Solving Single-Step Inequalities

9.2

***MathLinks 9*, pages 350–359**

Suggested Timing
50–60 minutes

Materials
- long strip of paper or number line
- ruler
- two different-coloured tokens or markers
- algebra tiles
- coloured counters

Blackline Masters
Master 2 Communication Peer Evaluation
BLM 9–3 Chapter 9 Warm-Up
BLM 9–7 Chapter 9 Number Line
BLM 9–8 Section 9.2 Extra Practice
BLM 9–9 Section 9.2 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

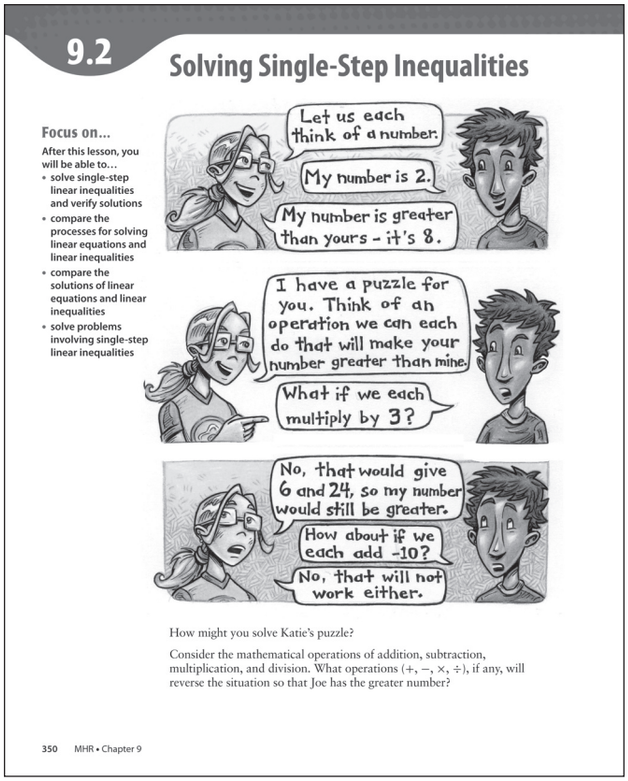
Specific Outcomes
PR4 Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 5a), 5d), 6a), 6b), 7a), 7d), 8, 9, 12, 14, 16a), 17
Typical	#1–4, 5a), 5d), 6a), 6b), 7a), 7d), 8, 9, 12, 14, 16, 17, 18
Extension/Enrichment	#1–4, 5c), 5d), 6b), 6c), 13, 17, 19–25

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce material learned in previous sections.

Tell students that the focus of this section is on solving one-step inequalities, which will involve many of the same solving skills used to solve one-step equations.


9.2 Solving Single-Step Inequalities

Focus on...
After this lesson, you will be able to...
- solve single-step linear inequalities and verify solutions
- compare the processes for solving linear equations and linear inequalities
- compare the solutions of linear equations and linear inequalities
- solve problems involving single-step linear inequalities

How might you solve Katie's puzzle?

Consider the mathematical operations of addition, subtraction, multiplication, and division. What operations (+, −, ×, ÷), if any, will reverse the situation so that Joe has the greater number?

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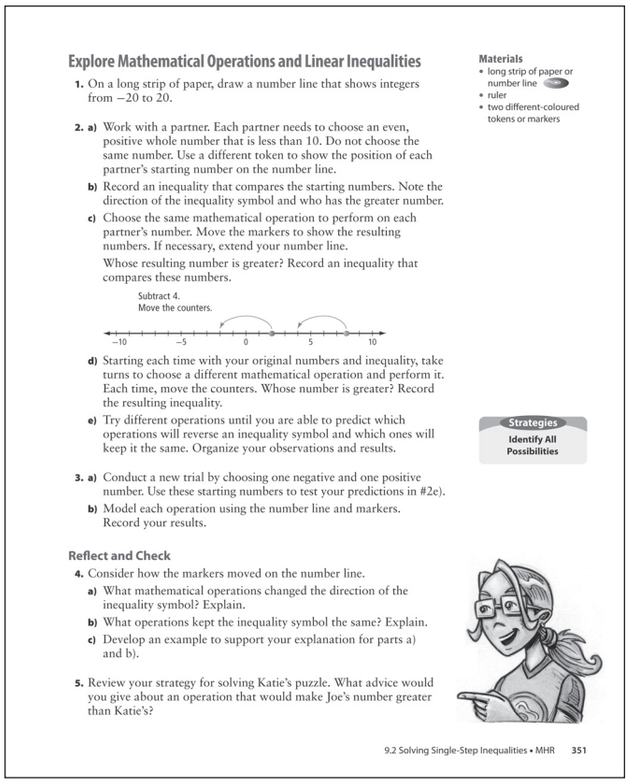
Explore Mathematical Operations and Linear Inequalities

Students investigate the results of various operations when performed on both sides of an inequality.

Method 1 Have students work in pairs to complete the Explore. Each pair needs to make a number line using a long strip of paper as described in the student resource, or students could be provided with **BLM 9–7 Chapter 9 Number Line**.

Have students read the discussion in the cartoon between Katie and Joe. Encourage students to model the operations described in the cartoon to help understand the activity. Circulate to help ensure each pair is able to get started.

Students need to think about how they plan to organize their results. Many students may design a table to record their findings. Some may need coaching. Ask:
- What information would be important to record? (e.g., starting inequality, operation, resulting inequality)



Explore Mathematical Operations and Linear Inequalities

Materials
- long strip of paper or number line
- ruler
- two different-coloured tokens or markers

1. On a long strip of paper, draw a number line that shows integers from −20 to 20.

2. a) Work with a partner. Each partner needs to choose an even, positive whole number that is less than 10. Do not choose the same number. Use a different token to show the position of each partner's starting number on the number line.

b) Record an inequality that compares the starting numbers. Note the direction of the inequality symbol and who has the greater number.

c) Choose the same mathematical operation to perform on each partner's number. Move the markers to show the resulting numbers. If necessary, extend your number line.

Whose resulting number is greater? Record an inequality that compares these numbers.

Subtract 4.
Move the counters.

−10 −5 0 5 10

d) Starting each time with your original numbers and inequality, take turns to choose a different mathematical operation and perform it. Each time, move the counters. Whose number is greater? Record the resulting inequality.

e) Try different operations until you are able to predict which operations will reverse an inequality symbol and which ones will keep it the same. Organize your observations and results.

Strategies
Identify All Possibilities

3. a) Conduct a new trial by choosing one negative and one positive number. Use these starting numbers to test your predictions in #2e).

b) Model each operation using the number line and markers. Record your results.

Reflect and Check

4. Consider how the markers moved on the number line.

a) What mathematical operations changed the direction of the inequality symbol? Explain.

b) What operations kept the inequality symbol the same? Explain.

c) Develop an example to support your explanation for parts a) and b).

5. Review your strategy for solving Katie's puzzle. What advice would you give about an operation that would make Joe's number greater than Katie's?

Some students will quickly discover that multiplying or dividing by a negative results in the inequality sign needing to be reversed, while other students will not. Provide such coaching questions as the following:

- What information could you record to solve the puzzle?
- What different operations might you try?
- What operations have you tried so far?
- Have any operations reversed the inequality sign? If so, explain.
- What types of numbers might you try each operation with?
- Have you tried all possible combinations of operations? all types of numbers?

Students may solve the puzzle with the first operation they find that reverses the sign. Encourage students to continue to check until they have exhausted all possibilities (i.e., four operations with both positive and negative numbers).

In #3, students repeat the activity using one positive and one negative number. Some students can be challenged to select odd integers or even non-integer values to use this second time around.

Method 2 Follow the same procedure as in Method 1. Once students have come to some possible conclusions, have them repeat the activity a third time using two negative numbers. Discuss as a class whether this repetition confirms or negates their results.

Meeting Student Needs

- Help students explore the concept of reversing the inequality sign when multiplying or dividing by a negative by giving a demonstration using a large number line and an inequality with a constant on each side (i.e., without a variable), similar to what students did in the Explore. As the class suggests operations for you to perform, use your hands to mark the values before and after the operation. Challenge students to come up with an operation that makes you switch your hands.

Gifted and Enrichment

- Ask students to explain why the inequality sign is reversed when they divide or multiply by a negative number. To answer the question, they can create a blog, such as "Little Known Facts About Inequalities."

Answers

Explore Mathematical Operations and Linear Inequalities

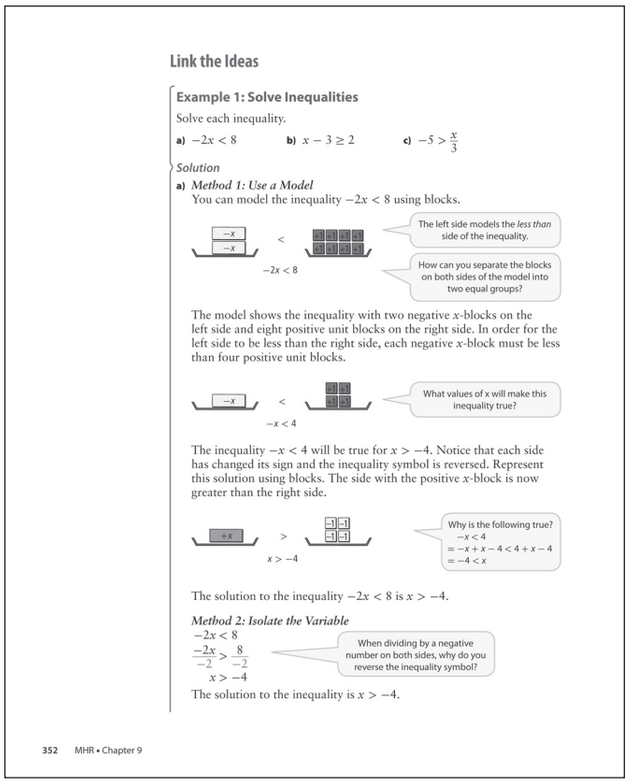
4. a) Multiplying or dividing by a negative number resulted in a change in the direction of the inequality symbol.

b) Adding, subtracting, and multiplying or dividing by a positive number all resulted in the inequality symbol staying the same.

c) Example: For part a), choose two original numbers 6 and 8, and a multiplier of −3. Six is less than 8. Then multiply by −3, which results in the respective products −18 and −24. Now, the first number, −18, is greater than the second number, −24. For part b), subtract 5 from each number. The resulting differences are 1 and 3, respectively. In this case, the first number is still less than the second number.

5. Example: Multiply both Katie's and Joe's number by −10.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore. Try to have students explain what specifically caused the inequality sign to change.	• Discussing the results with the class will benefit those students who did not arrive at a suitable conclusion. • Encourage students to write down two or three examples of their own in their Foldable.



Link the Ideas

Example 1: Solve Inequalities

Solve each inequality.

a) $-2x < 8$ b) $x - 3 \geq 2$ c) $-5 > \frac{x}{3}$

Solution

a) *Method 1: Use a Model*

You can model the inequality $-2x < 8$ using blocks.

The left side models the *less than* side of the inequality.

How can you separate the blocks on both sides of the model into two equal groups?

$-2x < 8$

The model shows the inequality with two negative x-blocks on the left side and eight positive unit blocks on the right side. In order for the left side to be less than the right side, each negative x-block must be less than four positive unit blocks.

What values of x will make this inequality true?

$-x < 4$

The inequality $-x < 4$ will be true for $x > -4$. Notice that each side has changed its sign and the inequality symbol is reversed. Represent this solution using blocks. The side with the positive x-block is now greater than the right side.

Why is the following true?
$-x < 4$
$= -x + x - 4 < 4 + x - 4$
$= -4 < x$

$x > -4$

The solution to the inequality $-2x < 8$ is $x > -4$.

Method 2: Isolate the Variable

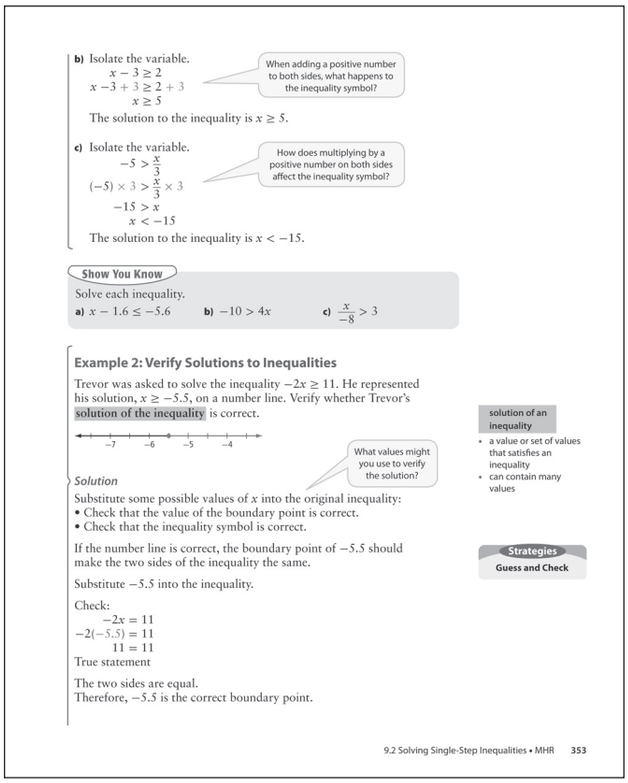
$-2x < 8$

$\frac{-2x}{-2} > \frac{8}{-2}$

When dividing by a negative number on both sides, why do you reverse the inequality symbol?

$x > -4$

The solution to the inequality is $x > -4$.



b) Isolate the variable.

$x - 3 \geq 2$

$x - 3 + 3 \geq 2 + 3$

$x \geq 5$

When adding a positive number to both sides, what happens to the inequality symbol?

The solution to the inequality is $x \geq 5$.

c) Isolate the variable.

$-5 > \frac{x}{3}$

$(-5) \times 3 > \frac{x}{3} \times 3$

$-15 > x$

$x < -15$

How does multiplying by a positive number on both sides affect the inequality symbol?

The solution to the inequality is $x < -15$.

Show You Know

Solve each inequality.

a) $x - 1.6 \leq -5.6$ b) $-10 > 4x$ c) $\frac{x}{-8} > 3$

Example 2: Verify Solutions to Inequalities

Trevor was asked to solve the inequality $-2x \geq 11$. He represented his solution, $x \geq -5.5$, on a number line. Verify whether Trevor's solution of the inequality is correct.

−7 −6 −5 −4

What values might you use to verify the solution?

Solution

Substitute some possible values of x into the original inequality:
- Check that the value of the boundary point is correct.
- Check that the inequality symbol is correct.

If the number line is correct, the boundary point of -5.5 should make the two sides of the inequality the same.

Substitute -5.5 into the inequality.

Check:

$-2x = 11$

$-2(-5.5) = 11$

$11 = 11$

True statement

The two sides are equal.
Therefore, -5.5 is the correct boundary point.

solution of an inequality
- a value or set of values that satisfies an inequality
- can contain many values

Strategies
Guess and Check

Link the Ideas

Example 1

Example 1 introduces students to solving one-step inequalities in a concrete manner. If possible, have students use algebra tiles or other concrete materials to model the example. Ask:

- When have you used blocks before?
- Why is there a less than sign in this case?
- How could you isolate the variable in this case?
- Why does it change to a greater than sign in the last step?
- How does this change correspond to the algebraic steps in solving?
- Will the less than or greater than sign always change when solving an inequality?

Help students compare the concrete model with the algebraic process.

Have students work in small groups on the Show You Know. Provide one set of manipulatives to each small group. Ask students to model what they are doing as they work on the algebraic solution.

Example 2

Example 2 focuses on verifying a solution to an inequality. It is important for students to realize that to check the solution to an inequality, they need to check two components: that both the boundary point and the inequality sign are correct. Put another way, they need to check that the dot is in the right place and that the arrow is in the correct direction on the number line. Help students by asking the following questions:

- How can you verify the solution for an equation?
- How is the solution to an inequality different from the solution to an equation?
- Can you verify the solution to an inequality in the same way as an equation?
- How can you check if the boundary point is located correctly?
- How can you check if the arrow is pointing in the correct direction?
- What values can you use to make it easy to verify the solution given?
- How is *verifying* different than *solving*?

If the number line is correct, any value greater than -5.5 should make a true statement.

Substitute one or more values greater than -5.5, such as -5 and 0, into the inequality.

Choose numbers that are easy to work with.

Check:

$-2x \geq 11$	$-2x \geq 11$
$-2(-5) \geq 11$	$-2(0) \geq 11$
$10 \geq 11$	$0 \geq 11$
False statement	False statement

The values -5 and 0 are non-solutions since they result in false statements.

Substituting numbers greater than -5.5 does not result in true statements.

Trevor has drawn the arrow facing the wrong way on the number line. He should have changed the direction of the inequality symbol in his solution. The solution should be $x \leq -5.5$.

Verify the correct solution by substituting one or more values less than -5.5, such as -8 and -6, into the inequality.

Check:

$-2x \geq 11$	$-2x \geq 11$
$-2(-8) \geq 11$	$-2(-6) \geq 11$
$16 \geq 11$	$12 \geq 11$
True statement	True statement

The values -8 and -6 are specific solutions since they result in true statements.

Trevor's solution is not correct. He forgot to reverse the inequality sign when dividing by a negative number.

Show You Know

Verify the solution for each inequality. If incorrect, what is the solution?

a) For the inequality $x - 12 \leq 20$, the solution is $x \leq 32$.

b) For the inequality $-5x < 30$, the solution is $x < -6$.

Example 3: Model and Solve a Problem

A games store is offering games on sale for $12.50, including tax. Sean has set his spending limit at $80. How many games can Sean buy and stay within his limit?

a) Write an inequality to model the problem.

b) Solve the inequality and interpret the solution.

Solution

a) If n represents the number of games that Sean can buy, the cost of n games is 12.5 times n. Sean must spend no more than $80.

The situation can be modelled with the inequality $12.5n \leq 80$.

b) $12.5n \leq 80$

$\frac{12.5n}{12.5} \leq \frac{80}{12.5}$

$n \leq 6.4$

$12 \times 6 = 72$ $12 \times 7 = 84$

The number of games Sean can buy is between 6 and 7.

Since it is not possible to buy part of a game, 6.4 is not a solution to the original problem, even though it is a solution to the inequality $12.5n \leq 80$. Only a whole number is a possible solution for this situation.

Sean can buy up to and including six games and stay within his spending limit.

Show You Know

Yvonne is planting trees as a summer job. She gets paid $0.10 per tree planted. She wants to earn at least $20/h. How many trees must she plant per hour in order to achieve her goal?

a) Write an inequality to model the number of trees Yvonne must plant to reach her goal.

b) Will the solution be a set of whole numbers or a set of integers? Explain.

c) Solve the inequality and interpret the solution.

Did You Know?

Piecework is work paid by the amount done, not by the time it takes. For example, tree planters are paid by the number of trees they plant.

Encourage students to choose values that are convenient to work with (such as zero) when choosing values to check the inequality sign. To help students see the benefit of checking several values, ask questions such as the following:

- How many values are there in the solution to a linear equation?
- How many values are there in the solution to an inequality?
- Which usually has more specific values in its solution: an equation or an inequality?

Have students work in pairs on the Show You Know. Challenge them to use at least two different values each to check their solution. Ask:

- Does it make a difference what values you use?
- How might it be helpful to use a number of values in your verification checks?

Example 3

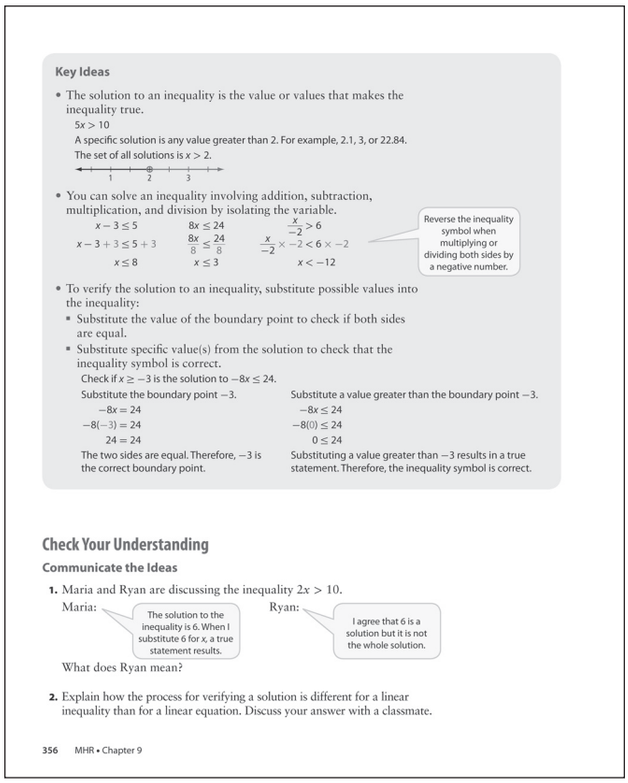
In Example 3, students examine a real-world problem situation that can be modelled and solved using an inequality. Although they will likely have had much practice applying equations to problem situations, they might not immediately see why an inequality is the most appropriate model to use in this situation. Help them by asking questions such as the following:

- What word or words in the problem might suggest that an inequality should be used?
- How many possible answers will there be to this problem?
- What types of values will the answers be?
- Without solving the problem, give an example of one value that you know is a possible answer.

Students might not see why the solution to the inequality used is not the solution to the original problem. When working independently, some students might stop at the point of having one answer to the inequality used rather than going back to the original problem. Help them by asking:

- How is the solution to the inequality used here related to the solution to the problem?
- How many values are there in the solution to the inequality used here?
- How many values are there in the solution to the original problem? How do you know?

Have students work in groups of three on the Show You Know. Have students work individually to do the three parts, and then to decide who will be a), b), and c). Students can explain to the other members in the group how they did the part related to their letter. Ask the listeners to consider what they are hearing and being shown and to decide whether the speaker is correct or needs some coaching.



Key Ideas

- The solution to an inequality is the value or values that makes the inequality true.

  $5x > 10$

  A specific solution is any value greater than 2. For example, 2.1, 3, or 22.84.

  The set of all solutions is $x > 2$.

- You can solve an inequality involving addition, subtraction, multiplication, and division by isolating the variable.

$x - 3 \leq 5$	$8x \leq 24$	$\frac{x}{-2} > 6$
$x - 3 + 3 \leq 5 + 3$	$\frac{8x}{8} \leq \frac{24}{8}$	$\frac{x}{-2} \times -2 < 6 \times -2$
$x \leq 8$	$x \leq 3$	$x < -12$

  Reverse the inequality symbol when multiplying or dividing both sides by a negative number.

- To verify the solution to an inequality, substitute possible values into the inequality:
  - Substitute the value of the boundary point to check if both sides are equal.
  - Substitute specific value(s) from the solution to check that the inequality symbol is correct.

  Check if $x \geq -3$ is the solution to $-8x \leq 24$.

  Substitute the boundary point -3.

  $-8x = 24$

  $-8(-3) = 24$

  $24 = 24$

  The two sides are equal. Therefore, -3 is the correct boundary point.

  Substitute a value greater than the boundary point -3.

  $-8x \leq 24$

  $-8(0) \leq 24$

  $0 \leq 24$

  Substituting a value greater than -3 results in a true statement. Therefore, the inequality symbol is correct.

Check Your Understanding

Communicate the Ideas

1. Maria and Ryan are discussing the inequality $2x > 10$.

Maria: The solution to the inequality is 6. When I substitute 6 for x, a true statement results.

Ryan: I agree that 6 is a solution but it is not the whole solution.

What does Ryan mean?

2. Explain how the process for verifying a solution is different for a linear inequality than for a linear equation. Discuss your answer with a classmate.

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Key Ideas

The Key Ideas summarize what constitutes a solution to an inequality, important concepts related to solving single-step inequalities, as well as verifying solutions to inequalities using elements from a given solution. Have students make notes to summarize the Key Ideas in their own way in their Foldable. Have students complete the definition oval on the concept map for *solution of an inequality*. Check their notes to ensure accuracy, and assist with revisions where necessary.

Meeting Student Needs

- For Example 2, some students will benefit from marking each value they check on a blank number line. As they check more and more values, they will see whether or not their checked values match the given solution.
- For Example 3, some students will benefit from making a number line that shows the solution to the inequality and another that shows only the solution to the original problem (i.e., closed circles only on the whole numbers from 0 to 6). They can then compare the two and see that one is really a subset of the other.

Answers

Example 1: Show You Know

a) $x \leq -4$

b) $-2.5 > x$

c) $x < -24$

Example 2: Show You Know

a) correct

b) incorrect; $x > -6$

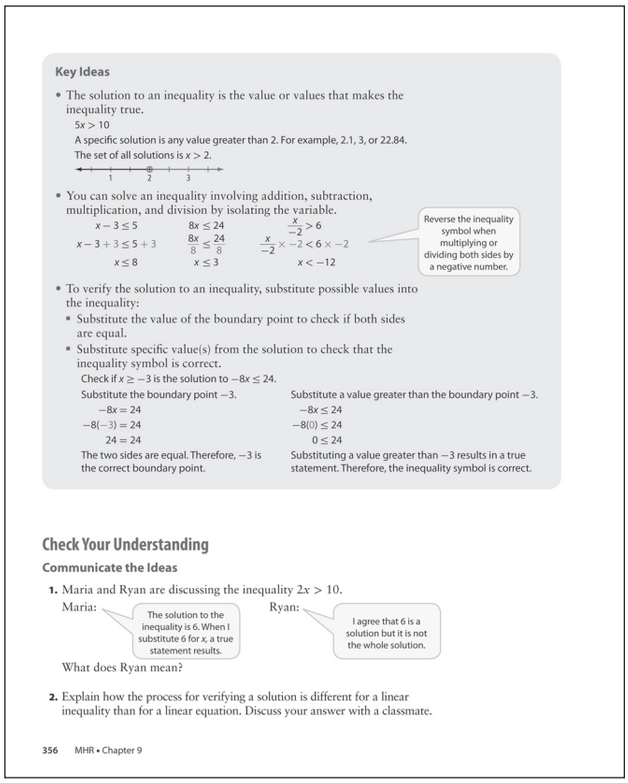
Example 3: Show You Know

a) $0.1t \geq 20$

b) The solution will be a set of integers because Yvonne cannot plant part of a tree.

c) $t > 200$; Yvonne must plant at least 200 trees every hour in order to earn at least \$20/h.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • All students will benefit from a demonstration of Method 1 using concrete materials such as algebra tiles. You can also use a metre stick or other object to model the balance tipping from one side to the other. This will help students connect with the algebraic method. • Students experiencing difficulty should be encouraged to model their questions for as long as necessary. • Remind students that the ultimate goal is to solve for a single positive variable.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students who need concrete manipulation should be encouraged to model their questions for as long as necessary. • Some students may benefit from drawing a number-line solution first in order to decide which values to test.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students will benefit from making two number lines and comparing one that shows the solution to the inequality used to another that shows only the solutions to the original problem. • Some students may benefit from discussing some of the key terminology associated with the problems. • It may benefit some students to start by solving the problem using a personal strategy other than an inequality.



Key Ideas

- The solution to an inequality is the value or values that makes the inequality true.
  $5x > 10$
  A specific solution is any value greater than 2. For example, 2.1, 3, or 22.84.
  The set of all solutions is $x > 2$.
- You can solve an inequality involving addition, subtraction, multiplication, and division by isolating the variable.
  $x - 3 \le 5$ → $x - 3 + 3 \le 5 + 3$ → $x \le 8$
  $8x \le 24$ → $\frac{8x}{8} \le \frac{24}{8}$ → $x \le 3$
  $\frac{x}{-2} > 6$ → $\frac{x}{-2} \times -2 < 6 \times -2$ → $x < -12$
  Reverse the inequality symbol when multiplying or dividing both sides by a negative number.
- To verify the solution to an inequality, substitute possible values into the inequality:
  - Substitute the value of the boundary point to check if both sides are equal.
  - Substitute specific value(s) from the solution to check that the inequality symbol is correct.

  Check if $x \ge -3$ is the solution to $-8x \le 24$.

  Substitute the boundary point -3.
  $-8x = 24$
  $-8(-3) = 24$
  $24 = 24$
  The two sides are equal. Therefore, -3 is the correct boundary point.

  Substitute a value greater than the boundary point -3.
  $-8x \le 24$
  $-8(0) \le 24$
  $0 \le 24$
  Substituting a value greater than -3 results in a true statement. Therefore, the inequality symbol is correct.

Check Your Understanding

Communicate the Ideas

1. Maria and Ryan are discussing the inequality $2x > 10$.
   Maria: The solution to the inequality is 6. When I substitute 6 for x, a true statement results.
   Ryan: I agree that 6 is a solution but it is not the whole solution.
   What does Ryan mean?
2. Explain how the process for verifying a solution is different for a linear inequality than for a linear equation. Discuss your answer with a classmate.

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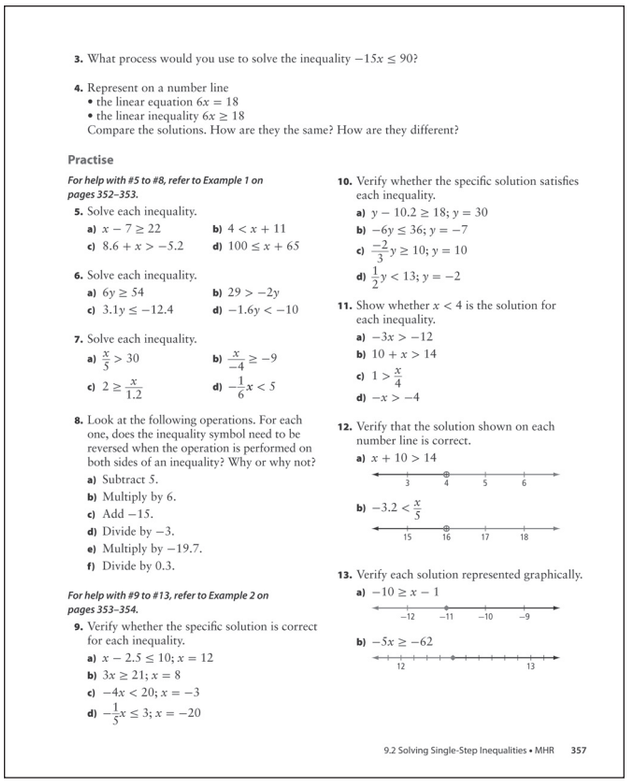

3. What process would you use to solve the inequality $-15x \le 90$?
4. Represent on a number line
   - the linear equation $6x = 18$
   - the linear inequality $6x \ge 18$

   Compare the solutions. How are they the same? How are they different?

Practise

For help with #5 to #8, refer to Example 1 on pages 352–353.

5. Solve each inequality.
   a) $x - 7 \ge 22$ b) $4 < x + 11$
   c) $8.6 + x > -5.2$ d) $100 \le x + 65$
6. Solve each inequality.
   a) $6y \ge 54$ b) $29 > -2y$
   c) $3.1y \le -12.4$ d) $-1.6y < -10$
7. Solve each inequality.
   a) $\frac{x}{5} > 30$ b) $\frac{x}{-4} \ge -9$
   c) $2 \ge \frac{x}{1.2}$ d) $-\frac{1}{6}x < 5$
8. Look at the following operations. For each one, does the inequality symbol need to be reversed when the operation is performed on both sides of an inequality? Why or why not?
   a) Subtract 5.
   b) Multiply by 6.
   c) Add -15.
   d) Divide by -3.
   e) Multiply by -19.7.
   f) Divide by 0.3.

For help with #9 to #13, refer to Example 2 on pages 353–354.

9. Verify whether the specific solution is correct for each inequality.
   a) $x - 2.5 \le 10$; $x = 12$
   b) $3x \ge 21$; $x = 8$
   c) $-4x < 20$; $x = -3$
   d) $-\frac{1}{5}x \le 3$; $x = -20$
10. Verify whether the specific solution satisfies each inequality.
    a) $y - 10.2 \ge 18$; $y = 30$
    b) $-6y \le 36$; $y = -7$
    c) $\frac{-2}{3}y \ge 10$; $y = 10$
    d) $\frac{1}{2}y < 13$; $y = -2$
11. Show whether $x < 4$ is the solution for each inequality.
    a) $-3x > -12$
    b) $10 + x > 14$
    c) $1 > \frac{x}{4}$
    d) $-x > -4$
12. Verify that the solution shown on each number line is correct.
    a) $x + 10 > 14$

    b) $-3.2 < \frac{x}{5}$

13. Verify each solution represented graphically.
    a) $-10 \ge x - 1$

    b) $-5x \ge -62$

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Check Your Understanding

Communicate the Ideas

Questions #1 and 4 will help students evaluate whether they understand what constitutes a solution when solving an inequality.

Use students' responses to #2 to assess how effectively they are able to verify a given solution for an inequality.

In #3, students should be able to solve the inequality correctly, including reversing the inequality sign, depending on the method they choose.

Practise

Students need to understand how to solve single-step inequalities, as well as how to verify given solutions or those they generate themselves. Let students know that unless otherwise specified, they can express their solutions either graphically or algebraically. As students work with questions in this section, check that they are able to solve inequalities and reverse the sign where necessary. Also, observe that they are able to verify given solutions correctly.

The final two Practise questions require students to model and solve a problem similar to Example 3. You may wish to discuss the scenarios in these questions. Depending on the interests of your students, you may wish to change the scenarios to something that particular classroom students do.

Apply

The Apply section allows students to focus on using single-step inequalities to model and solve problems.

Question #16 asks students to identify specific solutions and non-solutions. Students need to realize that the simplest way is to solve the inequality first.

In #17, students are required to clarify which negative sign is the reason that the inequality sign needs to be reversed.

You may wish to change the rates for #18 to similar rates from a local arena. Discuss why people would want to know the answer to a), and where else students might use such a skill.

Question #19 has students consider how to meet a personal goal. That goal could have been how much they could save toward a certain item if they saved 4% of their monthly income. How much would they then have to make in order to afford a particular item?

Students interested in mechanics might want to replace the fuel consumption in #20 with the fuel consumption on a vehicle of their choice.

For help with #14 and #15, refer to Example 3 on page 355.

14. The Super Fencing Company builds cedar fences for homes at a cost of $85 per section of fence, including tax. How many sections of fence could you buy if you could spend no more than $1400?

a) Model the problem using an inequality.

b) Solve the inequality.

c) Is the boundary point a reasonable solution for the number of fence sections? Explain.

15. Megan is competing in a series of mountain bike races this season. She gets 6 points for each race she wins. If she gets more than 50 points in total, she will move up to the next racing category. How many race wins this season will allow her to move up to the next category?

a) Use an inequality to represent the problem.

b) Determine the solution and use it to solve the problem.

c) Is the boundary point a reasonable solution for the number of race wins? Explain.

Apply

16. For each of the following inequalities, state three values that are specific solutions and three values that are non-solutions.

a) $-5 + x < -10$

b) $-3x < 24$

17. Colin's teacher asked him to solve the inequality $-5x \geq -15$. His solution was $x \leq 3$. He explained that he reversed the inequality symbol because of the negative number. Write a more accurate explanation.

18. A local sports complex offers the following options for sharpening skates.

Skate Sharpening Rates

Standard Rate: $5.75 per pair of skates

Special: $49 per month for unlimited sharpening

a) Estimate at what point the special would be the better option. Show the process you used. Why do you think your method provides a reasonable estimate?

b) Model and solve the problem using an inequality. Compare the answer to your estimate.

19. The owner of a craft store donates 3% of her profits to a local charity every month. If she wants to donate at least $250 this month, how much profit will the business need to earn?

a) Model and solve the problem using an inequality.

b) Verify your solution. Show your work.

20. Andrew's family is driving from Winnipeg to Saskatoon. Before leaving, they fill the gas tank with 57 L of fuel. The car uses fuel at an average rate of 8.4 L/100 km for highway driving. How many kilometres can they drive on this amount of fuel? What assumptions did you make?

21. Natalie is entering the 3200-m event at an upcoming meet. Each lap of the track is 400 m. Her goal is to beat the current record of 9 min 23 s. How fast must she run each lap, on average, to beat the record?

a) Explain why the situation can be modelled using the inequality $8x < 563$.

b) Solve the problem and verify your solution. Show your work.

22. Fiona has a rewards card that gives her a reward point for every $5 she spends. If she earns at least 120 points in a year, she gets a bonus. How much does she need to spend to get at least 120 points?

Extend

23. Chris has a weekend business building doghouses. Each doghouse takes 4 h to build and is sold for $115. Chris wants to earn at least $1000 per month. He wants to work no more than 50 h on his business per month.

a) Write two inequalities to model the situation.

b) Solve each inequality.

c) What possible numbers of doghouses can he build each month and stay within his guidelines?

24. Solve and check the inequality $-\frac{2}{5}x < \frac{1}{3}$. Show the solution on a number line.

25. If $-2x > 22$ and $-4x < 60$, determine the possible values of x that satisfy both inequalities. Show your solution on a number line.

26. A food company that is developing a new energy bar has not decided on the size of the bar. The recipe includes 9% protein and 13% fat. The company wants the bar to contain at least 6 g of protein and no more than 10 g of fat. Use two inequalities to determine the possible range of masses for the bar.

27. Consider the inequality $ax \leq 5a$.

a) Solve the inequality if $a > 0$.

b) Solve the inequality if $a < 0$.

28. Solve each combination of inequalities.

a) $-5 \leq x + 9$ and $x + 9 \leq 8$

b) $-2 < 2x$ and $2x < 12$

c) $-15 \leq -6x$ and $-6x < 9$

Math Link

Some amusement parks offer single-ride tickets, where you pay each time you ride, and all-day passes, where you pay once for unlimited rides. The prices for both types of tickets need to be high enough for the amusement park to earn a profit but low enough that people decide to come.

Search various media, such as newspapers, magazines, and the Internet. Look for information about ticket prices at amusement parks.

a) Choose a price for single-ride tickets and a price for all-day passes. Explain why your choices are reasonable.

b) Use an inequality to determine the number of rides that make one option a better deal than the other.

c) Your friends plan on going on seven rides in your amusement park. Which is the better option for them? Show your work.

In #21, students are given an inequality, but may not immediately recognize the numbers involved.

What rewards cards do students have? How might they use the points they receive on these rewards cards to develop a question similar to #22?

Extend

The Extend problems provide the opportunity for higher-level thinking.

In #23, students need to solve two given inequalities and use both solutions to specify a range of values. This concept is extended in #25 and 26. In each question, students need to create two inequalities of their own to solve the problem. Note that students can give answers to #23, 25, and 26 as a combination of inequalities.

In #24, students solve an inequality involving fractions.

In #27, students solve an inequality given the value for one of the variables.

Question #28 asks students to isolate the variable in a combination of inequalities. Note that part c) will require students to reverse both inequality signs as they divide by a negative value.

Math Link

The Math Link for this section gives students a chance to apply inequalities to model a real-world situation. They will need information about ticket prices for amusement parks. You may wish to have them use information from the Web Link that follows.

If students do not have access to the Internet or other media, you might ask them to do this research at home or at another location prior to class. If access to media is not readily available, students might be asked to come up with their own prices for single-ride tickets and all-day passes.

As students work through the Math Link, circulate to ensure that they have created inequalities correctly, and that they are able to use them to solve the given problems. Consider having students work in pairs or share and compare their solutions.

Meeting Student Needs

- Provide **BLM 9–8 Section 9.2 Extra Practice** to students who would benefit from more practice.

ELL

- Ensure students understand the term *rewards card*.

Common Errors

- Some students may forget to reverse the inequality sign when necessary.

R_x Have them revisit the Explore and Example 1. Have them verify a solution when the inequality sign has not been reversed to see that the values in the solution do not satisfy the original inequality.

Web Link

For information about single-ride tickets and all-day passes at amusement parks in Western Canada, go to www.mathlinks9.ca and follow the links.

Answers

Communicate the Ideas

1. Example: Ryan means that there is more than one solution to the inequality. The number 6 is one of many solutions to this inequality.

2. Example: A linear inequality has a set of solutions, whereas a linear equation has only one solution.

3. Example: Divide both sides by -15 and reverse the direction of the inequality.

4. Example: The value of the solution to the equation, 3, is the boundary value for the inequality. Three is a solution to both algebraic problems. However, in the case of the inequality the number 3 represents only part of the solution;

Math Link

Check that student answers include:
- prices for single-ride tickets and all-day passes
- an explanation of why prices are reasonable for their park
- an inequality demonstrating which option, single-rides vs. all-day, is better
- mathematical equations that show which option is better for seven rides

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You might have students share answers in a small group or as a class. Individuals or pairs can check their own answers for completeness of ideas. • Each question links directly to processes required in the questions that follow. Ensure that students demonstrate a good understanding of these problems before moving on. • Students who need coaching should make good use of the chart on page 342 and the helpful hints they wrote into their Foldable in the previous section. • Allow students to use manipulatives for as long as needed.
Assessment *for* Learning	
Practise Have students do #5a), 5d), 6a), 6b), 7a), 7d), 8, 9, 12, 14, 16a), 17. Students who have no problems with these questions can go on to the remaining Apply questions.	• Some students might benefit from a review of using inverse operations to isolate a variable when working on #5. • Questions #6 and 7 are similar; some students may wish to try #7 before working on #6. The suggested questions look at slightly different applications. Students who need additional practice might benefit from doing the parts of #5, 6, and 7 that are of a similar process to the question(s) that gave them some difficulty. • In #6d), 7b), and 7d), students will likely need to reverse the inequality sign. Encourage them to verify their solutions. • Students are asked to verify solutions in #9 to 13. In #9 to 11, the solutions are given algebraically, whereas in #12 and 13, the solutions are given graphically. Supplementary questions might benefit some students, especially if they match the ones that gave them difficulty. • Questions #14, 16, and 18 are problem-solving contexts in which the student must develop an inequality and/or apply what has been learned. Have the students review Example 3 and reference their Foldable for guidelines and terminology.
Math Link The Math Link on page 359 is intended to help students work toward the chapter problem wrap-up titled MathLink: Wrap It Up! on page 371.	• Students who need help getting started could use **BLM 9–9 Section 9.2 Math Link**, which provides scaffolding. • If your school has limited access to the Internet, you may wish to download ticket price information and provide it to students. • As an alternative to amusement park information, consider providing information about hockey games, such as individual ticket prices or season passes.
Assessment *as* Learning	
Literacy Link (page 337) Have students complete the part of the concept map related to this section. Have them provide two examples of single-step inequalities, solve them, and then prove that the solution is correct.	• Remind students that the same rules are used for solving inequalities as for solving one-step algebra equations. • You may need to remind some students to use the opposite operation to isolate the variable. • Some students may wish to use an inequality from the student resource or their notes. Ask them to use this as a model but not to duplicate the inequality in their concept map. • Some students may wish to use inequalities that involve adding or subtracting the constant, as those are easier than dividing and multiplying.
Math Learning Log Have students answer the following questions: • What does it mean to *verify* a solution to an inequality?	• Why does an inequality sign need to be reversed sometimes? • Encourage students to use the What I Need to Work On section of their Foldable to note what they continue having difficulties with.