Representing Inequalities

9.1

***MathLinks 9*, pages 340–349**

Suggested Timing
50–60 minutes

Materials
- grid paper
- ruler
- coloured pencils, straws, or small wood sticks

Blackline Masters
Master 2 Communication Peer Evaluation
BLM 9–3 Chapter 9 Warm-Up
BLM 9–5 Section 9.1 Extra Practice
BLM 9–6 Section 9.1 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

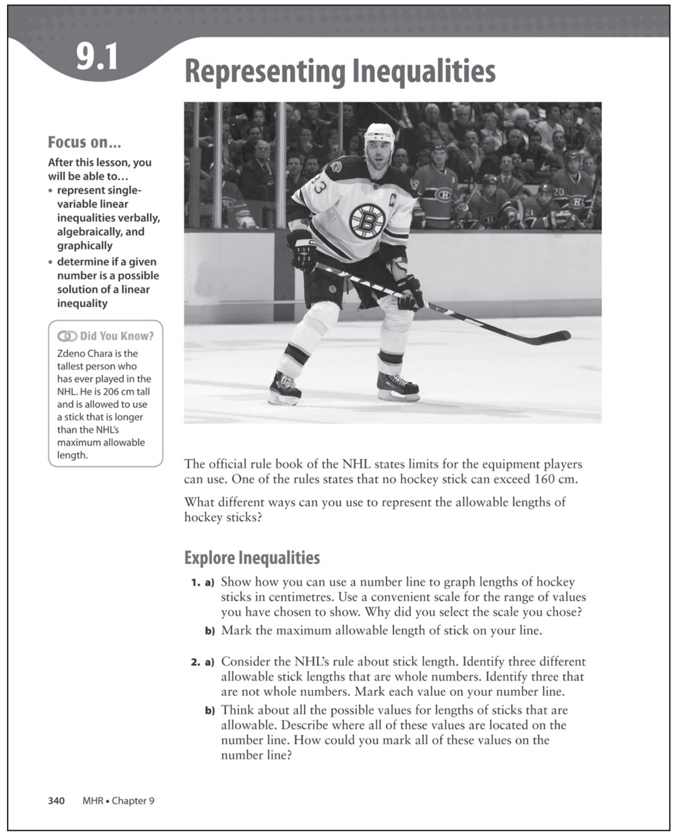
Specific Outcomes
PR4 Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5–7, 9, 10b), 11, 13, 15, 17, Math Link
Typical	#1–3, 5–7, 9, 10b), 11, 16, 19, 21, Math Link
Extension/Enrichment	#2–4, 8, 10, 13, 23–25, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce material learned in previous sections.

As a class, discuss the photo, taken in Duncan, B.C., and ask students why they think the NHL needs to have a rule for measuring hockey sticks. You might want to ask students to share similar types of rules for other sports or activities that they are involved in.



9.1 Representing Inequalities

Focus on…
After this lesson, you will be able to…
- represent single-variable linear inequalities verbally, algebraically, and graphically
- determine if a given number is a possible solution of a linear inequality

Did You Know?
Zdeno Chara is the tallest person who has ever played in the NHL. He is 206 cm tall and is allowed to use a stick that is longer than the NHL's maximum allowable length.

The official rule book of the NHL states limits for the equipment players can use. One of the rules states that no hockey stick can exceed 160 cm.

What different ways can you use to represent the allowable lengths of hockey sticks?

Explore Inequalities

1. a) Show how you can use a number line to graph lengths of hockey sticks in centimetres. Use a convenient scale for the range of values you have chosen to show. Why did you select the scale you chose?
   b) Mark the maximum allowable length of stick on your line.

2. a) Consider the NHL's rule about stick length. Identify three different allowable stick lengths that are whole numbers. Identify three that are not whole numbers. Mark each value on your number line.
   b) Think about all the possible values for lengths of sticks that are allowable. Describe where all of these values are located on the number line. How could you mark all of these values on the number line?

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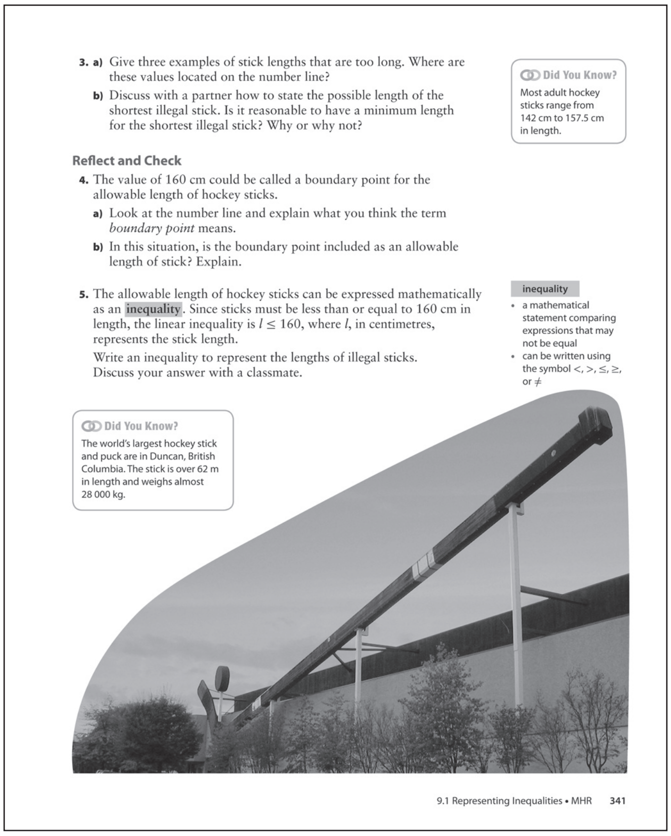
Explore Inequalities

In Explore Inequalities, students will examine how to visually represent a situation that involves an inequality.

Method 1 Have students work independently to design a number line and complete the questions. Circulate as they work on the activity. As students plan out number lines in #1, note how they have set up their number lines. Students may not choose a range of values and a scale that are convenient to work with—many might start by including a labelled tick mark for every integer value in the range they have chosen, for example. You might help them by asking the following questions:
- What range of values is relevant to show in this situation?
- What scale could be used with the range of values you have chosen?
- How can the number line be labelled to help show the values and information effectively?
- Is it necessary to label every tick mark on your number line?



3. a) Give three examples of stick lengths that are too long. Where are these values located on the number line?

   b) Discuss with a partner how to state the possible length of the shortest illegal stick. Is it reasonable to have a minimum length for the shortest illegal stick? Why or why not?

Did You Know?
Most adult hockey sticks range from 142 cm to 157.5 cm in length.

Reflect and Check

4. The value of 160 cm could be called a boundary point for the allowable length of hockey sticks.

   a) Look at the number line and explain what you think the term *boundary point* means.

   b) In this situation, is the boundary point included as an allowable length of stick? Explain.

5. The allowable length of hockey sticks can be expressed mathematically as an **inequality**. Since sticks must be less than or equal to 160 cm in length, the linear inequality is $l \leq 160$, where l, in centimetres, represents the stick length.

   Write an inequality to represent the lengths of illegal sticks. Discuss your answer with a classmate.

inequality
- a mathematical statement comparing expressions that may not be equal
- can be written using the symbol $<$, $>$, $\leq$, $\geq$, or $\neq$

Did You Know?
The world's largest hockey stick and puck are in Duncan, British Columbia. The stick is over 62 m in length and weighs almost 28 000 kg.

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As students plot the lengths for legal sticks in #2, encourage them to consider where all these values are in relation to the maximum allowable length. They need to see that the values are all on one side of 160 cm, and consider how they might show all of these values on their line. You might ask these questions:

- Where on your number line are the values of stick lengths that are legal?
- Where would all of the illegal stick lengths be?
- How could you show *all* possible lengths of legal sticks on your number line?

In #4 students are asked to consider what they think the term *boundary point* means, even though this term is not formally defined until later in the section. You might help them by using the following prompts:

- The term has two words. Can you break it down to help think about what it might mean?
- What does the word *boundary* mean? Is there a value that is a *boundary* in this situation?

After working through most or all of Explore Inequalities, draw students' attention to the table with examples of inequalities just below the Link the Ideas on page 342. Students will likely be familiar with most or all of the possible inequality signs. Encourage them to develop their own method for understanding what the different statements mean, rather than just memorizing the various signs.

After looking at the various inequality statements on this page, perhaps ask students why the definition for inequality says "may not be equal" rather than "are not equal." They should understand that the signs $\leq$ and $\geq$ include equivalence along with inequality.

You may wish to discuss the Did You Know? at the bottom of page 341 with students. How could they use an inequality to express the length of this stick? the mass?

Method 2 Alternatively, you might have students work in pairs or small groups. As each pair or group progresses through Explore Inequalities, students might be encouraged to check with another pair or group to see alternative methods or lines of thinking.

Meeting Student Needs

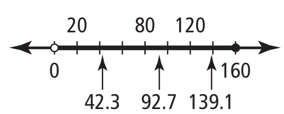
- Encourage students who need concrete representations to model inequalities with algebra tiles or blocks on a balance. The "heavier" side (greater than) can be shown as being lower down to emphasize the fact that the two sides are unequal.

Answers

Explore Inequalities

1. a), b)

2. a)



   b) All of the possible values are greater than 0 cm and less than or equal to 160 cm.

3. a) Example: The following three stick lengths would be too long: 175 cm, 190 cm, and 210 cm. All of these lengths would fall right of 160 cm on the number line.

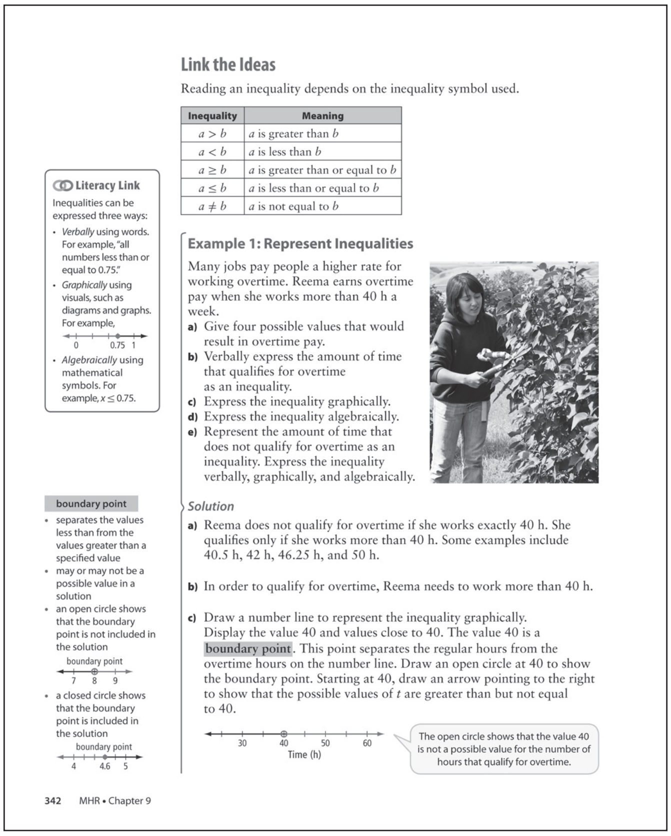
   b) Example: The shortest illegal stick would measure just over 160 cm. The length cannot be accurately shown.

4. a) Example: The value of 160 cm represents the boundary between acceptable lengths of hockey sticks to the left on the number line and unacceptable lengths to the right.

   b) Yes, the value of 160 cm is included because it is the maximum length allowable.

5. $l > 160$

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Try to have students explain the importance of the boundary point and how it links to an inequality.	• Some students may find it helpful to review solving an equation that has a whole-number solution. Have students verbalize what the solution means. • Using the same solution, replace the equality with an inequality sign and ask students to verbalize how the answer has changed. • It may be useful for some students to create the chart showing a description of the inequality signs in their Foldable for future reference.



Link the Ideas

Reading an inequality depends on the inequality symbol used.

Inequality	Meaning
$a > b$	a is greater than b
$a < b$	a is less than b
$a \geq b$	a is greater than or equal to b
$a \leq b$	a is less than or equal to b
$a \neq b$	a is not equal to b

Literacy Link

Inequalities can be expressed three ways:

- *Verbally* using words. For example, "all numbers less than or equal to 0.75."
- *Graphically* using visuals, such as diagrams and graphs. For example,
- *Algebraically* using mathematical symbols. For example, $x \leq 0.75$.

Example 1: Represent Inequalities

Many jobs pay people a higher rate for working overtime. Reema earns overtime pay when she works more than 40 h a week.

a) Give four possible values that would result in overtime pay.

b) Verbally express the amount of time that qualifies for overtime as an inequality.

c) Express the inequality graphically.

d) Express the inequality algebraically.

e) Represent the amount of time that does not qualify for overtime as an inequality. Express the inequality verbally, graphically, and algebraically.

Solution

a) Reema does not qualify for overtime if she works exactly 40 h. She qualifies only if she works more than 40 h. Some examples include 40.5 h, 42 h, 46.25 h, and 50 h.

b) In order to qualify for overtime, Reema needs to work more than 40 h.

c) Draw a number line to represent the inequality graphically. Display the value 40 and values close to 40. The value 40 is a boundary point. This point separates the regular hours from the overtime hours on the number line. Draw an open circle at 40 to show the boundary point. Starting at 40, draw an arrow pointing to the right to show that the possible values of t are greater than but not equal to 40.

The open circle shows that the value 40 is not a possible value for the number of hours that qualify for overtime.

boundary point

- separates the values less than from the values greater than a specified value
- may or may not be a possible value in a solution
- an open circle shows that the boundary point is not included in the solution
- a closed circle shows that the boundary point is included in the solution

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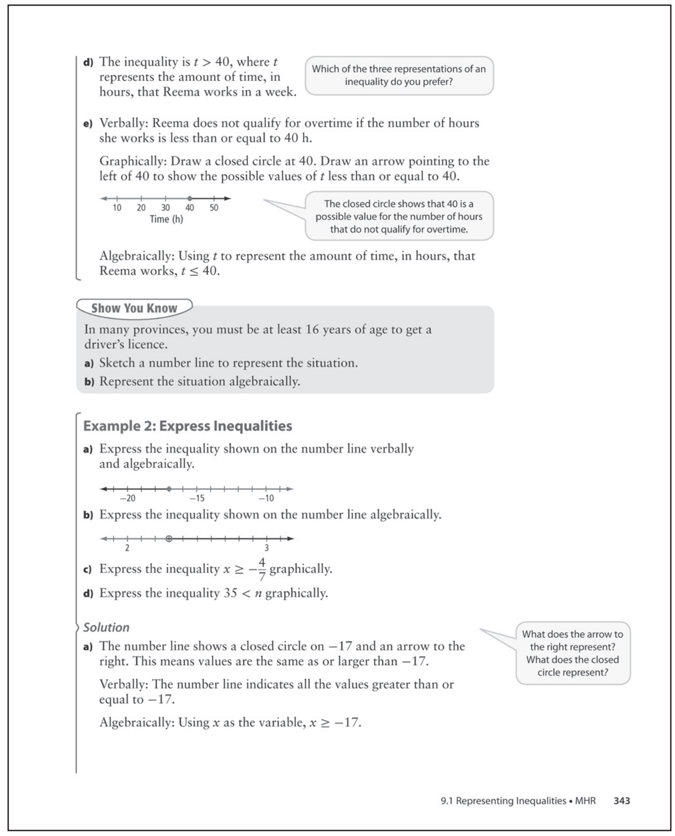

d) The inequality is $t > 40$, where t represents the amount of time, in hours, that Reema works in a week.

Which of the three representations of an inequality do you prefer?

e) Verbally: Reema does not qualify for overtime if the number of hours she works is less than or equal to 40 h.

Graphically: Draw a closed circle at 40. Draw an arrow pointing to the left of 40 to show the possible values of t less than or equal to 40.

The closed circle shows that 40 is a possible value for the number of hours that do not qualify for overtime.

Algebraically: Using t to represent the amount of time, in hours, that Reema works, $t \leq 40$.

Show You Know

In many provinces, you must be at least 16 years of age to get a driver's licence.

a) Sketch a number line to represent the situation.

b) Represent the situation algebraically.

Example 2: Express Inequalities

a) Express the inequality shown on the number line verbally and algebraically.

b) Express the inequality shown on the number line algebraically.

c) Express the inequality $x \geq -\frac{4}{7}$ graphically.

d) Express the inequality $35 < n$ graphically.

Solution

a) The number line shows a closed circle on −17 and an arrow to the right. This means values are the same as or larger than −17.

What does the arrow to the right represent? What does the closed circle represent?

Verbally: The number line indicates all the values greater than or equal to −17.

Algebraically: Using x as the variable, $x \geq -17$.

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Link the Ideas

If you did not have students discuss the table with examples of inequalities earlier, do it now. Students will likely be familiar with most or all of the possible inequality signs.

Literacy Link Make sure students read and understand the Literacy Link on page 342. The link defines and illustrates the three ways used in this chapter to represent the inequalities. Students can represent inequalities graphically with a number line, which is a one-dimensional graph. Representing algebraically with symbols, students can use numbers, variables, and operation signs. These words are used in Example 1. Use these words with students when asking questions; encourage your students to use these words as they work through this chapter.

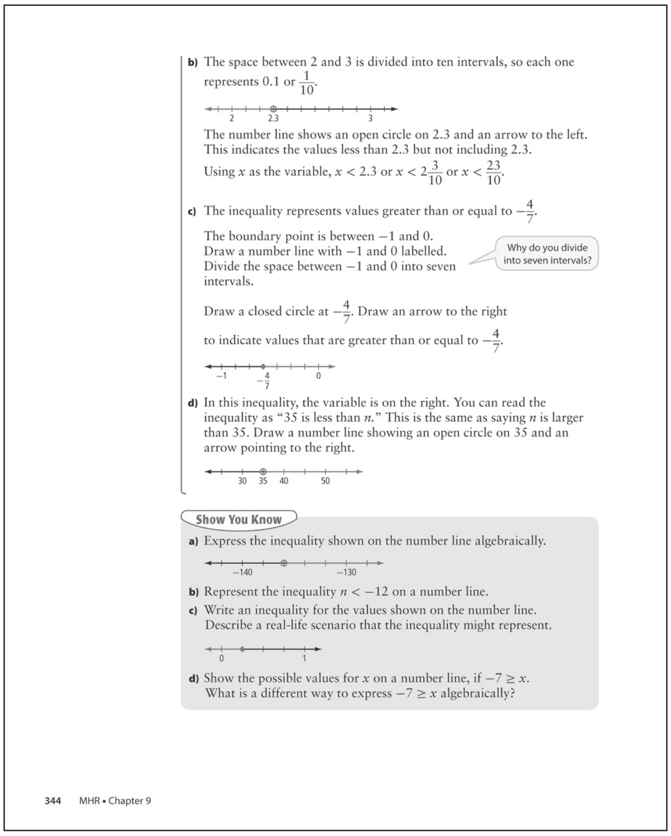
Example 1

This example uses a real-world situation to illustrate the three different ways inequalities are represented in this chapter. You might start by having students read the problem and discuss what their understanding of *overtime* pay is. Have students work in pairs or small groups to analyse what is being shown in the example. Ask questions such as the following:

- Which inequality sign should be used in this situation? What key words might indicate this?
- Why is an open circle used at 40 when showing the overtime hours?
- Why is the number line shaded above 40?
- Why is a closed circle used at 40 when showing the non-overtime hours?
- What is 40 a boundary between?
- What is the connection between the type of inequality sign and the type of circle used on a number line?

The solution for c) refers to the term *boundary point*. The margin definition outlines the use of open and closed circles on number lines to indicate whether the boundary point is included or not. Read this definition as a class and discuss the concept. How can students remember this idea?

As students complete the Show You Know, they might think of ages as discrete values (i.e., a person is 14 until the day he or she turns 15). Help by pointing out that age can also be thought of as continuous: a person can be $15\frac{1}{2}$ or 16.37 years old, etc.



b) The space between 2 and 3 is divided into ten intervals, so each one represents 0.1 or $\frac{1}{10}$.

The number line shows an open circle on 2.3 and an arrow to the left. This indicates the values less than 2.3 but not including 2.3.

Using x as the variable, $x < 2.3$ or $x < 2\frac{3}{10}$ or $x < \frac{23}{10}$.

c) The inequality represents values greater than or equal to $-\frac{4}{7}$.

The boundary point is between −1 and 0. Draw a number line with −1 and 0 labelled. Divide the space between −1 and 0 into seven intervals.

Draw a closed circle at $-\frac{4}{7}$. Draw an arrow to the right to indicate values that are greater than or equal to $-\frac{4}{7}$.

d) In this inequality, the variable is on the right. You can read the inequality as "35 is less than *n*." This is the same as saying *n* is larger than 35. Draw a number line showing an open circle on 35 and an arrow pointing to the right.

Show You Know

a) Express the inequality shown on the number line algebraically.

b) Represent the inequality $n < -12$ on a number line.

c) Write an inequality for the values shown on the number line. Describe a real-life scenario that the inequality might represent.

d) Show the possible values for x on a number line, if $-7 \geq x$. What is a different way to express $-7 \geq x$ algebraically?

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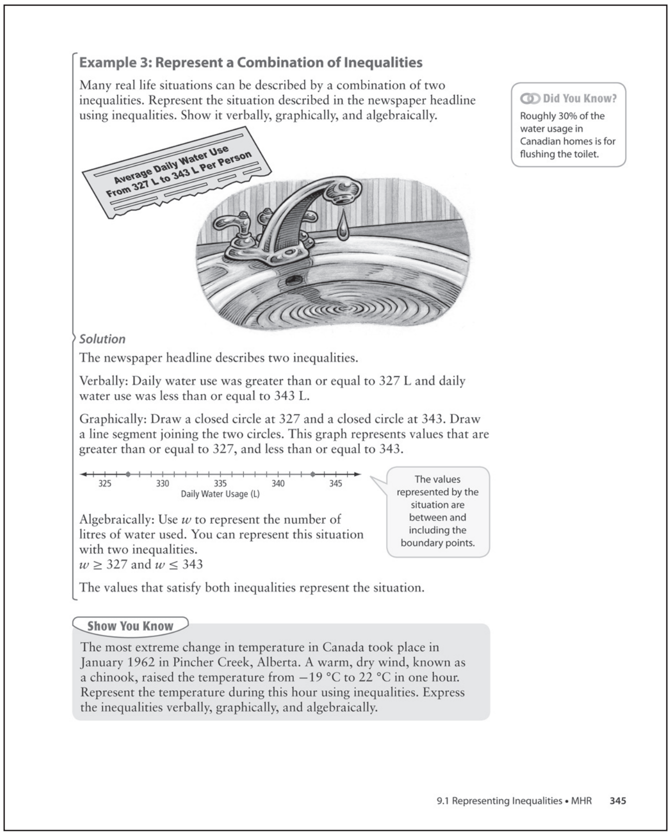

Example 3: Represent a Combination of Inequalities

Many real life situations can be described by a combination of two inequalities. Represent the situation described in the newspaper headline using inequalities. Show it verbally, graphically, and algebraically.

Did You Know?

Roughly 30% of the water usage in Canadian homes is for flushing the toilet.

Solution

The newspaper headline describes two inequalities.

Verbally: Daily water use was greater than or equal to 327 L and daily water use was less than or equal to 343 L.

Graphically: Draw a closed circle at 327 and a closed circle at 343. Draw a line segment joining the two circles. This graph represents values that are greater than or equal to 327, and less than or equal to 343.

Algebraically: Use w to represent the number of litres of water used. You can represent this situation with two inequalities.
$w \geq 327$ and $w \leq 343$

The values that satisfy both inequalities represent the situation.

Show You Know

The most extreme change in temperature in Canada took place in January 1962 in Pincher Creek, Alberta. A warm, dry wind, known as a chinook, raised the temperature from −19 °C to 22 °C in one hour. Represent the temperature during this hour using inequalities. Express the inequalities verbally, graphically, and algebraically.

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Example 2

This example helps strengthen the connection between graphical and algebraic representations of inequalities. When representing inequalities in various ways, observe that students have made the connection between the direction of the arrow and type of circle when representing graphically, the inequality sign used when representing algebraically, and the words used when expressing verbally. Ask students questions such as the following:

- How does the direction of the arrow relate to the inequality symbol used?
- How does the type of circle relate to the inequality symbol used?

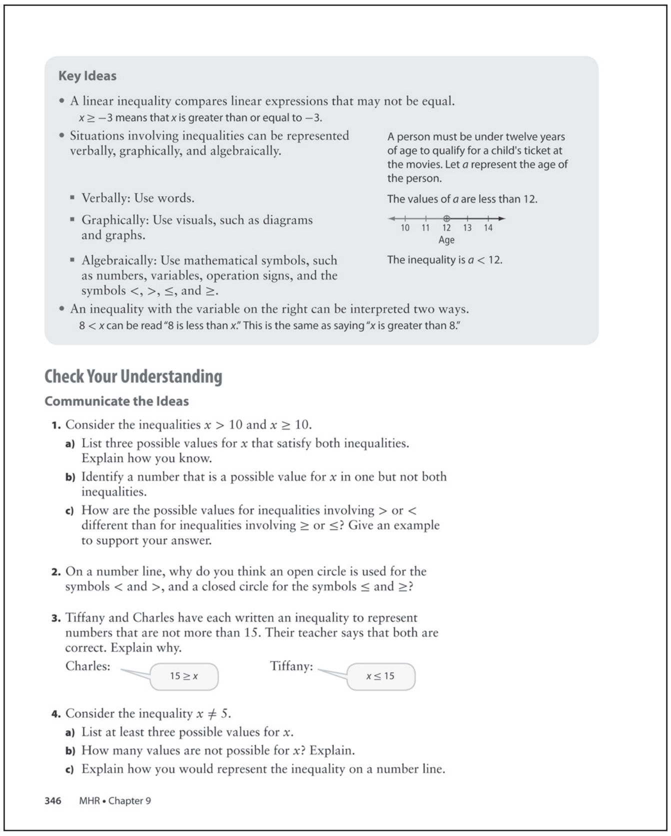
In part c), draw students' attention to the thought bubble beside the solution. Ensure that they can answer this question for themselves. You might ask these questions:

- Which part of the fraction might tell you how many subintervals you need?
- How can you decide how many intervals you need if you have to show a fractional value on a number line?

Students can work through the inequality in part d) to reinforce working with variables on the right. Have students read the inequality starting from the variable: $35 < n$ can be read as "*n* is greater than 35." Help students see that $35 < n$ and $n > 35$ are equivalent statements, even though the sign is the opposite way in each. This realization might help students understand that the meaning of an inequality sign depends on whether it is read left to right or right to left. This concept is critical before students go on to Example 3 involving combinations of inequalities. As students consider part d), ask them questions such as:

- What is different about this inequality?
- How can it be expressed verbally (when read left to right)?
- What if we read this inequality from right to left—what does it say?
- What does an inequality sign mean if it is read backward (from right to left)?
- How could this inequality be rewritten with the variable on the left so that it has the same meaning?

As students complete the Show You Know, check that they can identify the fractional value that is required in part c) and ensure that they correctly interpret the inequality with the variable on the right in part d).



Key Ideas

- A linear inequality compares linear expressions that may not be equal.
  $x \geq -3$ means that x is greater than or equal to -3.
- Situations involving inequalities can be represented verbally, graphically, and algebraically.
  A person must be under twelve years of age to qualify for a child's ticket at the movies. Let a represent the age of the person.
  - Verbally: Use words. — The values of a are less than 12.
  - Graphically: Use visuals, such as diagrams and graphs.

  - Algebraically: Use mathematical symbols, such as numbers, variables, operation signs, and the symbols $<$, $>$, $\leq$, and $\geq$. — The inequality is $a < 12$.
- An inequality with the variable on the right can be interpreted two ways.
  $8 < x$ can be read "8 is less than x." This is the same as saying "x is greater than 8."

Check Your Understanding

Communicate the Ideas

1. Consider the inequalities $x > 10$ and $x \geq 10$.
   a) List three possible values for x that satisfy both inequalities. Explain how you know.
   b) Identify a number that is a possible value for x in one but not both inequalities.
   c) How are the possible values for inequalities involving $>$ or $<$ different than for inequalities involving $\geq$ or $\leq$? Give an example to support your answer.
2. On a number line, why do you think an open circle is used for the symbols $<$ and $>$, and a closed circle for the symbols $\leq$ and $\geq$?
3. Tiffany and Charles have each written an inequality to represent numbers that are not more than 15. Their teacher says that both are correct. Explain why.
   Charles: $15 \geq x$ Tiffany: $x \leq 15$
4. Consider the inequality $x \neq 5$.
   a) List at least three possible values for x.
   b) How many values are not possible for x? Explain.
   c) Explain how you would represent the inequality on a number line.

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Example 3

This example asks students to consider a combination of inequalities, where there are two conditions given for a variable. If time permits, you might initiate a discussion of water use. Students might be surprised at how much water is used for various things in their homes, such as toilets and laundry! As students analyse the example, help them by asking a sequence of questions including the following:

- What conditions are given on the variable?
- How many conditions are there?
- How is this different than other inequalities you have looked at so far?
- How many boundary points are there?
- Where are all the possible values for average water use located in relation to the boundary points?
- How might this affect what the number line would look like?
- Why is the inequality in this situation called a combination of inequalities?

As students complete the Show You Know, after expressing the situation with an inequality verbally, they might choose whether they want to first use symbols to represent the situation and then draw the number line, or show it graphically first and use that to help them express it algebraically.

Key Ideas

The Key Ideas reinforce three different ways to represent inequalities: verbally, graphically, and algebraically. Have students make notes in their own words that outline the connections between the three methods. The Key Ideas also specifically show two concepts students might find more difficult: inequalities with the variable on the right and a combination of inequalities. To ensure that students understand the concepts in this section, you might ask them the following questions:

- How is the sign in an inequality expressed algebraically related to its graphical and verbal representations?
- How can you decide if an open or closed circle should be used?
- How can you decide which way the arrow on a number line should point?
- Why do some inequalities have an arrow pointing in one direction, while others have a line between two boundary points?

Have students represent the inequalities in the first, third, and fourth bullets verbally, graphically, and algebraically. These examples could be placed with their Foldable notes for this section.

Meeting Student Needs

- Consider reactivating students' prior knowledge of inequality symbols by having them write statements comparing pairs of values using $>$ or $<$. You might ask pairs of students to compare their ages, locker numbers, student numbers, or other values.
- Some students may need to develop a mnemonic to help them remember which symbol means *more than* and which means *less than*. Discuss possible memory devices with them, such as remembering that the *less than* symbol points to lower numbers on the number line and the *more than* symbol points to higher numbers.
- Some students may not be familiar with the symbols $\geq$ or $\leq$. Help them see that these can be thought of as combinations of two other symbols; for example, the symbol $\geq$ is like a combination of $>$ and $=$.
- Some students may benefit from drawing a simple sketch of a number line for an inequality before writing it algebraically (even if they are not asked or required to graph it).
- Some students may have difficulty with the concept of inequality. Coach these students through the

process of solving some simple equations such as: $2x + 4 = 8$ or $x - 6 = 8$. Showing students that there is only one exact answer could be a step to introducing the idea of an inequality having more that one solution.

ELL

- Ensure students understand the term *overtime*.

Gifted and Enrichment

- Have these students use information from the Web Links that follow to develop their own inequality scenarios for Example 3. They can exchange them and solve each other's scenarios.

Common Errors

- Students may incorrectly interpret inequalities that have the variable on the right.

R_x Have students read the inequality starting from the variable: $6 > x$ can be read as "x is less than six," starting from the right. Encourage students to rewrite these inequalities backwards with the inequality on the left before trying to interpret them.

- Some students may have difficulty with remembering which way the inequality sign goes.

R_x Refer students to the chart they placed into their Foldable. An alternative rule that may help some students is to remember that the "*L*'s go together," for the terms *left* and *less than*.

- Some students forget which form of graphing includes the boundary point and which does not.

R_x Some sayings that are helpful reminders include "on the mark" or "on the spot," literal terms for right on the value, which can be used to remember what a closed circle represents. Open circles can be described as "around but not on," implying that the value is not included.

WWW Web Link

For more information on water usage, go to www.mathlinks9.ca and follow the links.

For more examples of extreme weather information, go to www.mathlinks9.ca and follow the links.

Answers

Example 1: Show You Know

a)

b) $A \geq 16$

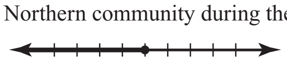
Example 2: Show You Know

a) $n > -136$

b)

c) $t \leq 0.25$. Example: The range of temperatures, in Celsius, in a Northern community during the month of January.

d)



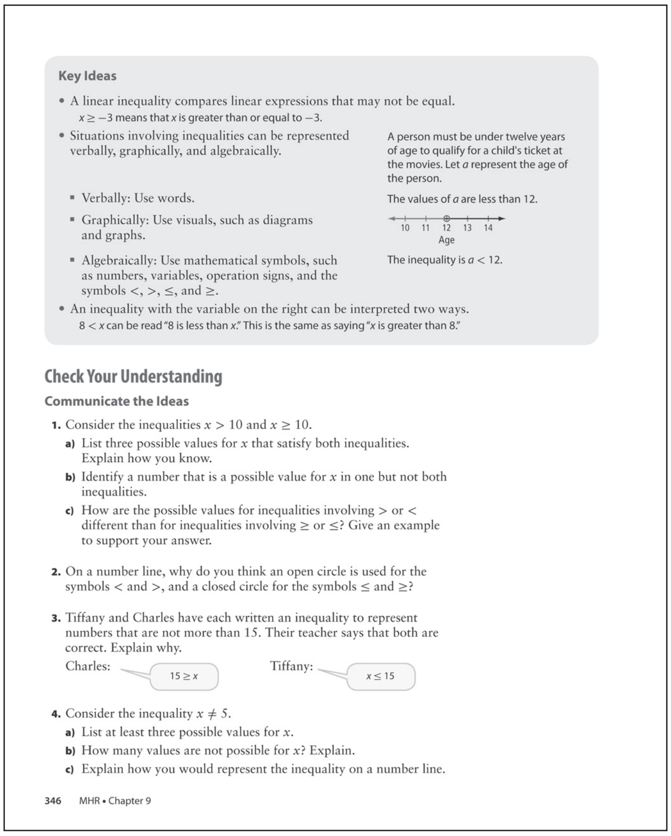
Example: $x \leq -7$

Example 3: Show You Know

Example: The temperature during that hour was greater than or equal to –19 °C and less than or equal to 22 °C.

$t \geq -19$ and $t \leq 22$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Ensure that students can verbalize and write algebraically the difference between an equation and an inequality. Some students may benefit from including examples of each in their Foldable.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students need extra experience with inequalities that have variables on the right side. Review the equivalent forms.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from additional coaching on combinations of inequalities. Review how to determine which value is included and which is not.



Key Ideas

- A linear inequality compares linear expressions that may not be equal.
  $x \geq -3$ means that x is greater than or equal to -3.
- Situations involving inequalities can be represented verbally, graphically, and algebraically.
  A person must be under twelve years of age to qualify for a child's ticket at the movies. Let a represent the age of the person.
  - Verbally: Use words.
    The values of a are less than 12.
  - Graphically: Use visuals, such as diagrams and graphs.
    Age
  - Algebraically: Use mathematical symbols, such as numbers, variables, operation signs, and the symbols $<$, $>$, $\leq$, and $\geq$.
    The inequality is $a < 12$.
- An inequality with the variable on the right can be interpreted two ways.
  $8 < x$ can be read "8 is less than x." This is the same as saying "x is greater than 8."

Check Your Understanding

Communicate the Ideas

1. Consider the inequalities $x > 10$ and $x \geq 10$.
   a) List three possible values for x that satisfy both inequalities. Explain how you know.
   b) Identify a number that is a possible value for x in one but not both inequalities.
   c) How are the possible values for inequalities involving $>$ or $<$ different than for inequalities involving $\geq$ or $\leq$? Give an example to support your answer.
2. On a number line, why do you think an open circle is used for the symbols $<$ and $>$, and a closed circle for the symbols $\leq$ and $\geq$?
3. Tiffany and Charles have each written an inequality to represent numbers that are not more than 15. Their teacher says that both are correct. Explain why.
   Charles: $15 \geq x$ Tiffany: $x \leq 15$
4. Consider the inequality $x \neq 5$.
   a) List at least three possible values for x.
   b) How many values are not possible for x? Explain.
   c) Explain how you would represent the inequality on a number line.

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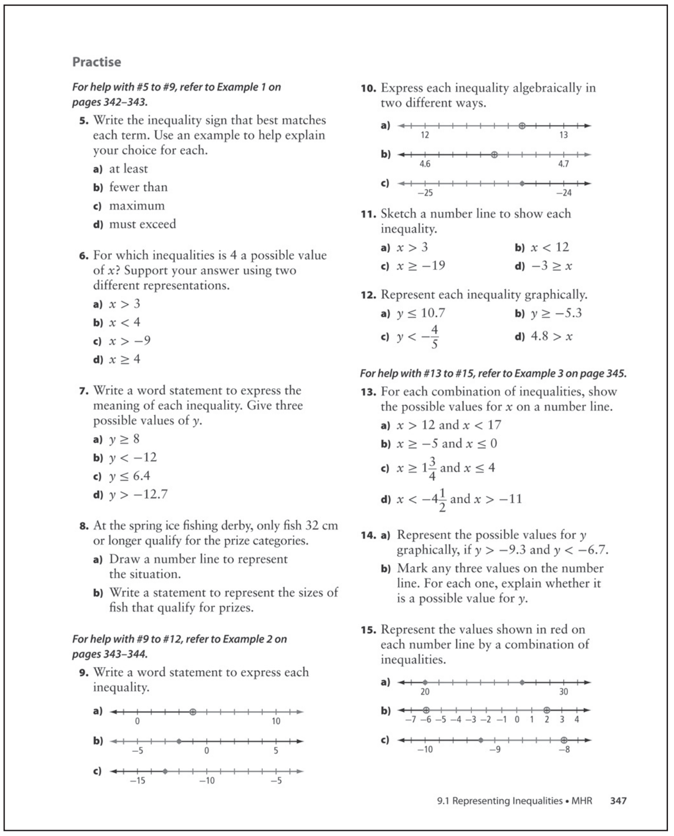

Practise

For help with #5 to #9, refer to Example 1 on pages 342–343.

5. Write the inequality sign that best matches each term. Use an example to help explain your choice for each.
   a) at least
   b) fewer than
   c) maximum
   d) must exceed
6. For which inequalities is 4 a possible value of x? Support your answer using two different representations.
   a) $x > 3$
   b) $x < 4$
   c) $x > -9$
   d) $x \geq 4$
7. Write a word statement to express the meaning of each inequality. Give three possible values of y.
   a) $y \geq 8$
   b) $y < -12$
   c) $y \leq 6.4$
   d) $y > -12.7$
8. At the spring ice fishing derby, only fish 32 cm or longer qualify for the prize categories.
   a) Draw a number line to represent the situation.
   b) Write a statement to represent the sizes of fish that qualify for prizes.

For help with #9 to #12, refer to Example 2 on pages 343–344.

9. Write a word statement to express each inequality.
   a)
   b)
   c)
10. Express each inequality algebraically in two different ways.
    a)
    b)
    c)
11. Sketch a number line to show each inequality.
    a) $x > 3$ b) $x < 12$
    c) $x \geq -19$ d) $-3 \geq x$
12. Represent each inequality graphically.
    a) $y \leq 10.7$ b) $y \geq -5.3$
    c) $y < -\frac{4}{5}$ d) $4.8 > x$

For help with #13 to #15, refer to Example 3 on page 345.

13. For each combination of inequalities, show the possible values for x on a number line.
    a) $x > 12$ and $x < 17$
    b) $x \geq -5$ and $x \leq 0$
    c) $x \geq 1\frac{3}{4}$ and $x \leq 4$
    d) $x < -4\frac{1}{2}$ and $x > -11$
14. a) Represent the possible values for y graphically, if $y > -9.3$ and $y < -6.7$.
    b) Mark any three values on the number line. For each one, explain whether it is a possible value for y.
15. Represent the values shown in red on each number line by a combination of inequalities.
    a)
    b)
    c)

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Check Your Understanding

Communicate the Ideas

Note whether students understand the difference between $>$ and $\geq$ in #1.

In #2, students are asked to explain why an open circle is used with *greater/less than*. Students might identify that it is because an open circle is hollow or empty to show that the value is not included.

Use students' responses to #3 to assess whether they can interpret an inequality with the variable on the right.

As students complete #4, you might help them by having them first think about how $x = 5$ might be shown on a number line.

Practise

The Practise questions focus on a variety of skills and concepts. In #5 to 7, students are asked to connect everyday expressions to corresponding inequality signs, and identify possible values of the variable, given an inequality.

For #9, students represent everyday situations with inequalities.

A critical skill students need is the ability to represent inequalities using any of the three methods in this chapter; students can practise these skills in #9 to 12.

Combinations of inequalities might prove to be a challenging concept for students. They can check their understanding in questions #13, 14, and 15.

Apply

The Apply section focuses on representing real-world situations using inequalities. Encourage students to create carefully labelled and titled number lines when representing graphically, and to choose a variable when representing algebraically.

For #16 to 18, have students consider the scenarios and how inequalities might be used to communicate information in a real-life situation.

For #19, students might develop their own scenario and inequality, then exchange it with a partner and solve each other's questions. You may wish to have students use the first Web Link that follows to research film classifications in Canada for their inequalities.

For #20, you may wish to have students replace the coupon on the student resource with one from a local restaurant.

Apply

16. The manager of a clothing store has set goals for her sales staff. Express each goal algebraically.

- **a)** The monthly total sales, m, will be a minimum of $18 000.
- **b)** At month end, the total time, t, spent counting store inventory will be at most 8 h.
- **c)** The value of total daily sales, d, will be more than $700.

17. If Emily keeps a daily balance of at least $1500 in her bank account, she will pay no monthly fees.

- **a)** Draw a number line to represent the situation.
- **b)** If x represents her daily balance, write an inequality that represents the possible values for x when she will pay no fees.

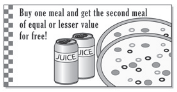
18. Paul is training for a race and hopes to beat the record time. The number line represents the finishing times that will allow him to beat the record.

- **a)** Write a statement to express the finishing times that will let Paul beat the record.
- **b)** Express the inequality algebraically.

19. **a)** Develop a problem that could be represented by an inequality. Express the inequality verbally.

- **b)** Graph the inequality.
- **c)** Express the inequality algebraically.

20. Owen has a coupon for a restaurant.



- **a)** Owen buys a meal for $10.75. If m is the cost of his second meal, write an inequality to represent the possible values of m that will allow him to use the coupon.
- **b)** Represent the inequality graphically.

21. Shanelle is buying insurance for a car to drive to and from work. The cost of insurance will be higher if she works farther than 15 km from home.

- **a)** Verbally express the inequality that represents the possible values for the distance for which Shanelle will have to pay more insurance.
- **b)** Sketch a number line to represent the inequality.

22. During winter, ice roads allow access to remote places in northern communities. The ice road to Aklavik, NWT is made through the Mackenzie River Delta. The ice road to Tuktoyuktuk travels up the Mackenzie River and out onto the sea ice. Ice roads are made by flooding the existing ice on a river or lake until it reaches the required thickness.

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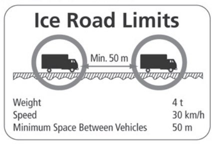
For safety reasons, there are restrictions such as the ones shown.



Represent each restriction

- **a)** graphically
- **b)** algebraically

Literacy Link
A metric tonne (t) is a measurement of mass that equals 1000 kg.

Extend

23. **a)** If the inequalities $x \geq 6$ and $x \leq 6$ are both true, describe the possible values for x.

- **b)** What would a number line showing possible values of x look like for this situation? Justify your answer.

24. Bluesky is building a wooden puzzle triangle. She has cut two sides that measure 30 cm and 80 cm, respectively. The longest side of the triangle is 80 cm. Write inequalities to represent the possible lengths for the third side of the triangle.

25. What values of x would each of the following combinations of inequalities represent? Explain verbally and show graphically.

- **a)** $x > 4$ and $x < 7$
- **b)** $x < 4$ and $x < 7$
- **c)** $x > 4$ and $x > 7$
- **d)** $x < 4$ and $x > 7$

Math Link

For safety reasons, some amusement park rides have age and height restrictions for riders.

- **a)** Choose an amusement park ride that you have seen or design one of your own. Describe your ride.
- **b)** For your ride, consider the safety restrictions or conditions that you might impose on riders. List at least three restrictions. Use terms of your choice.
- **c)** Represent each restriction algebraically using a different variable for each.
- **d)** Sketch a sign. Use words and graphics that clearly inform riders about each of your restrictions.

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For #22, have students locate Aklavik, Tuktoyuktuk, the Mackenzie River, and the Mackenzie River Delta on a map. They can start with the Dettah Ice Road, Great Slave Lake, N.W.T., the road pictured in the text.

Extend

The Extend section offers a variety of higher-level thinking opportunities for students.

In #23, students consider a value that satisfies two inequalities; in this case, there is only one such value.

Question #24 involves a combination of inequalities, but the values involved are not given in the problem and need to be determined. Students might benefit from using physical objects such as pencils or straws cut to the lengths given in the problem—this might help them determine the range of values for the third side.

Question #25 presents students with various combinations of inequalities involving the same values but different inequalities. This question gives students an opportunity to see how the various inequalities compare.

Literacy Link Using their concept map, have students attach an oval to the Definitions oval for each term they learned in this section. For the Expressing Inequalities oval, have students attach three ovals and use an example to show three different ways to express an inequality. For the Boundary Points oval, have students attach two ovals and draw an example of a boundary point showing an open circle and a closed circle.

Math Link

As students complete the Math Link, encourage them to represent restrictions they come up with using all three methods they used in this section. Students are bound to consider restrictions pertaining to age, height, weight (mass), etc. Encourage them to express these in an appropriate manner. The restrictions that students develop might be realistic or not—either is fine, as the focus is on how to represent them using the methods they looked at in this section.

You may wish to have students use the second Web Link that follows to research current ride restrictions.

Meeting Student Needs

- Provide **BLM 9–5 Section 9.1 Extra Practice** to students who would benefit from more practice.

ELL

- Ensure students understand the term *insurance*.

Gifted and Enrichment

- Challenge students to complete all of the Extend questions, and create their own questions with solutions.

Web Link

For information about film classifications in Canada, go to www.mathlinks9.ca and follow the links.

For information about restrictions for rides in Western Canada, go to www.mathlinks9.ca and follow the links.

Answers

Communicate the Ideas

1. a) Example: The following three numbers satisfy both inequalities because they are larger than the boundary value: 11, 12, and 15.

b) 10. The boundary value is not included in the inequality, $x > 10$, but it is included in the inequality, $x \geq 10$.

c) Inequalities involving $>$ or $<$ do not include the boundary value whereas inequalities with $\geq$ or $\leq$ do include the boundary value. For example, $x < 7$ does not include the boundary value of 7 whereas $x \leq 7$ does include the boundary value.

2. Example: The open circle indicates that the boundary value is not included but that the inequality includes all values near to the boundary value on the number line.

3. Example: Charles's expression can be read as "15 is greater than or equal to x" which is equivalent to Tiffany's expression that can be read as "x is less than or equal to 15."

4. a) Example: Three possible values for x are 3, 7, and 9.2.

b) Five is the only value that is not possible because the inequality includes all values except 5.

c) Draw a number line with an open circle at 5.

Math Link

Check that student answers include:
- a description of the ride
- safety restrictions or conditions
- an algebraic representation of each restriction
- a different variable for each restriction
- a sketch of a sign
- words and graphics that clearly communicate the restrictions

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1, 2, and 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. Students may use **Master 2 Communication Peer Evaluation** to assess each other's responses to the Communicate the Ideas questions. • Students who need assistance with #1 and 2 may need additional coaching on Examples 1 and 2. Making use of the chart in their Foldable may also benefit these students. • Review Example 3 and the examples in their Foldable with students who need assistance with #3.
Assessment *for* Learning	
Practise Have students do #6–7, 9a)–b), 10a)–b), 11a)–b), 12c)–d), 13a)–b), 16–17. Students who have no problems with these questions can go on to the rest of the Apply questions.	• Students working with #6 and 7 might benefit from considering what a number line for each part would look like, as well as by looking back at Exploring Inequalities and Example 1. • Before working on #9 to 12, students may benefit from reviewing the related Examples and the Key Ideas. • In #9, students have an opportunity to demonstrate their graphing skills. Provide coaching to students who make errors, and then have them try #10 on their own. A similar approach can be used for #11. Provide coaching to students who make errors, and then have them try questions from #12 based on the type of error made in #10. • Encourage students to start on #13, share their work with other students, and then complete #13.
Math Link The Math Link on page 349 is intended to help students work toward the chapter problem wrap-up titled MathLink: Wrap It Up! on page 371.	• Encourage students to find ways of expressing inequalities verbally, graphically, and algebraically. Think about ways that height restrictions are presented in public venues. Brainstorm as a class. • Have photographs of amusement parks on hand to stimulate discussions, and get students thinking about different kinds of amusement park rides. • Students who need help getting started could use **BLM 9–6 Section 9.1 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Literacy Link (page 337) By the end of section 9.1, have students fill in the concept map for Definitions, Expressing Inequalities, and Boundary Points.	• Some students may benefit from attaching another oval to each of the ovals that contains a term and writing in the definition of that term. • For Definitions, some students who need more space may benefit from attaching another oval to each of the ovals that contains a term and writing in the definition of that term. • For Expressing Inequalities, you might allow students to use a variation of an existing example, and then represent it three different ways. • For Boundary Points, encourage students to express the solution to the inequality for each boundary point shown on the number line. • Consider having students draw a number line showing the solution to an equation and to a related inequality.
Math Learning Log Have students answer the following question: • How can situations involving inequalities be represented in different ways?	• Encourage students to add definitions from this section to their Foldable. Advise them to record notes, examples, and Key Ideas also. • Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. • Some students may need a prompt to get started. Give them a sample, such as $5 < x$, to stimulate their thinking.