9 \\ 3d + 1.2 - 1.2 &= -3.9 - 1.2 \\ 3d &= -5.1 \\ \frac{3d}{3} &= \frac{-5.1}{3} \\ d &= -1.7 \end{aligned}$$

Literacy Link
The *distributive property* is:
$a(b + c) = ab + ac$

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- What formula relates the high temperature, the low temperature, and the daily average temperature? (From students' past experiences, the most likely answer is $\frac{H + L}{2} = D$ or its reverse, $D = \frac{H + L}{2}$.)
- In the first question in the second paragraph, what was the unknown in your formula?
- In the second question in the second paragraph, what was the unknown in your formula?
- How could you rewrite the formula so that it includes multiplication instead of division? (Example: $\frac{1}{2}(H + L) = D$, or perhaps $\frac{1}{2}H + \frac{1}{2}L = D$.)

  Having established the formula $\frac{1}{2}(H + L) = D$, you might point out that in answering the second question in the second paragraph, students provided the solution to an equation of the form $a(x + b) = c$, where $a = \frac{1}{2}$. Explain that in the exploration and the worked examples, students will learn other ways to solve equations of this type involving rational numbers.

Explore Equations With Grouping Symbols

In this exploration, students model and solve linear equations of the form $a(x + b) = c$, where a, b, and c are positive rational numbers. You may wish to provide **Master 14 Coin Models**, if actual coins are not available.

Method 1 Have students work on the exploration with a partner or in small groups, using manipulatives. Encourage students to compare their models and discuss their answers.

In #1, students observe a model of a solution to an equation of the form $a(x + b) = c$ involving decimals. You might ask:
- How does the first part of the diagram in #1 model the equation $2(x + 0.10) = 0.30$?
- How do you know that the solution modelled by the second part of the diagram is correct?

In #2, encourage students to use the manipulatives to model solutions in whatever way they prefer. The strategies they use may include inspection, Guess and Check, and methods that make use of their growing knowledge of algebra.

In #3a) and b), encourage students to engage in open discussion.

In #4, students apply their learning to solve a problem that involves the areas of fields, in a variation on the farming context introduced before the Explore. Have students compare their equations and solutions.

Students may solve the equation $4(x + 0.1) = 2.4$ in various ways, including inspection, Guess and Check, modelling with cups, coins, and paper clips, and solving algebraically. Encourage students to think of two ways of solving the equation algebraically and to compare the solutions with other ways of reasoning.

After students have shared their ideas, you might ask:
- Which solution method do you prefer in #4? Why?
- Do you find any of the solution methods difficult to understand? Why?
- How can you check that your answer to #4 is correct?

Method 2 Have students complete #1 and 2 in the exploration by sketching diagrams.

Meeting Student Needs

- Some students will benefit from a review of the concept of like terms.
- Before starting this section, all students will benefit from a refresher, using whole numbers, on how the distributive property can be used to remove brackets.
- In the chapter opener, some students may need help in realizing that $\frac{H + L}{2} = D$ and $\frac{1}{2}(H + L) = D$

are equivalent. If so, you might show that they are by using some numerical examples.

- It may be better for your class for you to work through the Explore as a whole-class activity.
- Encourage students to see connections between using manipulatives and algebraic techniques.
- In #2b), some students may have difficulty in understanding how to model fractions with coins. If so, encourage them to rewrite the equation in decimal form before modelling it with manipulatives.
- If students would benefit from modelling and solving more equations like the ones in #2, you might have them try $2(x + 0.15) = 0.50$, $3(x + 0.25) = 1.20$, $2\left(x + \frac{1}{5}\right) = \frac{3}{5}$, and $3\left(x + \frac{1}{4}\right) = 1\frac{1}{2}$.
- Consider holding a class discussion around questions #3a) and b) so that students have the opportunity to share any creative models that they developed.
- In discussing #3b), stress that students are not limited to models that involve cups, coins, and paper clips.
- Encourage algebraic reasoning by asking students to describe the operations they would perform in solving each equation in the Explore symbolically.
- Some students may benefit from drawing arrows from the coefficient to each of the terms in the brackets to help remind them of the sequence when applying the distributive property.

ELL

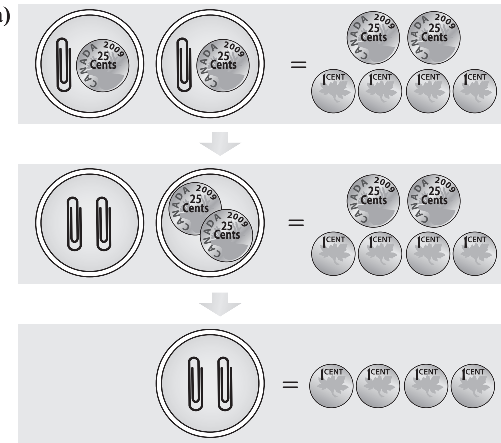
- Teach the following words in context: *average*, *wheat*, *barley*, *canola*, *heat*, *indicator of the quantity*, *botanists*, *climate*, and *typical day*.
- Very new Canadians will not know the name of the coins yet. Clarify the value of each type of coin. Provide them with **BLM 8–5 Canadian Coins and Their Values**.
- For the Reflect and Check, allow English language learners to write the answers in their first language. Then, have them try to explain their answer, helping them by giving them the words when necessary.

Answers

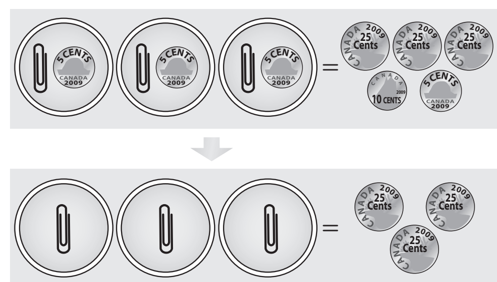
Explore Equations With Grouping Symbols

1. Example: The diagram models that two cups, each with a paper clip and a dime inside, have a value of 30¢. The value of each paper clip is 5¢.

2. a)



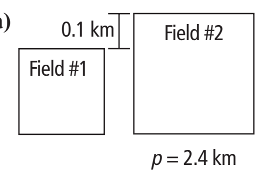
b)



3. a) Using coins, cups, and paper clips

b) Using algebra tiles or number lines

4. a)



$4(x + 0.1) = 2.4$, $x = 5$. This equation may be solved by dividing by 4 first or by distributing the 4. Each method gives the correct solution.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore the Math. Try to have students generalize the conclusion about their findings.	• Students may benefit from a class discussion in response to #3. Make a list of methods that students generate on the board and encourage students to write them into their Foldable for possible reference later on. • Encourage students to draw a diagram for #4 and label it accordingly. Have students work with a partner to determine the equation.

2. Work with a partner to explore how to model the solutions to the following equations using manipulatives or diagrams. Share your models with other classmates.

a) $2(x + 0.25) = 0.54$

b) $3\left(x + \frac{1}{20}\right) = \frac{9}{10}$

Reflect and Check

3. **a)** How can you model solutions to equations of the form $a(x + b) = c$ using manipulatives or diagrams?

b) Think of other ways to model the solutions. Explain how you would use them.

4. Stefan works on a farm in the Fraser Valley of British Columbia. Two of the fields are square. The perimeter of the larger field is 2.4 km. The side length of the larger field is 0.1 km more than the side length of the smaller field. Create a labelled drawing of this situation. Suggest ways of determining the side length of the smaller field. Include a suggestion for solving an equation of the form $a(x + b) = c$. Share your ideas with your classmates.

Link the Ideas

Example 1: Solve Equations With Grouping Symbols

Solve and check.

a) $3(d + 0.4) = -3.9$

b) $\frac{t-1}{5} = \frac{3}{2}$

Solution

a) *Method 1: Use the Distributive Property First*

Use the distributive property to remove the brackets.

$$\begin{aligned} 3(d + 0.4) &= -3.9 \\ (3 \times d) + (3 \times 0.4) &= -3.9 \\ 3d + 1.2 &= -3.9 \\ 3d + 1.2 - 1.2 &= -3.9 - 1.2 \\ 3d &= -5.1 \\ \frac{3d}{3} &= \frac{-5.1}{3} \\ d &= -1.7 \end{aligned}$$

Literacy Link

The *distributive property* is:

$a(b + c) = ab + ac$

8.3 Solving Equations: $a(x + b) = c$ • MHR 315

Method 2: Divide First

$$\begin{aligned} 3(d + 0.4) &= -3.9 \\ \frac{3(d + 0.4)}{3} &= \frac{-3.9}{3} \\ d + 0.4 &= -1.3 \\ d + 0.4 - 0.4 &= -1.3 - 0.4 \\ d &= -1.7 \end{aligned}$$

Why do you divide both sides by 3?

Check:

Left Side $= 3(d + 0.4)$ Right Side $= -3.9$

$= 3(-1.7 + 0.4)$

$= 3(-1.3)$

$= -3.9$

Left Side = Right Side

The solution, $d = -1.7$, is correct.

Literacy Link

A *fraction bar* acts as a grouping symbol and as a division symbol. The expression $\frac{t-1}{5}$ can be written as $\frac{1}{5}(t - 1)$ or as $(t - 1) \div 5$.

b)

$$\begin{aligned} \frac{t-1}{5} &= \frac{3}{2} \\ 10 \times \frac{t-1}{5} &= 10 \times \frac{3}{2} \\ 2(t - 1) &= 15 \\ 2t - 2 &= 15 \\ 2t - 2 + 2 &= 15 + 2 \\ 2t &= 17 \\ \frac{2t}{2} &= \frac{17}{2} \\ t &= \frac{17}{2} \end{aligned}$$

Why do you multiply both sides by 10? Is there a different way to solve the equation?

Check:

Left Side $= \frac{t-1}{5}$ Right Side $= \frac{3}{2}$

$= \left(\frac{17}{2} - 1\right) \div 5$

$= \left(\frac{17}{2} - \frac{2}{2}\right) \div 5$

$= \frac{15}{2} \div \frac{5}{1}$

$= \frac{15}{2} \times \frac{1}{5}$

$= \frac{3}{2}$

Left Side = Right Side

The solution, $t = \frac{17}{2}$, is correct.

Show You Know

Solve and check.

a) $2(e - 0.6) = 4.2$ **b)** $\frac{c+2}{3} = -\frac{5}{2}$

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Link the Ideas

Example 1

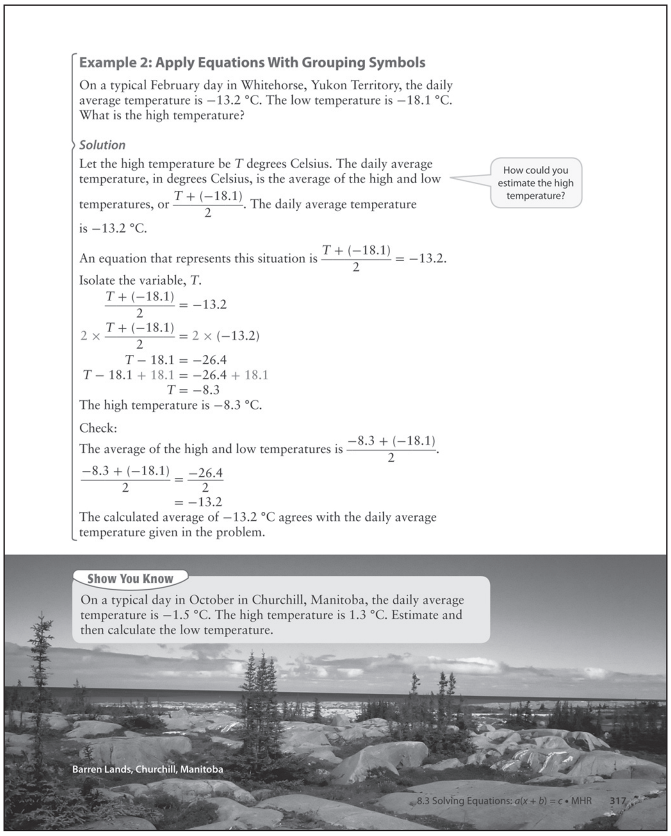
This example presents algebraic solutions for equations of the form $a(x + b) = c$ involving rational numbers. Part a) involves decimals and part b) involves fractions.

Literacy Link Point out the Literacy Link beside Method 1 of the solution for part a). You may want to check students' understanding of the distributive property by asking them to remove the brackets from each of the following expressions: $2(x + 3)$, $-3(5 + y)$, $4(g - 2)$, $-2(a - 4)$.

Ask students to answer the question in the thought bubble in Method 2. Make it clear that the check and summary statement after Method 2 apply to both methods. Emphasize that both methods give the same answer and that students are free to use whichever method they prefer.

Literacy Link Point out the Literacy Link in part b). Students can derive from it that $\frac{t-1}{5} = \frac{3}{2}$ is an equation of the form $a(x + b) = c$, where $a = \frac{1}{5}$, $b = -1$, and $c = \frac{3}{2}$. To check for understanding, you might have students state the values of a, b, and c in $\frac{x+5}{4} = -2\frac{1}{3}$. Then, to check that students understand that a fraction bar acts as a grouping symbol, you might have them evaluate $\frac{6+4}{2}$ and $\frac{9-6}{3}$.

Encourage discussion around the question in the thought bubble in part b). Students may suggest multiplying by other common multiples of 5 and 2. If so, you might have them try using one of the common multiples, such as 20, to show that the answer is the same.



Example 2: Apply Equations With Grouping Symbols

On a typical February day in Whitehorse, Yukon Territory, the daily average temperature is −13.2 °C. The low temperature is −18.1 °C. What is the high temperature?

Solution

Let the high temperature be T degrees Celsius. The daily average temperature, in degrees Celsius, is the average of the high and low temperatures, or $\frac{T + (-18.1)}{2}$. The daily average temperature is −13.2 °C.

How could you estimate the high temperature?

An equation that represents this situation is $\frac{T + (-18.1)}{2} = -13.2$.

Isolate the variable, T.

$$\frac{T + (-18.1)}{2} = -13.2$$
$$2 \times \frac{T + (-18.1)}{2} = 2 \times (-13.2)$$
$$T - 18.1 = -26.4$$
$$T - 18.1 + 18.1 = -26.4 + 18.1$$
$$T = -8.3$$

The high temperature is −8.3 °C.

Check:

The average of the high and low temperatures is $\frac{-8.3 + (-18.1)}{2}$.

$$\frac{-8.3 + (-18.1)}{2} = \frac{-26.4}{2}$$
$$= -13.2$$

The calculated average of −13.2 °C agrees with the daily average temperature given in the problem.

Show You Know

On a typical day in October in Churchill, Manitoba, the daily average temperature is −1.5 °C. The high temperature is 1.3 °C. Estimate and then calculate the low temperature.

Barren Lands, Churchill, Manitoba

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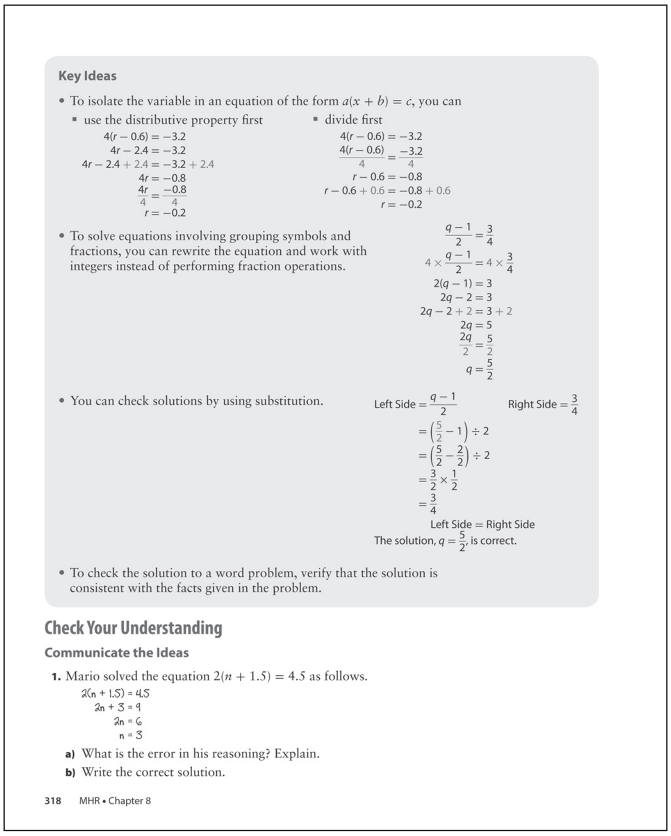

Key Ideas

- To isolate the variable in an equation of the form $a(x + b) = c$, you can
  - use the distributive property first
  $$4(r - 0.6) = -3.2$$
  $$4r - 2.4 = -3.2$$
  $$4r - 2.4 + 2.4 = -3.2 + 2.4$$
  $$4r = -0.8$$
  $$\frac{4r}{4} = \frac{-0.8}{4}$$
  $$r = -0.2$$
  - divide first
  $$4(r - 0.6) = -3.2$$
  $$\frac{4(r - 0.6)}{4} = \frac{-3.2}{4}$$
  $$r - 0.6 = -0.8$$
  $$r - 0.6 + 0.6 = -0.8 + 0.6$$
  $$r = -0.2$$
- To solve equations involving grouping symbols and fractions, you can rewrite the equation and work with integers instead of performing fraction operations.
  $$\frac{q - 1}{2} = \frac{3}{4}$$
  $$4 \times \frac{q - 1}{2} = 4 \times \frac{3}{4}$$
  $$2(q - 1) = 3$$
  $$2q - 2 = 3$$
  $$2q - 2 + 2 = 3 + 2$$
  $$2q = 5$$
  $$\frac{2q}{2} = \frac{5}{2}$$
  $$q = \frac{5}{2}$$
- You can check solutions by using substitution.
  Left Side $= \frac{q - 1}{2}$ Right Side $= \frac{3}{4}$
  $$= \left(\frac{5}{2} - 1\right) \div 2$$
  $$= \left(\frac{5}{2} - \frac{2}{2}\right) \div 2$$
  $$= \frac{3}{2} \times \frac{1}{2}$$
  $$= \frac{3}{4}$$
  Left Side = Right Side
  The solution, $q = \frac{5}{2}$, is correct.
- To check the solution to a word problem, verify that the solution is consistent with the facts given in the problem.

Check Your Understanding

Communicate the Ideas

1. Mario solved the equation $2(n + 1.5) = 4.5$ as follows.
   $2(n + 1.5) = 4.5$
   $2n + 3 = 9$
   $2n = 6$
   $n = 3$
   a) What is the error in his reasoning? Explain.
   b) Write the correct solution.

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You might ask students if they see any advantage in using 10 instead of a different common multiple of the denominators. On the basis of the Literacy Link, students should realize that they could rewrite the equation as $\frac{1}{5}(t - 1) = \frac{3}{2}$ and use either solution method from part a). You may wish to have students solve the equation by both of those methods using fraction operations, and to follow up with a discussion of which solution method students prefer and why.

Example 2

This example shows the writing and solving of an equation in the context of daily average temperatures introduced at the beginning of this section.

Encourage discussion around the question in the thought bubble. If students need support, you might ask:

- Would you expect the high temperature to be greater than or less than the average temperature? Why?
- About how many degrees less than the average temperature is the low temperature?
- About how many degrees more than the average temperature is the high temperature? How do you know?
- What is the approximate value of the high temperature?

Emphasize that the method shown for isolating the variable is just one possible solution. You might ask students to solve the equation in the form $\frac{1}{2}(T - 18.1) = -13.2$ by first using the distributive property, and then have them describe any advantages and disadvantages of this method in comparison with the method in the example.

Key Ideas

This section summarizes solution methods for equations of the form $a(x + b) = c$ involving rational numbers. Have students describe in their own words the steps used in the algebraic solutions shown and the reasoning behind each step. You might also have students compare the two solutions for $4(r - 0.6) = -3.2$ and explain any preference they have. Students might also solve $\frac{q - 1}{2} = \frac{3}{4}$ by using fraction operations, instead of working with integers as shown. Again, have them explain any preference they have.

Have students review the Key Ideas for any additional material they would like to add to their Foldable.

Meeting Student Needs

- Encourage students to complete the Show You Know using the method(s) of their choice and to check and correct each other's solutions.

Common Errors

- Some students may have difficulty in understanding the result of multiplying an expression with a fraction bar by a common multiple, such as $10 \times \frac{t-1}{5} = 2(t-1)$in Example 1b).

R*x* Point out that the expression $\frac{t-1}{5}$ can be written with brackets as $\frac{1}{5}(t-1)$. Then, represent the multiplication as $10 \times \frac{1}{5}(t-1)$. Check for understanding by asking students to multiply $6 \times \frac{x+1}{2}$ and $12 \times \frac{y-2}{3}$.

Answers

Example 1: Show You Know

a) $e = 2.7$ **b)** $c = -\frac{19}{2}$

Example 2: Show You Know

$t = -4.3$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may want to check students' understanding of the distributive property by asking them to remove the brackets from each of the following expressions before they solve: $2(x + 3)$, $-3(5 + y)$, $4(g - 2)$, $-2(a - 4)$. Ensure students' understanding before they attempt the Show You Know. • Some students may benefit from a review of common denominator and its relevance to these questions.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to look back at the list of methods to solve that were generated in the Explore. Remind them that the method shown is just one way to solve. Ask them to identify their preferred method. Have them solve using this method. • Remind students of the importance of checking their answers.

Key Ideas

- To isolate the variable in an equation of the form $a(x + b) = c$, you can
  - use the distributive property first

$$\begin{aligned}4(r - 0.6) &= -3.2\\ 4r - 2.4 &= -3.2\\ 4r - 2.4 + 2.4 &= -3.2 + 2.4\\ 4r &= -0.8\\ \frac{4r}{4} &= \frac{-0.8}{4}\\ r &= -0.2\end{aligned}$$

  - divide first

$$\begin{aligned}4(r - 0.6) &= -3.2\\ \frac{4(r - 0.6)}{4} &= \frac{-3.2}{4}\\ r - 0.6 &= -0.8\\ r - 0.6 + 0.6 &= -0.8 + 0.6\\ r &= -0.2\end{aligned}$$

- To solve equations involving grouping symbols and fractions, you can rewrite the equation and work with integers instead of performing fraction operations.

$$\begin{aligned}\frac{q - 1}{2} &= \frac{3}{4}\\ 4 \times \frac{q - 1}{2} &= 4 \times \frac{3}{4}\\ 2(q - 1) &= 3\\ 2q - 2 &= 3\\ 2q - 2 + 2 &= 3 + 2\\ 2q &= 5\\ \frac{2q}{2} &= \frac{5}{2}\\ q &= \frac{5}{2}\end{aligned}$$

- You can check solutions by using substitution.

Left Side $= \frac{q - 1}{2}$ Right Side $= \frac{3}{4}$

$$\begin{aligned}&= \left(\frac{5}{2} - 1\right) \div 2\\ &= \left(\frac{5}{2} - \frac{2}{2}\right) \div 2\\ &= \frac{3}{2} \times \frac{1}{2}\\ &= \frac{3}{4}\end{aligned}$$

Left Side = Right Side

The solution, $q = \frac{5}{2}$, is correct.

- To check the solution to a word problem, verify that the solution is consistent with the facts given in the problem.

Check Your Understanding

Communicate the Ideas

1. Mario solved the equation $2(n + 1.5) = 4.5$ as follows.

$2(n + 1.5) = 4.5$
$2n + 3 = 9$
$2n = 6$
$n = 3$

a) What is the error in his reasoning? Explain.
b) Write the correct solution.

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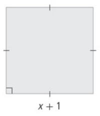
2. Cal and Tyana solved the equation $3(k - 4.3) = -2.7$ in different ways. Cal used the distributive property first, while Tyana divided first.
a) Show both of their methods.
b) Whose method do you prefer? Explain.
c) If the equation was $3(k - 4.3) = -2.5$, which method would you use? Explain.

3. Renée and Paul used different methods to solve $\frac{x + 1}{2} = \frac{3}{5}$.
a) Renée first multiplied both sides by a common multiple.
$10 \times \frac{x + 1}{2} = 10 \times \frac{3}{5}$
Show the rest of her solution.
b) Paul first rewrote the equation as $\frac{1}{2}(x + 1) = \frac{3}{5}$ and used the distributive property. Show the rest of his solution.
$\frac{1}{2}x + \frac{1}{2} = \frac{3}{5}$
c) Which solution do you prefer? Explain.

4. Viktor and Ashni were solving the following problem together.
A square with a side length of $x + 1$ has a perimeter of 18.6 units. What is the value of x?
They disagreed over how to model the situation with an equation.

Viktor's Equation $4(x + 1) = 18.6$
Ashni's Equation $4x + 1 = 18.6$



a) Which equation is correct? Explain.
b) What is the value of x?
c) How can you check whether your value for x is correct?

Practise

5. Write an equation that is represented by the following. Then, solve the equation.

For help with #6 to #9, refer to Example 1 on pages 315–316.

6. Solve and check.
a) $2(x + 1.5) = 7.6$
b) $-2.8 = -1(c - 0.65)$
c) $-3.57 = 3(a + 4.51)$
d) $-3.6(0.25 - r) = 0.18$

7. Solve. Express each solution to the nearest hundredth.
a) $3(u - 12.5) = -3.41$
b) $14.01 = -7(1.93 + m)$
c) $6(0.15 + v) = 10.97$
d) $-9.5(x - 4.2) = 7.5$

8. Solve and check.
a) $\frac{n + 1}{2} = -\frac{3}{4}$
b) $\frac{5}{2} = \frac{1}{3}(x - 2)$
c) $\frac{3}{4}(w + 2) = 1\frac{1}{3}$
d) $\frac{7}{6} = \frac{2(5 - g)}{3}$

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Check Your Understanding

Communicate the Ideas

These questions allow students to explain aspects of solving equations with grouping symbols using rational numbers.

In #1, you might ask students how they can show that there must be an error in the reasoning before they determine what it is. (They can show by substitution that the solution, $n = 3$, is incorrect.)

Encourage discussion after students complete #2c), where the answer is a non-terminating decimal. If students divide first, a non-terminating decimal appears in the solution at an earlier stage than if students first use the distributive property. Some students may see a difficulty with the earlier introduction of a non-terminating decimal. Others may not, especially if they are using a calculator. You might ask students how they would record the non-terminating decimal in the solution. Some may round it, while others may use the bar notation for repeating decimals. You might point out that problems involving such solutions often specify the number of decimal places required in the answer. You might ask students to express the solution to the nearest hundredth.

In #3, have students compare and explain their preferences. Emphasize that students should use whichever method they feel more comfortable with.

If students have difficulty with #4a), you might suggest that they complete #4b) for both equations. Then, in #4c), they can check each solution for consistency with the information given in the problem. By identifying the correct solution, they can deduce which equation is correct. They can then think about why this is the case.

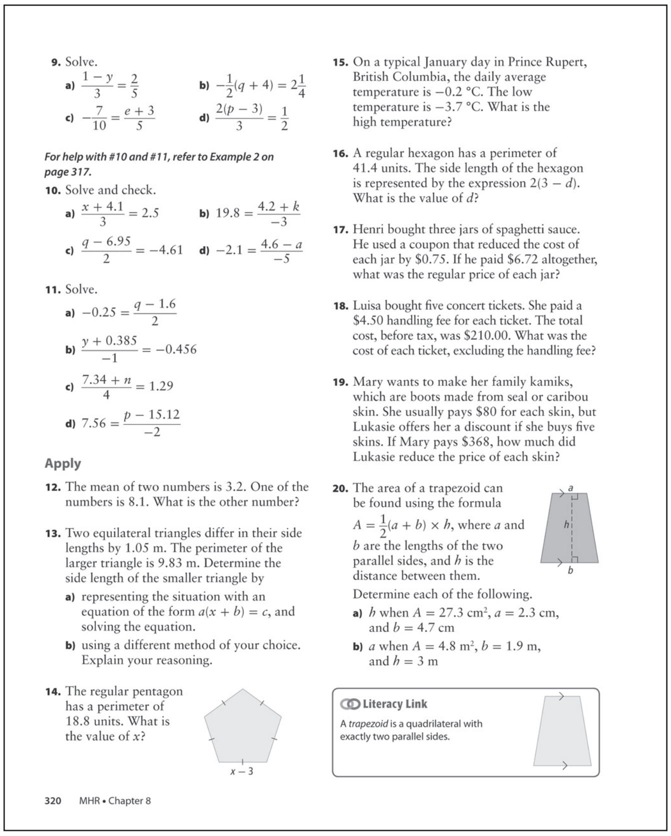
Practise

You may wish to have students complete the Practise questions individually, in pairs, or in small groups. Encourage students to compare their solution methods, check each other's answers, and suggest corrections or other improvements.

In #5, some students may write the equation $3x + 0.15 = 0.60$. Encourage them to write and solve an equation in the form $a(x + b) = c$ instead.

Apply

Encourage students to suggest and discuss alternative solutions for the same problem. For example, #12 lends itself to a Guess and Check approach as well as to an algebraic solution. Students may solve #17



9. Solve.

a) $\frac{1-y}{3} = \frac{2}{5}$
b) $-\frac{1}{2}(q + 4) = 2\frac{1}{4}$
c) $-\frac{7}{10} = \frac{e + 3}{5}$
d) $\frac{2(p - 3)}{3} = \frac{1}{2}$

For help with #10 and #11, refer to Example 2 on page 317.

10. Solve and check.

a) $\frac{x + 4.1}{3} = 2.5$
b) $19.8 = \frac{4.2 + k}{-3}$
c) $\frac{q - 6.95}{2} = -4.61$
d) $-2.1 = \frac{4.6 - a}{-5}$

11. Solve.

a) $-0.25 = \frac{q - 1.6}{2}$
b) $\frac{y + 0.385}{-1} = -0.456$
c) $\frac{7.34 + n}{4} = 1.29$
d) $7.56 = \frac{p - 15.12}{-2}$

Apply

12. The mean of two numbers is 3.2. One of the numbers is 8.1. What is the other number?

13. Two equilateral triangles differ in their side lengths by 1.05 m. The perimeter of the larger triangle is 9.83 m. Determine the side length of the smaller triangle by
a) representing the situation with an equation of the form $a(x + b) = c$, and solving the equation.
b) using a different method of your choice. Explain your reasoning.

14. The regular pentagon has a perimeter of 18.8 units. What is the value of x?

15. On a typical January day in Prince Rupert, British Columbia, the daily average temperature is −0.2 °C. The low temperature is −3.7 °C. What is the high temperature?

16. A regular hexagon has a perimeter of 41.4 units. The side length of the hexagon is represented by the expression $2(3 - d)$. What is the value of d?

17. Henri bought three jars of spaghetti sauce. He used a coupon that reduced the cost of each jar by $0.75. If he paid $6.72 altogether, what was the regular price of each jar?

18. Luisa bought five concert tickets. She paid a $4.50 handling fee for each ticket. The total cost, before tax, was $210.00. What was the cost of each ticket, excluding the handling fee?

19. Mary wants to make her family kamiks, which are boots made from seal or caribou skin. She usually pays $80 for each skin, but Lukasie offers her a discount if she buys five skins. If Mary pays $368, how much did Lukasie reduce the price of each skin?

20. The area of a trapezoid can be found using the formula $A = \frac{1}{2}(a + b) \times h$, where a and b are the lengths of the two parallel sides, and h is the distance between them.

Determine each of the following.
a) h when $A = 27.3$ cm², $a = 2.3$ cm, and $b = 4.7$ cm
b) a when $A = 4.8$ m², $b = 1.9$ m, and $h = 3$ m

Literacy Link
A *trapezoid* is a quadrilateral with exactly two parallel sides.

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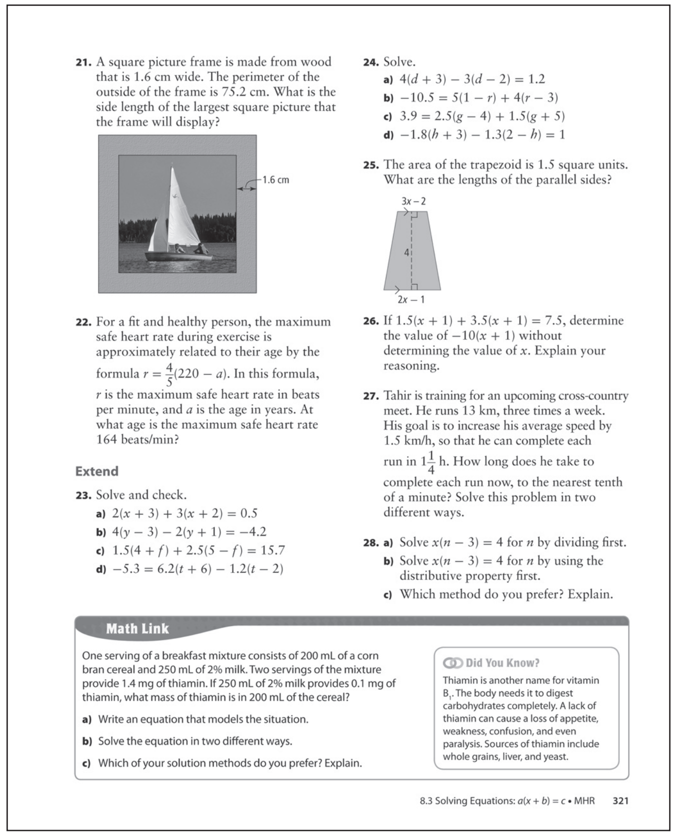

21. A square picture frame is made from wood that is 1.6 cm wide. The perimeter of the outside of the frame is 75.2 cm. What is the side length of the largest square picture that the frame will display?

22. For a fit and healthy person, the maximum safe heart rate during exercise is approximately related to their age by the formula $r = \frac{4}{5}(220 - a)$. In this formula, r is the maximum safe heart rate in beats per minute, and a is the age in years. At what age is the maximum safe heart rate 164 beats/min?

Extend

23. Solve and check.
a) $2(x + 3) + 3(x + 2) = 0.5$
b) $4(y - 3) - 2(y + 1) = -4.2$
c) $1.5(4 + f) + 2.5(5 - f) = 15.7$
d) $-5.3 = 6.2(t + 6) - 1.2(t - 2)$

24. Solve.
a) $4(d + 3) - 3(d - 2) = 1.2$
b) $-10.5 = 5(1 - r) + 4(r - 3)$
c) $3.9 = 2.5(g - 4) + 1.5(g + 5)$
d) $-1.8(h + 3) - 1.3(2 - h) = 1$

25. The area of the trapezoid is 1.5 square units. What are the lengths of the parallel sides?

26. If $1.5(x + 1) + 3.5(x + 1) = 7.5$, determine the value of $-10(x + 1)$ without determining the value of x. Explain your reasoning.

27. Tahir is training for an upcoming cross-country meet. He runs 13 km, three times a week. His goal is to increase his average speed by 1.5 km/h, so that he can complete each run in $1\frac{1}{4}$ h. How long does he take to complete each run now, to the nearest tenth of a minute? Solve this problem in two different ways.

28. a) Solve $x(n - 3) = 4$ for n by dividing first.
b) Solve $x(n - 3) = 4$ for n by using the distributive property first.
c) Which method do you prefer? Explain.

Math Link

One serving of a breakfast mixture consists of 200 mL of a corn bran cereal and 250 mL of 2% milk. Two servings of the mixture provide 1.4 mg of thiamin. If 250 mL of 2% milk provides 0.1 mg of thiamin, what mass of thiamin is in 200 mL of the cereal?

a) Write an equation that models the situation.
b) Solve the equation in two different ways.
c) Which of your solution methods do you prefer? Explain.

Did You Know?
Thiamin is another name for vitamin B_1. The body needs it to digest carbohydrates completely. A lack of thiamin can cause a loss of appetite, weakness, confusion, and even paralysis. Sources of thiamin include whole grains, liver, and yeast.

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more easily by working backward from the reduced price per jar to the regular price per jar, rather than by writing and solving an equation.

For #15, an alternative approach is to determine how many degrees the average is above the low, and then to add this number of degrees to the average.

For some problems, students may write and solve different equations. For example, in #17, the expected equation of the form $a(x + b) = c$ is $3(r - 0.75) = 6.72$. However, some students may reason that the total reduction for the three jars was $2.25, and then write $3r - 2.25 = 6.72$ If so, you might take the opportunity to compare the two solutions and show their equivalence.

Literacy Link Before students complete #20, draw their attention to the Literacy Link that follows it. To check for understanding, you might ask students to describe the difference between a trapezoid and a parallelogram.

Extend

In #23 and 24, students need to perform additional steps in solving equations. You might have them discuss ways of solving the equations most easily.

For #25, remind students that the formula for the area of a trapezoid appears in #20.

In #26, some students may solve by applying the distributive property and then thinking about how the values of $5x + 5$ and $-10x - 10$ compare. Other students may first determine that $1.5(x + 1) + 3.5(x + 1) = 5(x + 1)$ and avoid the need for the distributive property. Have students compare the efficiency of their solutions.

In #27, you might ask students if any information in the problem is unnecessary (i.e., to solve the problem, they do not need to know that Tahir runs three times a week). Then, students may write and solve different equations to determine Tahir's present average speed.

In #28, solving by dividing first results in fewer steps than solving by applying the distributive property first. However, students may not base their preference on the number of steps. If the two solutions result in expressions for n that look different (i.e., $\frac{4}{x} + 3$ and $\frac{4 + 3x}{x}$), check that students recognize their equivalence.

Literacy Link Remind students to complete the third oval in their concept map, which is labelled *with grouping symbols*. They should provide an example of the equation form, and outline the steps required to solve the equation, using a strategy of their choice.

Math Link

In this Math Link, students can apply their skills in writing and solving equations of the form $a(x + b)$ to the field of nutrition. After students have answered part b), you might challenge them to think of and discuss more ways of solving the equation $2(c + 0.1) = 1.4$. Possible solution methods include modelling with cups, coins, and paper clips, and solving by inspection or Guess and Check.

Draw students' attention to the Did You Know? within the Math Link. You may wish to have students do some research on dietary sources of other B vitamins and on the functions of these vitamins in the body. Students will encounter vitamin B2, riboflavin, in the Math Link at the end of section 8.4.

Meeting Student Needs

- Provide **BLM 8–10 Section 8.3 Extra Practice** to students who would benefit from more practice.
- As an alternative to #27, invite in a coach or a member of the North American Indigenous Games (NAIG) to talk about how the athletes qualify to play in the NAIG. Have the speaker bring along some of the statistics of the players. Then, set up a problem based on these statistics that students can solve with their new knowledge.

ELL

- Before beginning the Check Your Understanding section, ensure that students understand the words *error*, and *correct solution*. Use an *X* and a checkmark on the board to help define these words.
- In the Apply section, give fewer word problems to English language learners. Clarify with students any words they do not understand in the word problems. Draw a visual wherever possible.
- In #13, the phrase, *differ in their side lengths by 1.05 m* might make this question difficult for English language learners. In the students' notebook or on the board, sketch the two triangles. Show the sides of the smaller triangle with a question mark, and indicate that the second triangle's side length is the question mark plus another 1.05 m.
- In #18, the concept of a *handling fee* may have to be explained. English language learners need to understand that this is a one-time cost, not a cost for each ticket.

Gifted and Enrichment

- Ask students to investigate the expression $\left(\frac{1}{x} + \frac{1}{x}\right) - 1 = 4$ with a view to finding which values of x are a) inappropriate, and b) the actual value of x. Next, have them consider $\left(\frac{1}{x}\right) - 1 = x$ for inappropriate values of x, and have them explain their thinking.

Common Errors

- Some students may not multiply all the terms inside the brackets by the number outside the brackets when applying the distributive property.

R_x Encourage students to draw curved arrows as a reminder, as shown in Method 1 of Example 1a). Also, emphasize the need to check solutions in order to detect errors.

- Some students may have difficulty in writing a correct equation for #21. For example, if the side length of the outside of the frame is represented by s, they may think that the side length of the picture is represented by $s - 1.6$.

R_x Encourage students to copy the diagram and mark the value of the width of the frame more times, so that the side length of the picture becomes more apparent. You might also have students predict the width of the border around a 5 cm by 5 cm square that is centred within a 7 cm by 7 cm square. If students predict a 2-cm wide border, demonstrate that the border is only 1 cm wide and explain why.

Answers

Communicate the Ideas

1. a) Mario distributed the 2 to all three terms in the equation instead of just the terms inside the brackets.

b) $n = 0.75$

2. a) Cal's method:
the distributive property

$$\begin{aligned} 3(k - 4.3) &= -2.7 \\ 3k - 12.9 &= -2.7 \\ 3k &= -2.7 + 12.9 \\ 3k &= 10.2 \\ \frac{3k}{3} &= \frac{10.2}{3} \\ k &= 3.4 \end{aligned}$$

Tyana's method:
dividing by 3 first

$$\begin{aligned} \frac{3(k - 4.3)}{3} &= \frac{-2.7}{3} \\ k - 4.3 &= -0.9 \\ k - 4.3 + 4.3 &= -0.9 + 4.3 \\ k &= 3.4 \end{aligned}$$

b) Example: I would prefer dividing first as it is a shorter method.

c) The division method is not a good method to use because 3 does not divide evenly into -2.5.

3. a) Renee's solution:

$$\begin{aligned} 10\left(\frac{x + 1}{2}\right) &= 10\left(\frac{3}{5}\right) \\ 5(x + 1) &= 6 \\ 5x + 5 &= 6 \\ 5x + 5 - 5 &= 6 - 5 \\ 5x &= 1 \\ \frac{5x}{5} &= \frac{1}{5} \\ x &= \frac{1}{5} \end{aligned}$$

b) Paul's solution:

$$\begin{aligned} \frac{1}{2}x + \frac{1}{2} &= \frac{3}{5} \\ 10\left(\frac{1}{2}x\right) + 10\left(\frac{1}{2}\right) &= 10\left(\frac{3}{5}\right) \\ 5x + 5 &= 6 \\ 5x + 5 - 5 &= 6 - 5 \\ 5x &= 1 \\ \frac{5x}{5} &= \frac{1}{5} \\ x &= \frac{1}{5} \end{aligned}$$

4. a) Viktor's equation is correct because $4(x + 1)$ represents the length of all four sides. Each side equals $x + 1$.

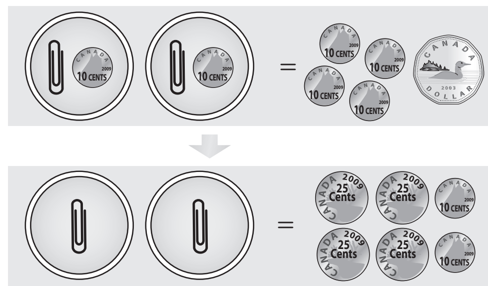
b) $x = 3.65$

c) This answer can be checked using substitution.

Math Link

a) $2(c + 0.1) = 1.4$

b)
$$\begin{aligned} 2(c + m) &= 1.4 \\ 2(c + 0.1) &= 1.4 \\ 2c + 0.2 &= 1.4 \\ 2c &= 1.2 \end{aligned}$$



c) Example: By dividing first; fewer steps and decimal numbers

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Note that #1 allows students to check their understanding of the distributive property. Students having difficulty should be coached in the meaning of *distributive* as well as what the coefficient actually multiplies in a question. • You may wish to hand out **Master 2 Communication Peer Evaluation** for students to assess each other's responses to the questions.
Assessment *for* Learning	
Practise and Apply Have students do #5, 6, 8, 10, 12, 14, 15, and 17. Students who have no problems with these questions can go on to the remaining Apply problems.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • For students having difficulty with #5, have them look at #1 of the Explore. Have them verbalize what each element represents. Then, see if they are able to transfer this understanding to explaining the model in #5. • For #6 and 8, struggling learners may need additional coaching in the distributive property and/or common denominators. Model the solution for students and then assign questions from #7 and 9, respectively. • Note that #12 may require students to review the meaning of the word *mean* and how we calculate the mean of 4 and 8, for example. Have them apply their understanding to #12. • Some students may not understand the meaning of *regular pentagon* in #14. Ensure they know that all sides are the same. Review the meaning of the markings on the sides of the pentagon. • If students need help with #15, refer them to Example 2, which includes a similar problem and its solution. Some students may benefit from a temperature chart or a number line so that they can visually identify the temperatures. You may wish to Provide **Master 4 Number Lines**. • Note that #17 is a real-life problem that students may well encounter in their own life. It is important that they realize each jar was reduced. They may benefit from partner work in setting up their equation.
Math Link The Math Link on page 321 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 333.	• Students who need help getting started could use **BLM 8–11 Section 8.3 Math Link**, which provides scaffolding. • Some students may benefit from small-group work or a class discussion about what students chose as equations. • This equation could be used to help students complete a similar one in the Wrap It Up! at the end of the chapter.
Assessment *as* Learning	
Literacy Link At the end of section 8.3, have students work in pairs to complete the oval of the concept map, labelled *with grouping symbols*.	• After completing section 8.3, brainstorm and discuss as a class the information needed to complete this oval.
Math Learning Log Have students respond to the following questions: • To solve the equation of the form $a(x + b) = c$, there are several methods you can use to get the answer. They might include inspection; Guess and Check; modelling with cups, coins, and paper clips; and solving algebraically. Which method do you find the easiest? Explain. Which do you find the hardest? Explain.	• Encourage students to think carefully about the process they prefer. They may have more than one that they feel particularly comfortable with. Have them identify as many as they wish. • For the one they find most difficult, listen or read for their explanation as to why. Is it process or conceptual understanding that causes them confusion with a particular method?