8.4 Solving Equations: $ax = b + cx$, $ax + b = cx + d$, $a(bx + c) = d(ex + f)$

***MathLinks 9*, pages 322–329**

Suggested Timing

80–100 minutes

Materials

- coins or items to represent coins of different denominations
- paper cups or small containers
- paper clips

Blackline Masters

Master 2 Communication Peer Evaluation
Master 14 Coin Models
BLM 8–3 Chapter 8 Warm-Up
BLM 8–5 Canadian Coins and Their Values
BLM 8–12 Section 8.4 Extra Practice
BLM 8–13 Section 8.4 Math Link

Mathematical Processes

- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes

PR3 Model and solve problems using linear equations of the form:

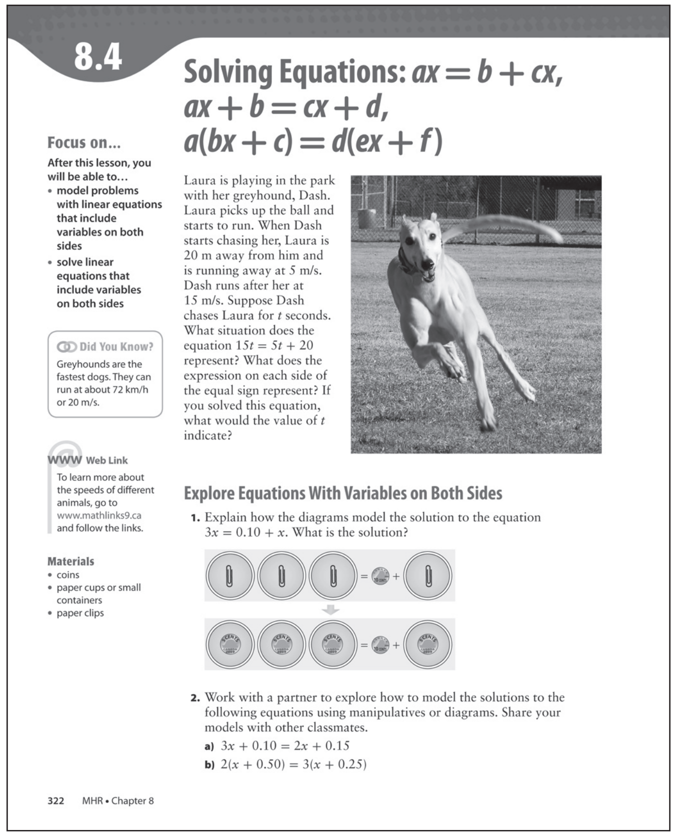
- $ax = b + cx$
- $ax + b = cx + d$
- $a(bx + c) = d(ex + f)$

where a, b, c, d, e and f are rational numbers.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–6, 8, 10, 14, 15, Math Link
Typical	#1–6, 8, 10, 14, 15, 18, 25, Math Link
Extension/Enrichment	#1–3, 16, 18, 22, 23, 24, 26–30

Planning Notes

Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce material learned in previous sections.



8.4 Solving Equations: $ax = b + cx$, $ax + b = cx + d$, $a(bx + c) = d(ex + f)$

Focus on...

After this lesson, you will be able to...
- model problems with linear equations that include variables on both sides
- solve linear equations that include variables on both sides

Laura is playing in the park with her greyhound, Dash. Laura picks up the ball and starts to run. When Dash starts chasing her, Laura is 20 m away from him and is running away at 5 m/s. Dash runs after her at 15 m/s. Suppose Dash chases Laura for t seconds. What situation does the equation $15t = 5t + 20$ represent? What does the expression on each side of the equal sign represent? If you solved this equation, what would the value of t indicate?

Did You Know?

Greyhounds are the fastest dogs. They can run at about 72 km/h or 20 m/s.

Web Link

To learn more about the speeds of different animals, go to www.mathlinks9.ca and follow the links.

Explore Equations With Variables on Both Sides

Materials
- coins
- paper cups or small containers
- paper clips

1. Explain how the diagrams model the solution to the equation $3x = 0.10 + x$. What is the solution?

2. Work with a partner to explore how to model the solutions to the following equations using manipulatives or diagrams. Share your models with other classmates.

a) $3x + 0.10 = 2x + 0.15$

b) $2(x + 0.50) = 3(x + 0.25)$

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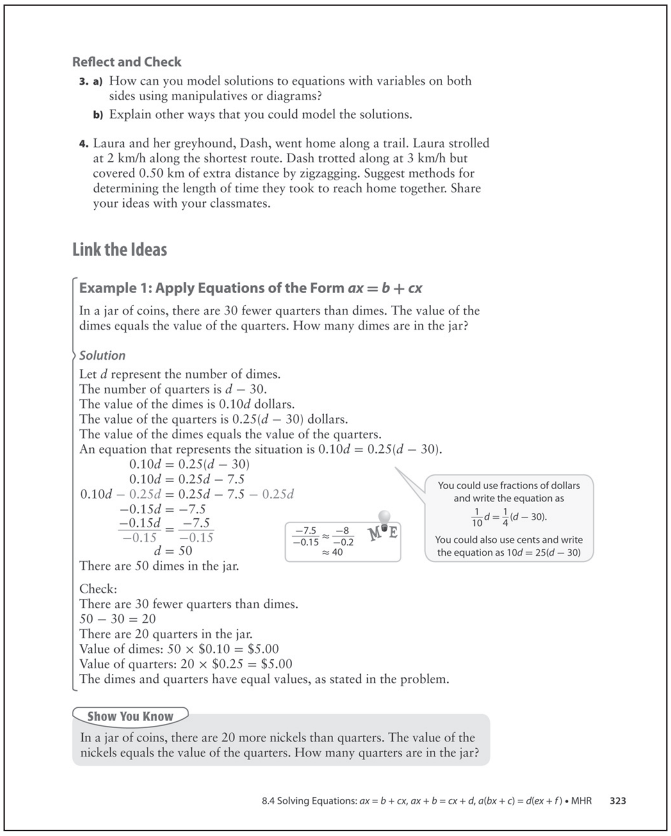
The opening paragraph on page 322 introduces a category of linear equation with which students are not familiar: an equation with the same variable on both sides. Have students read the storyline about Laura and her greyhound, Dash. Have students consider the three questions at the end of the paragraph. Students will need to decide what each expression represents before they can understand the equation as a whole.

To provide support, you might ask:

- How is distance travelled related to speed and time?
- If Pierre walks at 1.2 m/s for 5 s, how far is he from where he started?
- Pierre was standing 30 m from his home. He walked away from it at 1.2 m/s for 5 s. What is his total distance away from his home?
- If Sharon jogs at 4 m/s for t seconds, how far is she from where she started?
- Sharon was standing 40 m from her home. She then jogged away from it at 4 m/s for t seconds. What is her total distance away from her home?



Reflect and Check

3. a) How can you model solutions to equations with variables on both sides using manipulatives or diagrams?
   b) Explain other ways that you could model the solutions.

4. Laura and her greyhound, Dash, went home along a trail. Laura strolled at 2 km/h along the shortest route. Dash trotted along at 3 km/h but covered 0.50 km of extra distance by zigzagging. Suggest methods for determining the length of time they took to reach home together. Share your ideas with your classmates.

Link the Ideas

Example 1: Apply Equations of the Form $ax = b + cx$

In a jar of coins, there are 30 fewer quarters than dimes. The value of the dimes equals the value of the quarters. How many dimes are in the jar?

Solution

Let d represent the number of dimes.
The number of quarters is $d - 30$.
The value of the dimes is $0.10d$ dollars.
The value of the quarters is $0.25(d - 30)$ dollars.
The value of the dimes equals the value of the quarters.
An equation that represents the situation is $0.10d = 0.25(d - 30)$.

$$0.10d = 0.25(d - 30)$$
$$0.10d = 0.25d - 7.5$$
$$0.10d - 0.25d = 0.25d - 7.5 - 0.25d$$
$$-0.15d = -7.5$$
$$\frac{-0.15d}{-0.15} = \frac{-7.5}{-0.15}$$
$$d = 50$$

$\frac{-7.5}{-0.15} \approx \frac{-8}{-0.2}$
≈ 40

ME

You could use fractions of dollars and write the equation as $\frac{1}{10}d = \frac{1}{4}(d - 30)$.

You could also use cents and write the equation as $10d = 25(d - 30)$

There are 50 dimes in the jar.

Check:
There are 30 fewer quarters than dimes.
$50 - 30 = 20$
There are 20 quarters in the jar.
Value of dimes: $50 \times \$0.10 = \$5.00$
Value of quarters: $20 \times \$0.25 = \$5.00$
The dimes and quarters have equal values, as stated in the problem.

Show You Know

In a jar of coins, there are 20 more nickels than quarters. The value of the nickels equals the value of the quarters. How many quarters are in the jar?

8.4 Solving Equations: $ax = b + cx$, $ax + b = cx + d$, $a(bx + c) = d(ex + f)$ • MHR 323

When students realize that $15t$ represents Dash's distance from his starting location, and that $5t + 20$ represents Laura's distance from Dash's starting location (not from her own starting location), you might ask:

- What does the equation tell you about the distances of Laura and Dash from Dash's starting location?
- How do you know that t can only have one value in the equation? (In both expressions, t represents the same length of time, i.e., the time that Dash took to catch up with Laura. You might make the general point that in any situation represented by a linear equation with the same variable on both sides, the variable can only have one meaning and one value.)

Explore Equations With Variables on Both Sides

In this exploration, students begin to model and solve linear equations with forms $ax = b + cx$, $ax + b = cx + d$, and $a(bx + c) = d(ex + f)$. You may wish to provide students with **Master 14 Coin Models** if actual coins are not available.

Method 1 Have students work on the exploration with a partner or in small groups by using manipulatives. Encourage students to compare their models and discuss their answers.

In #1, students observe a model of a solution to an equation of the form $ax = b + cx$ involving decimals. You might ask:

- How does the first part of the diagram in #1 model the equation $3x = 0.10 + x$?
- How do you know that the solution modelled by the second part of the diagram is correct?
- Instead of replacing the four paper clips with coins, is there another way to model the solution in #1? (By this stage of the chapter, many students can be expected to engage in reasoning that resembles an algebraic method, i.e., beginning by removing one cup and paper clip from each side of the model.)

If you feel that students would benefit from modelling and solving more equations of the type $ax = b + cx$, you might have them try $2x = 0.20 + x$ and $4x = 0.12 + 2x$.

In #2, encourage students to use the manipulatives to model solutions in whatever way they prefer. In #2a), some students may use Guess and Check. Others may rely on reasoning that more closely resembles an algebraic process. More students may need to rely on the Guess and Check strategy in #2b). If they are unsure of how to simplify the model by removing coins and/or cups with paper clips from both sides, you may wish to point out that they are free to take the coins out of the cups on each side of the model of the original equation. You might ask students how doing so is related to applying the distributive property to the original equation.

If you feel that students would benefit from modelling and solving more equations such as the ones in #2, you might have them try $x + 0.35 = 2x + 0.25$, $2x + 1.25 = 4x + 0.75$, $2(x + 0.10) = x + 0.25$, and $4(x + 0.02) = 2(x + 0.05)$.

For #3a) and b), consider holding a class discussion around these questions so that students have the opportunity to share any creative models they developed. When discussing #3b), encourage algebraic reasoning by asking students to describe the operations they would perform in solving each equation in the Explore symbolically.

In #4, students return to the context of Laura and her greyhound, Dash. Some students may first use a Guess and Check to approach the problem without writing an equation. Others can be expected to write an equation first and then consider different ways of solving it.

After students have written the equation $2t + 0.50 = 3t$, have them solve it in different ways, share their methods, choose their preferred method, and explain their choice.

Method 1 Have students complete #1 and 2 in the Explore by sketching diagrams. Engage and encourage students as described in Method 1, above.

Meeting Student Needs

- Because students have not seen equations of the type presented in the introduction to the section, they are not asked to solve $15t = 5t + 20$. However, students can solve it by Guess and Check. You may wish to have students return to the equation after they have completed the Explore section or examined Example 1.
- It may be better for your class to work through the Explore as a whole-class activity.
- Invite in a respected Elder from one of the communities to give a talk on balance and the importance of balance in life. The concept of balance is one of the natural spiritual laws of many Aboriginal people. The powwow outfits are balanced in their designs and bead patterns, the powwow dancers are judged by how well they use balance in their routines, and the traditional teaching of the medicine wheel is based on balance.
- Have students research First Nations algebra on the Internet. Have a class discussion about what they find.

ELL

- To assist English language learners, clarify the meaning of the word *greyhound*.
- Have English language learners answer the Reflect and Check question in their first language to allow access to prior knowledge. Then, have students translate their ideas. Provide them with English words where necessary.

Common Errors

- Some students may misinterpret the wording of the problem in #4 and suggest the equation $2t = 3t + 0.5$.

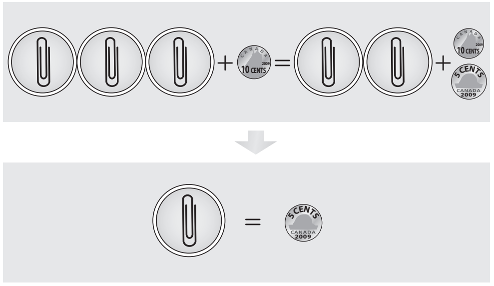
Rx You might ask students to consider whether Dash or Laura covered the greater distance. You might also point out that this equation implies that $2t$ is more than $3t$. Since t must be positive, this relationship is impossible.

Answers

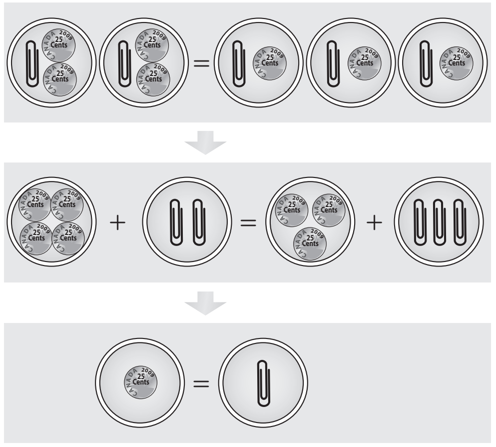
Explore Equations With Variables on Both Sides

1. Each paper clip represents x or 5¢.

2. a)



b)



3. a) Cups, paper clips, and coins
b) Algebra tiles or number lines

4. Example: Algebraically. If d = distance, in kilometres, then $\frac{d}{2} = \frac{d + 0.5}{3}$. So, $d = 1$ km.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Try to have students generalize the conclusion about their findings.	• Note that #3 might best be addressed as a class discussion so that students share their strategies or models for solving. That way, struggling learners will find an approach they are comfortable with, and other students may build on them to formulate their own approach. • Encourage students to use algebraic methods alongside their model. • For #4, some students may have a difficult time starting if they are unable to form the equation. Coach them on how to break down each piece of the problem to develop their equation. For example, ask what $2t$ represents; then $3t$, then $2t + 0.50$, etc. • After students have written the equation $2t + 0.50 = 3t$, have them solve it in different ways, share their methods, choose their preferred method, and explain their choice. Have them explain their thinking.

Reflect and Check

3. a) How can you model solutions to equations with variables on both sides using manipulatives or diagrams?
   b) Explain other ways that you could model the solutions.
4. Laura and her greyhound, Dash, went home along a trail. Laura strolled at 2 km/h along the shortest route. Dash trotted along at 3 km/h but covered 0.50 km of extra distance by zigzagging. Suggest methods for determining the length of time they took to reach home together. Share your ideas with your classmates.

Link the Ideas

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The value of the quarters is $0.25(d - 30)$ dollars.
The value of the dimes equals the value of the quarters.
An equation that represents the situation is $0.10d = 0.25(d - 30)$.

$$\begin{aligned} 0.10d &= 0.25(d - 30) \\ 0.10d &= 0.25d - 7.5 \\ 0.10d - 0.25d &= 0.25d - 7.5 - 0.25d \\ -0.15d &= -7.5 \\ \frac{-0.15d}{-0.15} &= \frac{-7.5}{-0.15} \\ d &= 50 \end{aligned}$$

There are 50 dimes in the jar.

$\frac{-7.5}{-0.15} \approx \frac{-8}{-0.2} \approx 40$ ME

You could use fractions of dollars and write the equation as $\frac{1}{10}d = \frac{1}{4}(d - 30)$.

You could also use cents and write the equation as $10d = 25(d - 30)$

Check:
There are 30 fewer quarters than dimes.
$50 - 30 = 20$
There are 20 quarters in the jar.
Value of dimes: $50 \times \$0.10 = \$5.00$
Value of quarters: $20 \times \$0.25 = \$5.00$
The dimes and quarters have equal values, as stated in the problem.

Show You Know

In a jar of coins, there are 20 more nickels than quarters. The value of the nickels equals the value of the quarters. How many quarters are in the jar?

8.4 Solving Equations: $ax = b + cx$, $ax + b = cx + d$, $a(bx + c) = d(ex + f)$ • MHR 323

Example 2: Apply Equations of the Form $ax + b = cx + d$

Alain has \$35.50 and is saving \$4.25/week. Eva has \$24.25 and is saving \$5.50/week. In how many weeks from now will they have the same amount of money?

Solution

Let the number of weeks from now be w.
In w weeks, Alain will have $35.50 + 4.25w$ dollars.
In w weeks, Eva will have $24.25 + 5.50w$ dollars.
In w weeks, Alain and Eva will have the same amount of money.
An equation that describes the situation is $35.50 + 4.25w = 24.25 + 5.50w$.

Isolate the variable.

$$\begin{aligned} 35.50 + 4.25w &= 24.25 + 5.50w \\ 35.50 + 4.25w - 4.25w &= 24.25 + 5.50w - 4.25w \\ 35.50 &= 24.25 + 1.25w \\ 35.50 - 24.25 &= 24.25 + 1.25w - 24.25 \\ 11.25 &= 1.25w \\ \frac{11.25}{1.25} &= \frac{1.25w}{1.25} \\ 9 &= w \end{aligned}$$

Try a different first step than subtracting $4.25w$ from both sides. Complete your solution and check that you can get the correct final answer.

Alain and Eva will have the same amount of money in nine weeks.

Check:
In nine weeks, Alain will have $\$35.50 + 9 \times \$4.25$, or \$73.75.
In nine weeks, Eva will have $\$24.25 + 9 \times \$5.50$, or \$73.75.
So, they will have the same amount of money in nine weeks from now.
The solution is correct.

Show You Know

One Internet café charges \$1 for 15 min and \$0.20 per page for printing. A second Internet café charges \$2 per hour and \$0.25 per page for printing. Suppose you want to use the Internet for one hour. How many pages would you need to print in order to make the two cafés equal in price?

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Link the Ideas

Example 1

This example shows the writing and solving of an equation of the form $ax = b + cx$ in a monetary context. To promote understanding, you might ask:

- Why is the value of the dimes $0.10d$ dollars?
- Why is the value of the quarters $0.25(d - 30)$ dollars?
- Why does $0.25(d - 30) = 0.25d - 7.5$?

Next, you might refer students to the thought bubble in the solution and ask:

- Which of the three forms of the equation would you prefer to solve? Why?
- How can you check that solving each form of the equation gives the same value of d?

Emphasize the importance of the summary statement and the need to check that the calculated answer is consistent with the information given in the problem.

Example 2

This example shows the writing and solving of an equation of the form $ax + b = cx + d$ in a monetary context. Point out the thought bubble beside the solution, and have students solve the equation individually by trying a different first step.

Students may first choose to subtract 35.50, 24.25, or $5.50w$ from both sides. Check that students are able to obtain the correct answer. Ask if they have any preference about which step is performed first.

You might also ask:

- What would the equation be if you worked in cents instead of dollars?
- Do you have a preference for working in cents or dollars? Why?

Example 3

This example introduces equations with grouping symbols on both sides. To promote understanding, you might ask:

- Why are both sides of the equation multiplied by 6 in the first step?
- Is there an alternative first step you could use? Would the solution with this alternative first step be easier or harder than the solution shown? Why?
- Why are square brackets used in the expressions $\frac{1}{3}[2 \times (-1) - 1]$ and $\frac{1}{2}[3 \times (-1) + 1]$ in the check?
- How is the order of operations used in the check?

Example 3: Solve Equations of the Form $a(bx + c) = d(ex + f)$

Solve $\frac{1}{3}(2x - 1) = \frac{1}{2}(3x + 1)$ and check.

Solution

$$\begin{aligned}6 \times \frac{1}{3}(2x - 1) &= 6 \times \frac{1}{2}(3x + 1)\\ 2(2x - 1) &= 3(3x + 1)\\ 4x - 2 &= 9x + 3\\ 4x - 2 - 4x &= 9x + 3 - 4x\\ -2 &= 5x + 3\\ -2 - 3 &= 5x + 3 - 3\\ -5 &= 5x\\ \frac{-5}{5} &= \frac{5x}{5}\\ -1 &= x\end{aligned}$$

Check:

$$\begin{aligned}\text{Left Side} &= \frac{1}{3}(2x - 1) & \text{Right Side} &= \frac{1}{2}(3x + 1)\\ &= \frac{1}{3}[2(-1) - 1] & &= \frac{1}{2}[3(-1) + 1]\\ &= \frac{1}{3}(-2 - 1) & &= \frac{1}{2}(-3 + 1)\\ &= \frac{1}{3}(-3) & &= \frac{1}{2}(-2)\\ &= -1 & &= -1\end{aligned}$$

Left Side = Right Side

The solution, $x = -1$, is correct.

Show You Know

Solve $\frac{3f + 1}{4} = \frac{3 + 2f}{2}$ and check.

Key Ideas

- You can solve and check equations with variables on both sides by applying the algebraic techniques learned in earlier sections.

$$\begin{aligned}3(0.5t + 1.3) &= 2(0.4t - 0.85)\\ 1.5t + 3.9 &= 0.8t - 1.7\\ 1.5t + 3.9 - 0.8t &= 0.8t - 1.7 - 0.8t\\ 0.7t + 3.9 &= -1.7\\ 0.7t + 3.9 - 3.9 &= -1.7 - 3.9\\ 0.7t &= -5.6\\ \frac{0.7t}{0.7} &= \frac{-5.6}{0.7}\\ t &= -8\end{aligned}$$

Check:

$$\begin{aligned}\text{Left Side} &= 3(0.5t + 1.3) & \text{Right Side} &= 2(0.4t - 0.85)\\ &= 3[0.5(-8) + 1.3] & &= 2[0.4(-8) - 0.85]\\ &= 3(-4 + 1.3) & &= 2(-3.2 - 0.85)\\ &= 3(-2.7) & &= 2(-4.05)\\ &= -8.1 & &= -8.1\end{aligned}$$

Left Side = Right Side

The solution, $t = -8$, is correct.

8.4 Solving Equations: $ax = b + cx$, $ax + b = cx + d$, $a(bx + c) = d(ex + f)$ • MHR 325

Key Ideas

This section shows an algebraic solution method for an equation of the form $a(bx + c) = d(ex + f)$ involving rational numbers. This solution includes algebraic techniques that are used to solve the other types of equations introduced in section 8.4. Have students describe in their own words the steps used in the solution, including the steps in the check, and the reasoning behind each step.

Have students review the Key Ideas for any additional material they would like to add to their Foldable.

Meeting Student Needs

- Some students may benefit from changing all subtraction questions by adding the opposite before multiplying through by a leading coefficient.
- Some students will benefit from modelling the steps using whole numbers before advancing to decimals.
- A review of common denominators will be useful for some students before trying the fraction questions.
- Point out to students that when "removing" the x on one side, it should be moved to the side with the larger coefficient to avoid a $-x$.
- For the Show You Know in Example 1, encourage students to write the equation in whichever way they prefer: using decimals, fractions, or integers. Have students compare the equations they used and their solutions.
- For the Show You Know in Example 3, some students might not recognize $\frac{3f + 1}{4} = \frac{3 + 2f}{2}$ as being of the form $a(bx + c) = d(ex + f)$. You can clarify this for them by pointing out that this equation can be rewritten as $\frac{1}{4}(3f + 1) = \frac{1}{2}(3 + 2f)$. However, you should also make it clear that it is not necessary to rewrite the equation in this form in order to solve it.

ELL

- Very new Canadians will not know the name of the coins yet. Clarify the value of each type of coin. Provide them with **BLM 8–5 Canadian Coins and Their Values**.

Answers

Example 1: Show You Know

There would be five quarters.

Example 2: Show You Know

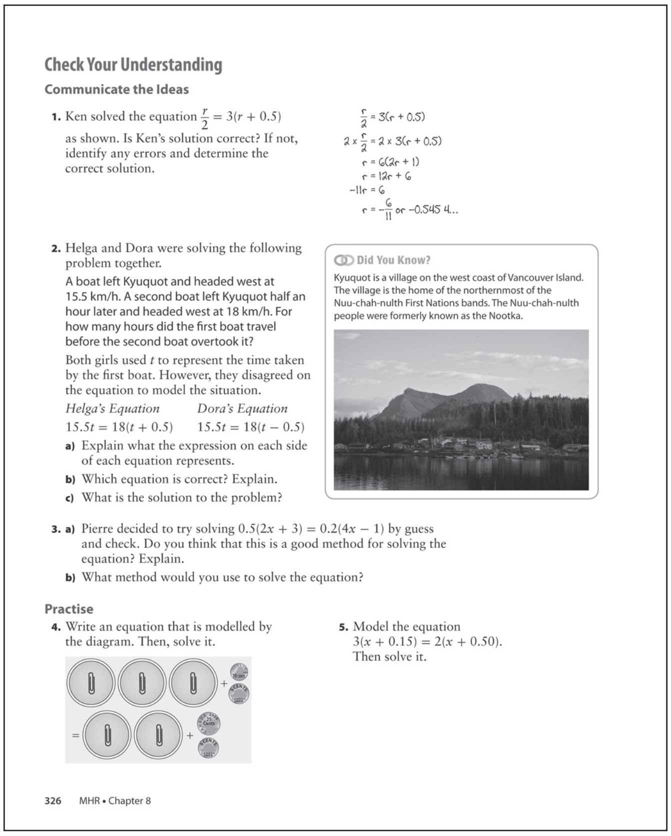
$$\begin{aligned}4 + 0.20x &= 2 + 0.25x\\ 2 &= 0.05x\\ 2(20) &= x\end{aligned}$$

The cost would be equal if you had to print 40 pages.

Example 3: Show You Know

$f = -5$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may require further coaching to complete the Show You Know. Suggest that students refer back to the solution presented in Example 1 and use the same type of scaffolding approach for designing the equation. Coach students through each step and ask them to draw the parallels between the Show You Know values and the Example 1 values.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may require further coaching to complete the Show You Know. As above, suggest that students refer back to the solution presented in Example 2 and scaffold in the same way to design their own equation.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Most students will find it easier to work with integer values. Ask prompting questions to coach students in eliminating the fractions. • Remind students that they will need to isolate the variable to one side and complete the order of operations in reverse. • Remind students of the importance of checking their solution when they are done to verify that the left side and right side are equal. • Once students have removed the fractions, some students may benefit from using a model to solve the questions. Provide whatever materials that are needed.



Check Your Understanding

Communicate the Ideas

1. Ken solved the equation $\frac{r}{2} = 3(r + 0.5)$ as shown. Is Ken's solution correct? If not, identify any errors and determine the correct solution.

$\frac{r}{2} = 3(r + 0.5)$
$2 \times \frac{r}{2} = 2 \times 3(r + 0.5)$
$r = 6(2r + 1)$
$r = 12r + 6$
$-11r = 6$
$r = -\frac{6}{11}$ or $-0.545\ 4...$

2. Helga and Dora were solving the following problem together.

A boat left Kyuquot and headed west at 15.5 km/h. A second boat left Kyuquot half an hour later and headed west at 18 km/h. For how many hours did the first boat travel before the second boat overtook it?

Both girls used t to represent the time taken by the first boat. However, they disagreed on the equation to model the situation.

Helga's Equation $15.5t = 18(t + 0.5)$
Dora's Equation $15.5t = 18(t - 0.5)$

a) Explain what the expression on each side of each equation represents.
b) Which equation is correct? Explain.
c) What is the solution to the problem?

Did You Know?
Kyuquot is a village on the west coast of Vancouver Island. The village is the home of the northernmost of the Nuu-chah-nulth First Nations bands. The Nuu-chah-nulth people were formerly known as the Nootka.

3. a) Pierre decided to try solving $0.5(2x + 3) = 0.2(4x - 1)$ by guess and check. Do you think that this is a good method for solving the equation? Explain.
b) What method would you use to solve the equation?

Practise

4. Write an equation that is modelled by the diagram. Then, solve it.

5. Model the equation $3(x + 0.15) = 2(x + 0.50)$. Then solve it.

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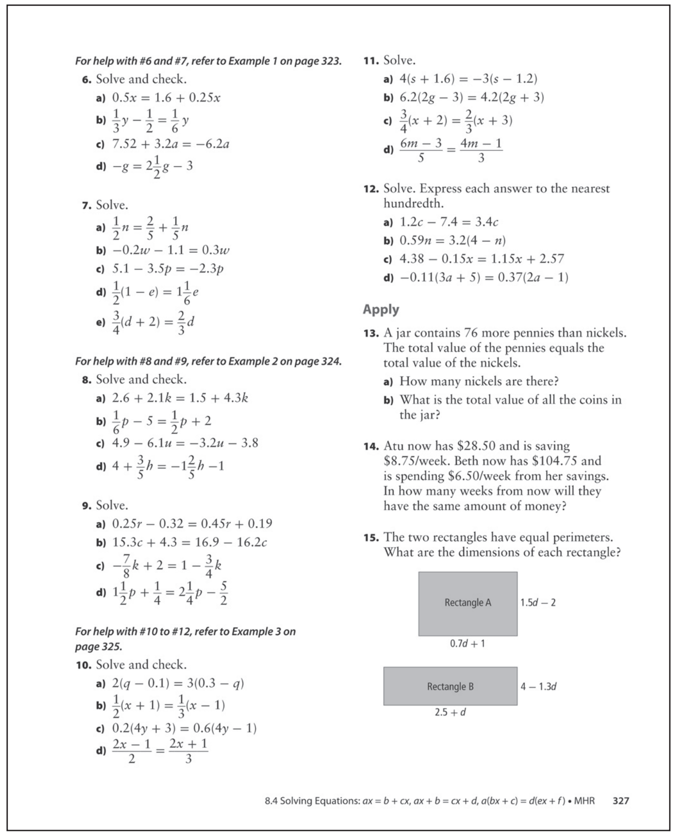


For help with #6 and #7, refer to Example 1 on page 323.

6. Solve and check.
a) $0.5x = 1.6 + 0.25x$
b) $\frac{1}{3}y - \frac{1}{2} = \frac{1}{6}y$
c) $7.52 + 3.2a = -6.2a$
d) $-g = 2\frac{1}{2}g - 3$

7. Solve.
a) $\frac{1}{2}n = \frac{2}{5} + \frac{1}{5}n$
b) $-0.2w - 1.1 = 0.3w$
c) $5.1 - 3.5p = -2.3p$
d) $\frac{1}{2}(1 - e) = 1\frac{1}{6}e$
e) $\frac{3}{4}(d + 2) = \frac{2}{3}d$

For help with #8 and #9, refer to Example 2 on page 324.

8. Solve and check.
a) $2.6 + 2.1k = 1.5 + 4.3k$
b) $\frac{1}{6}p - 5 = \frac{1}{2}p + 2$
c) $4.9 - 6.1u = -3.2u - 3.8$
d) $4 + \frac{3}{5}h = -1\frac{2}{5}h - 1$

9. Solve.
a) $0.25r - 0.32 = 0.45r + 0.19$
b) $15.3c + 4.3 = 16.9 - 16.2c$
c) $-\frac{7}{8}k + 2 = 1 - \frac{3}{4}k$
d) $1\frac{1}{2}p + \frac{1}{4} = 2\frac{1}{4}p - \frac{5}{2}$

For help with #10 to #12, refer to Example 3 on page 325.

10. Solve and check.
a) $2(q - 0.1) = 3(0.3 - q)$
b) $\frac{1}{2}(x + 1) = \frac{1}{3}(x - 1)$
c) $0.2(4y + 3) = 0.6(4y - 1)$
d) $\frac{2x - 1}{2} = \frac{2x + 1}{3}$

11. Solve.
a) $4(s + 1.6) = -3(s - 1.2)$
b) $6.2(2g - 3) = 4.2(2g + 3)$
c) $\frac{3}{4}(x + 2) = \frac{2}{3}(x + 3)$
d) $\frac{6m - 3}{5} = \frac{4m - 1}{3}$

12. Solve. Express each answer to the nearest hundredth.
a) $1.2c - 7.4 = 3.4c$
b) $0.59n = 3.2(4 - n)$
c) $4.38 - 0.15x = 1.15x + 2.57$
d) $-0.11(3a + 5) = 0.37(2a - 1)$

Apply

13. A jar contains 76 more pennies than nickels. The total value of the pennies equals the total value of the nickels.
a) How many nickels are there?
b) What is the total value of all the coins in the jar?

14. Atu now has \$28.50 and is saving \$8.75/week. Beth now has \$104.75 and is spending \$6.50/week from her savings. In how many weeks from now will they have the same amount of money?

15. The two rectangles have equal perimeters. What are the dimensions of each rectangle?

Rectangle A: $1.5d - 2$ by $0.7d + 1$
Rectangle B: $4 - 1.3d$ by $2.5 + d$

8.4 Solving Equations: $ax = b + cx$, $ax + b = cx + d$, $a(bx + c) = d(ex + f)$ • MHR 327

Check Your Understanding

Communicate the Ideas

These questions allow students to explain aspects of solving equations with variables on both sides using rational numbers.

In #1, you might ask students how they can show that there must be an error in the reasoning before they determine what it is. They can show by substitution that the solution is incorrect. You might ask whether they would check by substituting for r, using the fraction $-\frac{6}{11}$ or the decimal $-0.545\ 4\ldots$. Have them explain their preference.

Before students complete #2, refer them to the Did You Know? beside it to provide additional context for the problem. After students read the problem, you might poll the class to determine how many students agree with each equation. Have students explain and justify their choice and point out any errors in their classmates' reasoning.

In #3, encourage discussion. This will serve to make the point that, though the use of certain a strategy may eventually solve a given problem, there may be more efficient options. In this case, a Guess and Check approach is likely to take much longer than an algebraic solution.

Practise

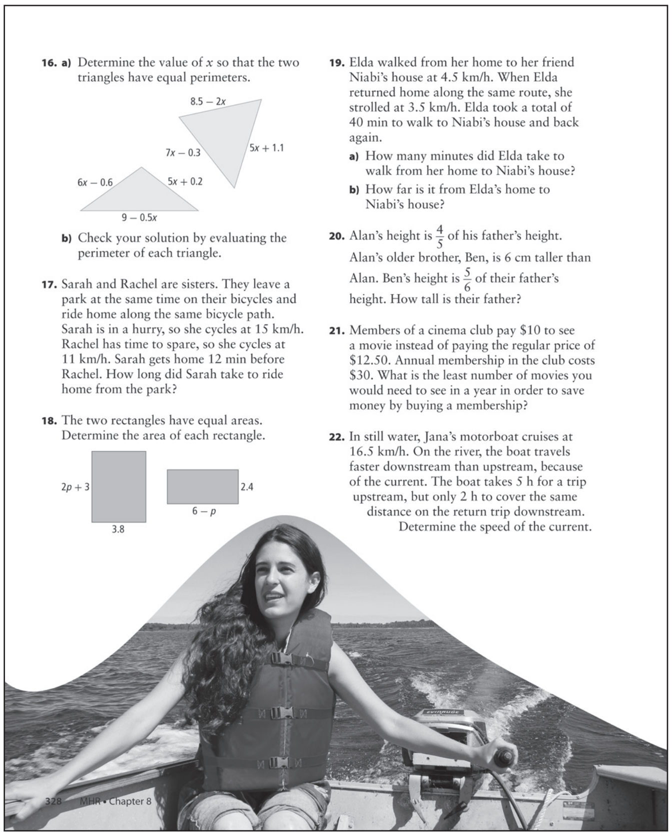
You may wish to have students complete the Practise questions individually, in pairs, or in small groups. Encourage students to compare their solution methods, check each other's answers, and suggest corrections or other improvements.

For parts of #9 to 11, you might consider having students complete solutions jointly with a partner. The first partner could write the first line of the solution for the second partner to check. Then, the partners could switch roles for the second line of the solution. They would continue to switch roles until all lines of the solution, including the check by substitution, were completed.

Apply

Encourage students to suggest and discuss alternative solutions for the same problem. For example, #13 readily lends itself to a Guess and Check approach as well as to an algebraic solution. Students may solve #14 by making a table.

The easiest way to write a word problem in #25 is to model it on a previous problem, such as #13. Emphasize that students must check that they can solve their own problem before giving it to a classmate to solve. Encourage students to use original contexts. You may wish to share particularly creative problems with the whole class.



16. a) Determine the value of x so that the two triangles have equal perimeters.

b) Check your solution by evaluating the perimeter of each triangle.

17. Sarah and Rachel are sisters. They leave a park at the same time on their bicycles and ride home along the same bicycle path. Sarah is in a hurry, so she cycles at 15 km/h. Rachel has time to spare, so she cycles at 11 km/h. Sarah gets home 12 min before Rachel. How long did Sarah take to ride home from the park?

18. The two rectangles have equal areas. Determine the area of each rectangle.

19. Elda walked from her home to her friend Niabi's house at 4.5 km/h. When Elda returned home along the same route, she strolled at 3.5 km/h. Elda took a total of 40 min to walk to Niabi's house and back again.

a) How many minutes did Elda take to walk from her home to Niabi's house?

b) How far is it from Elda's home to Niabi's house?

20. Alan's height is $\frac{4}{5}$ of his father's height. Alan's older brother, Ben, is 6 cm taller than Alan. Ben's height is $\frac{5}{6}$ of their father's height. How tall is their father?

21. Members of a cinema club pay \$10 to see a movie instead of paying the regular price of \$12.50. Annual membership in the club costs \$30. What is the least number of movies you would need to see in a year in order to save money by buying a membership?

22. In still water, Jana's motorboat cruises at 16.5 km/h. On the river, the boat travels faster downstream than upstream, because of the current. The boat takes 5 h for a trip upstream, but only 2 h to cover the same distance on the return trip downstream. Determine the speed of the current.

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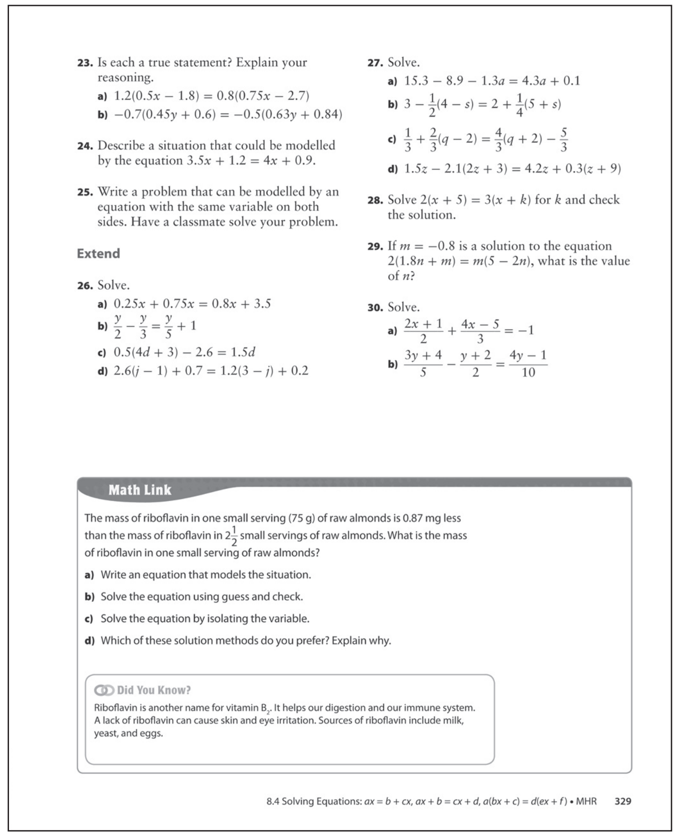

23. Is each a true statement? Explain your reasoning.

a) $1.2(0.5x - 1.8) = 0.8(0.75x - 2.7)$

b) $-0.7(0.45y + 0.6) = -0.5(0.63y + 0.84)$

24. Describe a situation that could be modelled by the equation $3.5x + 1.2 = 4x + 0.9$.

25. Write a problem that can be modelled by an equation with the same variable on both sides. Have a classmate solve your problem.

Extend

26. Solve.

a) $0.25x + 0.75x = 0.8x + 3.5$

b) $\frac{y}{2} - \frac{y}{3} = \frac{y}{5} + 1$

c) $0.5(4d + 3) - 2.6 = 1.5d$

d) $2.6(j - 1) + 0.7 = 1.2(3 - j) + 0.2$

27. Solve.

a) $15.3 - 8.9 - 1.3a = 4.3a + 0.1$

b) $3 - \frac{1}{2}(4 - s) = 2 + \frac{1}{4}(5 + s)$

c) $\frac{1}{3} + \frac{2}{3}(q - 2) = \frac{4}{3}(q + 2) - \frac{5}{3}$

d) $1.5z - 2.1(2z + 3) = 4.2z + 0.3(z + 9)$

28. Solve $2(x + 5) = 3(x + k)$ for k and check the solution.

29. If $m = -0.8$ is a solution to the equation $2(1.8n + m) = m(5 - 2n)$, what is the value of n?

30. Solve.

a) $\frac{2x + 1}{2} + \frac{4x - 5}{3} = -1$

b) $\frac{3y + 4}{5} - \frac{y + 2}{2} = \frac{4y - 1}{10}$

Math Link

The mass of riboflavin in one small serving (75 g) of raw almonds is 0.87 mg less than the mass of riboflavin in $2\frac{1}{2}$ small servings of raw almonds. What is the mass of riboflavin in one small serving of raw almonds?

a) Write an equation that models the situation.

b) Solve the equation using guess and check.

c) Solve the equation by isolating the variable.

d) Which of these solution methods do you prefer? Explain why.

Did You Know?

Riboflavin is another name for vitamin B_2. It helps our digestion and our immune system. A lack of riboflavin can cause skin and eye irritation. Sources of riboflavin include milk, yeast, and eggs.

8.4 Solving Equations: $ax = b + cx$, $ax + b = cx + d$, $a(bx + c) = d(ex + f)$ • MHR 329

Extend

In #26 and #27, students need to perform additional steps in solving equations. You might have them discuss ways of solving the equations most easily.

In #28, some students may use different approaches (e.g., using the distributive property first or dividing both sides by 3 first). Their solutions may appear to be different. For example, $k = \frac{-x + 10}{3}$ and $k = \frac{2(x + 5)}{3}$. Challenge students to show that correct solutions that look different are actually the same.

In #29, students are likely to substitute the value of m into the given equation, and then solve for n. You might challenge them to try the alternative method of solving the given equation for n, and then substituting the value of m. You might ask students which approach would be more efficient if they were being asked to determine the value of n for several different values of m.

If students have difficulty with #30, you might ask if the solution to Example 3 suggests a possible approach.

Literacy Link Remind students to complete the fourth oval in their concept map, which is labelled *with variables on both sides*. They should provide an example of the equation form, and outline the steps required to solve the equation, using a strategy of their choice.

Math Link

In this Math Link, students can apply their skills in writing and solving equations with variables on both sides to the field of nutrition. Check that students include the appropriate units in their answers for parts b) and c). Encourage students to state and explain their opinions in a discussion of part d).

Draw students' attention to the Did You Know? within the Math Link. You may wish to have students do some research on dietary sources of other B vitamins, or of other vitamins in general, and on the functions of these vitamins in the body. Students already encountered vitamin B_1, thiamin, in the Math Link at the end of section 8.3.

Meeting Student Needs

- Provide **BLM 8–12 Section 8.4 Extra Practice** to students who would benefit from more practice.
- If students need help with the algebraic solution to #14, refer them to Example 2, but point out that the two situations are a little different.
- In #20, some students may find it helpful to draw a diagram.
- If students need assistance with #23, you might ask questions involving simpler statements first. For example:
  - Is the equation $5 \times 5 = 25$ a true statement? Why or why not?
  - Is the equation $3x - x = x + x$ a true statement? Why or why not?
  - Is the equation $x + 2 = x + 3$ a true statement? Why or why not?

ELL

- Have English language learners partner with English-speaking students to answer the questions in the Check Your Understanding section.
- Clarify the meaning of the questions for English language learners by drawing pictures and rephrasing the questions.
- Note that #6 to 12 provide an opportunity to check if English language learners understand the math concepts. If they are successful with these questions and not the word problems, it probably still means they are getting stuck on the language in the math questions. If, however, students are having problems on questions #6 to 12, it will be necessary to coach them on these concepts.

Gifted and Enrichment

- Ask students to create an unsolvable equation with a variable on both sides of the equal sign. Have them analyse and report on characteristics of an unsolvable equation.

Common Errors

- Students may be unable to describe an appropriate situation in #24.

R_x Ask students to adapt the wording in #14, or the dimensions of the rectangles in #15, to create a situation that could be modelled by the equation in #24.

Answers

Communicate the Ideas

1. No, his solution is incorrect. He should have multiplied the 2 by only the 3 in the second step, not by what is in brackets as well.

2. **a)** Speed multiplied by time equals distance; therefore, the expression on each side of each equation equals the distance before the second boat overtook the first.
b) Dora's is correct.
c) $t = 3.6$ h, or 216 min

3. **a)** No, this method could be very time-consuming.
b) Example: An algebraic method

Math Link

a) $x = \frac{5}{2}x - 0.87$

c) $x = 0.58$

d) Example: The algebraic method because it is quicker.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1–3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • All three questions provide students with an opportunity to check their understanding of all the different approaches used in solving the equations in this section. • Students may have difficulty with #1 because it may not be completed in a method of their choosing. Encourage students to complete #1 on their own and compare their solution with the one in the student resource. • Note that #2 requires students to identify whether Helga or Dora has set up an equation correctly. Direct learners who are finding this question challenging to the scaffolding approach that was used in Examples 1 and 2. • Note that #3 identifies whether students can handle brackets on both sides of the equation. It may help some students to simplify each side separately and then isolate the variable. Have them verbalize the order they will use to complete the question, prior to solving. • You may wish to have students assess each other's responses to one or more of the questions using **Master 2 Communication Peer Evaluation**.
Assessment *for* Learning	
Practise and Apply Have students do #4–6, 8, 10, 14, and 15. Students who have no problems with these questions can go on to the remaining Apply questions.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Note that #4 and 5 offer students opposite presentations in solving equations with variables on both sides. Encourage them to look between the two questions to help complete each. For #5, tell students to use whatever model they find easier to work with. • For #6, 8, and 10, remind students that they wish to have the variable isolated on one side and that they will use opposite operations to achieve this. Coach them through the opposite operations in each question and have them recognize that they are the order of operations in reverse. Have students complete parts of #7, 9, and 11 to check their understanding. • For # 14, encourage students to use the scaffolding approach modelled in Example 2. Have them verbalize the similarities between the example values and the ones in #14. • For students having difficulty with #15, review the meaning of the word *perimeter*. Have them redraw the rectangles and label all sides. Students may find it easier to write out the total expression for the perimeter of each rectangle, simplify them, and then equate them.
Math Link The Math Link on page 329 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 333.	• Students who need help getting started could use **BLM 8–13 Section 8.4 Math Link**, which provides scaffolding. • Ensure students understand this process and make use of the scaffolding as this question will serve as a model for the Wrap It Up! at the end of the chapter.
Assessment *as* Learning	
Literacy Link At the end of this section, have students work in pairs to complete the last oval of the concept map, entitled *with variables on both sides*.	• After completing this section, brainstorm and discuss as a class the information needed to complete this oval. • Have students revisit the concept map before they do the practice test. When it is finished, it should provide a useful overview of the important concepts covered in this chapter.
Math Learning Log Have students respond to the following prompts: • When solving equations, the method I prefer is … because… • When writing equations, the method I find easiest to understand is … An example of this is …	• Students should be encouraged to identify multiple methods they are comfortable using in solving equations; however, one solid method is sufficient. • Encourage students to use an equation that they have written on their own and identify the steps they use to write the equation. Remind them that their steps will be a useful reminder when for review purposes. They may also wish to include these steps in their Foldable.