Solving Multi-Step Inequalities

9.3

***MathLinks 9*, pages 360–367**

Suggested Timing
50–60 minutes

Materials
- small blocks
- balance scale

Blackline Masters
Master 2 Communication Peer Evaluation
BLM 9–3 Chapter 9 Warm-Up
BLM 9–10 Section 9.3 Extra Practice
BLM 9–11 Section 9.3 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☐ Visualization (V)

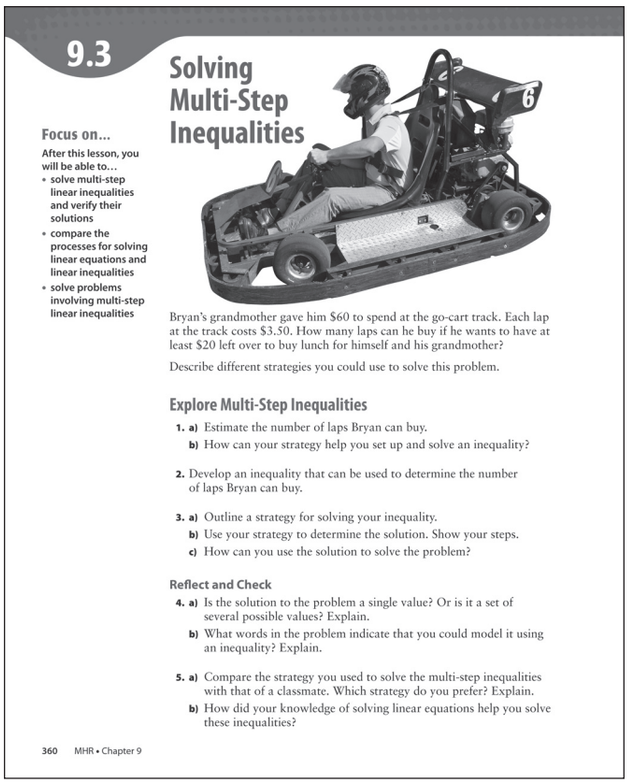
Specific Outcomes
PR4 Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 6, 8, 11
Typical	#1–3, 6, 8, 10–14
Extension/Enrichment	#2, 7, 8, 11, 13–14, 16–18, 20

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce material learned in previous sections.

Use the opening photo, taken at the Evergreen Go-Kart facility in Lethbridge, Alberta, and accompanying situation as an introduction to multi-step inequalities and Explore Multi-Step Inequalities.



9.3

Solving Multi-Step Inequalities

Focus on…

After this lesson, you will be able to…
- solve multi-step linear inequalities and verify their solutions
- compare the processes for solving linear equations and linear inequalities
- solve problems involving multi-step linear inequalities

Bryan's grandmother gave him $60 to spend at the go-cart track. Each lap at the track costs $3.50. How many laps can he buy if he wants to have at least $20 left over to buy lunch for himself and his grandmother?

Describe different strategies you could use to solve this problem.

Explore Multi-Step Inequalities

1. a) Estimate the number of laps Bryan can buy.
   b) How can your strategy help you set up and solve an inequality?
2. Develop an inequality that can be used to determine the number of laps Bryan can buy.
3. a) Outline a strategy for solving your inequality.
   b) Use your strategy to determine the solution. Show your steps.
   c) How can you use the solution to solve the problem?

Reflect and Check

4. a) Is the solution to the problem a single value? Or is it a set of several possible values? Explain.
   b) What words in the problem indicate that you could model it using an inequality? Explain.
5. a) Compare the strategy you used to solve the multi-step inequalities with that of a classmate. Which strategy do you prefer? Explain.
   b) How did your knowledge of solving linear equations help you solve these inequalities?

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Explore Multi-Step Inequalities

In this Explore, students are asked to model a situation using multi-step inequalities. Note that students are asked to come up with two different inequalities that can each be used to solve the problem.

Method 1 Have students work in pairs to generate as many different inequalities as they can to model the situation. Ask them questions such as:
- What words in this situation suggest that it can be modelled with an inequality?
- Can you write an inequality verbally that will help you represent the situation algebraically?
- What is the condition given in the situation that would help determine which inequality sign might be used?

Have students work in pairs or groups and discuss what types of numbers the variable might be in this situation. Ask them to consider questions such as:
- Can it be any rational number, or are there certain restrictions?

- Given the solutions and types of numbers allowed for n, explain how many laps Bryan can buy and still have enough money left for lunch.

Some students are likely to subtract mentally (i.e., \$60 – \$20 for lunch leaves at most \$40 for racing), and consequently will write a one-step inequality such as $3.5n \leq 40$. Others will be able to write one or more multi-step inequalities (such as $60 - 3.5n \geq 20$ or $3.5n + 20 \leq 60$). Encourage students to try to write at least one inequality that requires several steps to solve.

If students do write an inequality such as $60 - 3.5n \geq 20$ where the variable term is negative, it can serve as a concrete way to help them understand why the sign needs to be reversed in some cases. The expression on the left (involving the number of laps) needs to be *greater than* \$20, and the result ends up being the number of laps on the left needs to be *less than* a certain value. If we start by saying that we want *more than* a certain amount to be leftover, *less than* a certain amount needs to be used.

Method 2 Have students use a model of blocks on a tipped balance as they attempt to develop inequalities that will help solve the problem. You might ask them questions such as:

- What might be on each side of the balance?
- How do the two sides of the balance compare to each other?
- Does one side need to be more or less that the other side?
- Might the two sides be equal to each other?
- Can you rearrange the blocks on your balance to help find a different inequality that will work in this situation?

Note that as students complete the Reflect and Check, they will need to realize that they can use the solution they find in solving their inequalities to identify specific whole-number values that are solutions to the original problem.

Meeting Student Needs

- Some students may not readily see the connection between solving inequalities and solving equations. Help by modelling each with a balance, and illustrating the similarities. Point out that the only differences are that the balance is tipped one way when solving an inequality and it sometimes needs to be switched to tip the other way.

Common Errors

- Some students may need to be reminded why the inequality sign needs to be reversed sometimes when solving inequalities.

R_x The Explore offers an excellent opportunity to illustrate this in a concrete, real context. Ask students if it makes sense for an inequality such as $60 - 3.5n \geq 20$ to have a *greater than* sign to produce a solution with a *less than* sign. If Bryan wants to have *greater than* a certain amount left over, he needs to keep his laps to *less than* a certain amount.

Answers

Explore Solving Multi-Step Inequalities

1. a) Example: 10

b) Example: Subtract 20 from 60 to find the amount available to spend at the go-cart track. Estimate the cost of 1 lap as \$4. Divide 40 by 4 to estimate the number of laps: 10.

2. $3.5l \leq 60 - 20$

3. a) Example: Perform the subtraction on the right side of the inequality, and then divide both sides by 3.5.

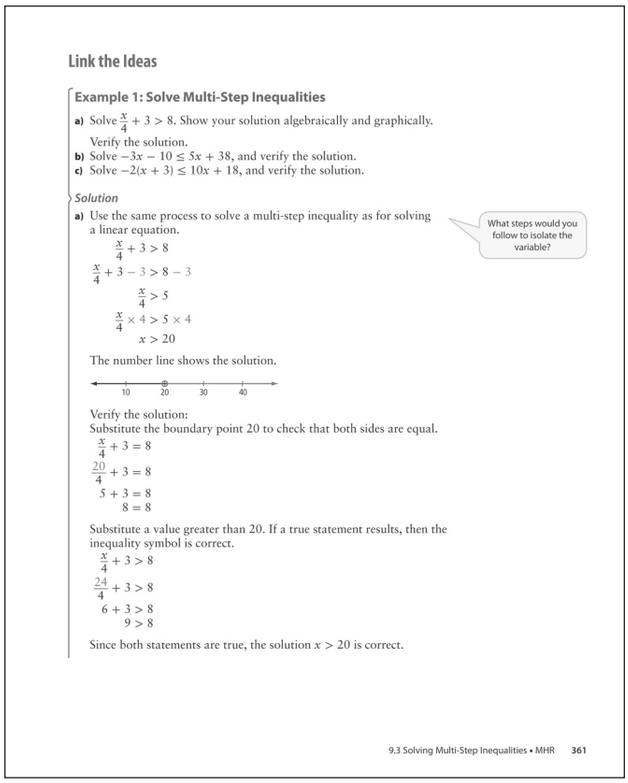
b) $l \leq 11.4$

c) Example: The number of laps must be a whole number, so Bryan can buy no more than 11 laps.

4. a) The solution is a set of several possible values. The number of laps that Bryan can purchase is between 0 and 11 inclusive.

b) The phrase "…at least \$20 left over.." indicates that an inequality can model this situation.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Monitor as students work through the Explore to see that they are able to model the situation with several different inequalities, as well as solve the inequalities they generate. Listen as students discuss what they discovered during the Explore.	• Have students model inequalities using concrete objects or diagrams of a tipped balance. • Encourage discussion and partnering among students, and sharing strategies with the class. • Encourage students to use their reference material and examples from their Foldable.



Link the Ideas

Example 1: Solve Multi-Step Inequalities

a) Solve $\frac{x}{4} + 3 > 8$. Show your solution algebraically and graphically.
Verify the solution.
b) Solve $-3x - 10 \leq 5x + 38$, and verify the solution.
c) Solve $-2(x + 3) \leq 10x + 18$, and verify the solution.

Solution

a) Use the same process to solve a multi-step inequality as for solving a linear equation.

What steps would you follow to isolate the variable?

$$\frac{x}{4} + 3 > 8$$
$$\frac{x}{4} + 3 - 3 > 8 - 3$$
$$\frac{x}{4} > 5$$
$$\frac{x}{4} \times 4 > 5 \times 4$$
$$x > 20$$

The number line shows the solution.

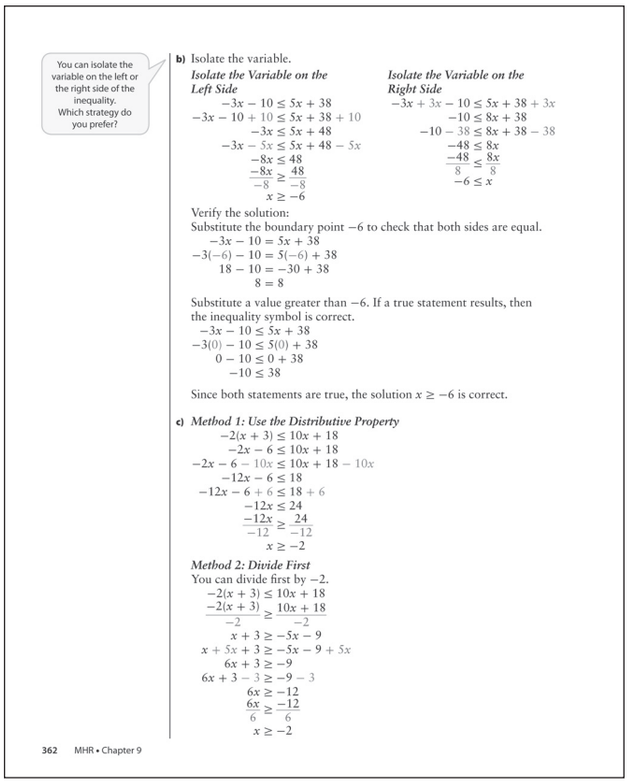
Verify the solution:
Substitute the boundary point 20 to check that both sides are equal.

$$\frac{x}{4} + 3 = 8$$
$$\frac{20}{4} + 3 = 8$$
$$5 + 3 = 8$$
$$8 = 8$$

Substitute a value greater than 20. If a true statement results, then the inequality symbol is correct.

$$\frac{x}{4} + 3 > 8$$
$$\frac{24}{4} + 3 > 8$$
$$6 + 3 > 8$$
$$9 > 8$$

Since both statements are true, the solution $x > 20$ is correct.



You can isolate the variable on the left or the right side of the inequality. Which strategy do you prefer?

b) Isolate the variable.

Isolate the Variable on the Left Side	*Isolate the Variable on the Right Side*
$-3x - 10 \leq 5x + 38$	$-3x + 3x - 10 \leq 5x + 38 + 3x$
$-3x - 10 + 10 \leq 5x + 38 + 10$	$-10 \leq 8x + 38$
$-3x \leq 5x + 48$	$-10 - 38 \leq 8x + 38 - 38$
$-3x - 5x \leq 5x + 48 - 5x$	$-48 \leq 8x$
$-8x \leq 48$	$\frac{-48}{8} \leq \frac{8x}{8}$
$\frac{-8x}{-8} \geq \frac{48}{-8}$	$-6 \leq x$
$x \geq -6$	

Verify the solution:
Substitute the boundary point -6 to check that both sides are equal.

$$-3x - 10 = 5x + 38$$
$$-3(-6) - 10 = 5(-6) + 38$$
$$18 - 10 = -30 + 38$$
$$8 = 8$$

Substitute a value greater than -6. If a true statement results, then the inequality symbol is correct.

$$-3x - 10 \leq 5x + 38$$
$$-3(0) - 10 \leq 5(0) + 38$$
$$0 - 10 \leq 0 + 38$$
$$-10 \leq 38$$

Since both statements are true, the solution $x \geq -6$ is correct.

c) *Method 1: Use the Distributive Property*

$$-2(x + 3) \leq 10x + 18$$
$$-2x - 6 \leq 10x + 18$$
$$-2x - 6 - 10x \leq 10x + 18 - 10x$$
$$-12x - 6 \leq 18$$
$$-12x - 6 + 6 \leq 18 + 6$$
$$-12x \leq 24$$
$$\frac{-12x}{-12} \geq \frac{24}{-12}$$
$$x \geq -2$$

Method 2: Divide First
You can divide first by -2.

$$-2(x + 3) \leq 10x + 18$$
$$\frac{-2(x + 3)}{-2} \geq \frac{10x + 18}{-2}$$
$$x + 3 \geq -5x - 9$$
$$x + 5x + 3 \geq -5x - 9 + 5x$$
$$6x + 3 \geq -9$$
$$6x + 3 - 3 \geq -9 - 3$$
$$6x \geq -12$$
$$\frac{6x}{6} \geq \frac{-12}{6}$$
$$x \geq -2$$

Link the Ideas

Example 1

Example 1 illustrates how multi-step inequalities can be solved and verified. Explain that students can make use of their prior knowledge of solving multi-step equations they have done during this course and in prior years.

The three parts to this example have students progress from an equation requiring two steps to solve to one that has the variable on both sides and contains an expression with brackets. Tell students that they can simplify and/or use inverse operations to solve an inequality in the same way as an equation, as long as they remember to reverse the inequality sign when multiplying or dividing by a negative number. Some students will see fairly quickly that they can avoid having to do this if they are careful to always isolate the variable on the side on which it will have a positive coefficient.

Solutions are verified here using the method that students were introduced to in Section 9.2.

As students consider the two methods shown for parts b) and c), ask them questions such as:

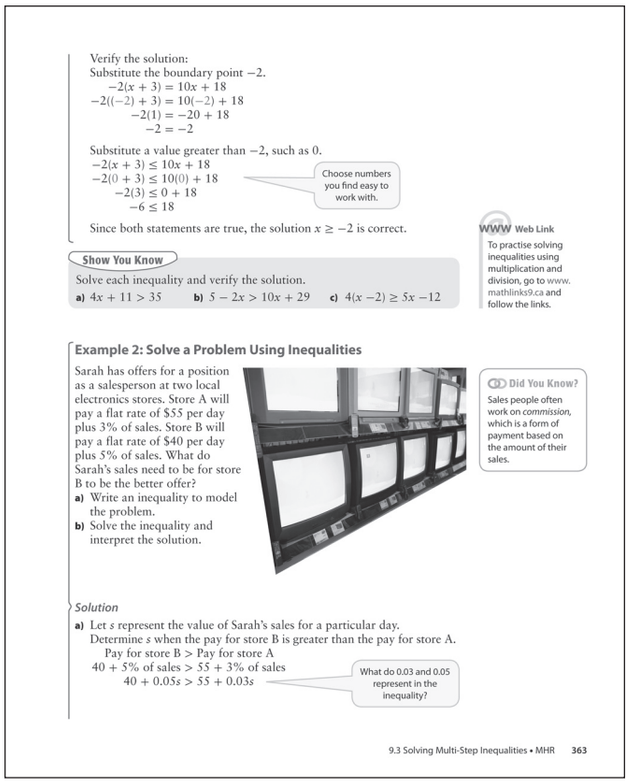
- Which method do you prefer?
- Will both methods shown here always be equally convenient to use?

Note that in Method 2 for part c), it is convenient to divide by –2 first because of the numbers involved. Students may realize as they complete the Show You Know that this is not always the best first step to use.

Have students work in pairs to do the Show You Know. Encourage students to use different methods to isolate the variable, and then to compare their answers. If their answers differ, have them examine each other's solutions to see why. Alternatively, they may wish to use a third way to isolate the variable and see if they get the same answer as one of their original ones.

Example 2

Students see in Example 2 how situations that involve comparisons can be analysed using an inequality. Not all students will be familiar with working on commission. After students read the problem, draw their attention to the Did You Know? in the margin and discuss commission as a form of payment. You may wish to discuss times when small flat rate plus larger commission would be preferable to large flat rate plus smaller commission (such as in an electronics store with high sales and large ticket prices).



Verify the solution:
Substitute the boundary point -2.
$-2(x + 3) = 10x + 18$
$-2((-2) + 3) = 10(-2) + 18$
$-2(1) = -20 + 18$
$-2 = -2$

Substitute a value greater than -2, such as 0.
$-2(x + 3) \leq 10x + 18$
$-2(0 + 3) \leq 10(0) + 18$
$-2(3) \leq 0 + 18$
$-6 \leq 18$

Choose numbers you find easy to work with.

Since both statements are true, the solution $x \geq -2$ is correct.

Show You Know

Solve each inequality and verify the solution.

a) $4x + 11 > 35$ **b)** $5 - 2x > 10x + 29$ **c)** $4(x - 2) \geq 5x - 12$

WWW Web Link

To practise solving inequalities using multiplication and division, go to www.mathlinks9.ca and follow the links.

Example 2: Solve a Problem Using Inequalities

Sarah has offers for a position as a salesperson at two local electronics stores. Store A will pay a flat rate of $55 per day plus 3% of sales. Store B will pay a flat rate of $40 per day plus 5% of sales. What do Sarah's sales need to be for store B to be the better offer?

a) Write an inequality to model the problem.

b) Solve the inequality and interpret the solution.

Did You Know?

Sales people often work on *commission*, which is a form of payment based on the amount of their sales.

Solution

a) Let s represent the value of Sarah's sales for a particular day.
Determine s when the pay for store B is greater than the pay for store A.
Pay for store B > Pay for store A
40 + 5% of sales > 55 + 3% of sales
$40 + 0.05s > 55 + 0.03s$

What do 0.03 and 0.05 represent in the inequality?

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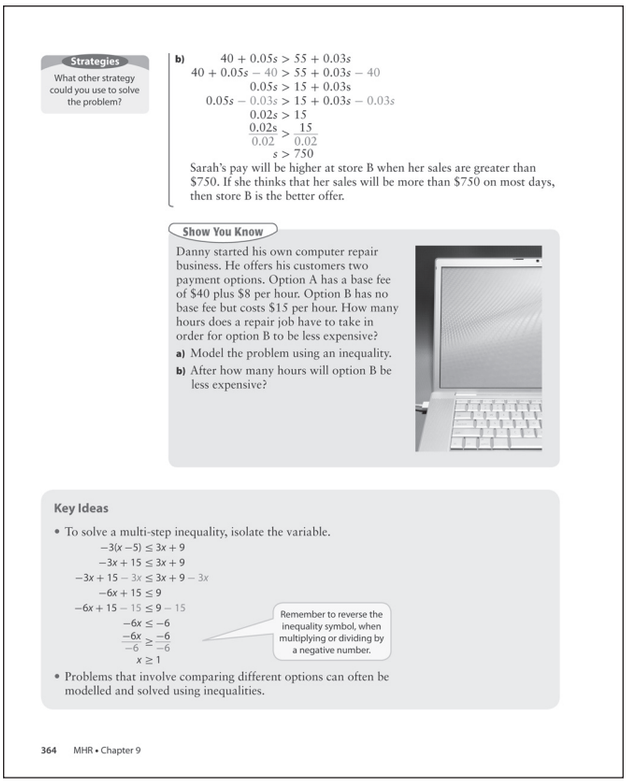

Strategies

What other strategy could you use to solve the problem?

b) $40 + 0.05s > 55 + 0.03s$
$40 + 0.05s - 40 > 55 + 0.03s - 40$
$0.05s > 15 + 0.03s$
$0.05s - 0.03s > 15 + 0.03s - 0.03s$
$0.02s > 15$
$\frac{0.02s}{0.02} > \frac{15}{0.02}$
$s > 750$

Sarah's pay will be higher at store B when her sales are greater than $750. If she thinks that her sales will be more than $750 on most days, then store B is the better offer.

Show You Know

Danny started his own computer repair business. He offers his customers two payment options. Option A has a base fee of $40 plus $8 per hour. Option B has no base fee but costs $15 per hour. How many hours does a repair job have to take in order for option B to be less expensive?

a) Model the problem using an inequality.

b) After how many hours will option B be less expensive?

Key Ideas

- To solve a multi-step inequality, isolate the variable.

$-3(x - 5) \leq 3x + 9$
$-3x + 15 \leq 3x + 9$
$-3x + 15 - 3x \leq 3x + 9 - 3x$
$-6x + 15 \leq 9$
$-6x + 15 - 15 \leq 9 - 15$
$-6x \leq -6$
$\frac{-6x}{-6} \geq \frac{-6}{-6}$
$x \geq 1$

Remember to reverse the inequality symbol, when multiplying or dividing by a negative number.

- Problems that involve comparing different options can often be modelled and solved using inequalities.

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After studying the example, have students complete the Show You Know to check for understanding. You may wish to have students work in pairs and compare their answers. If they do not agree, have them join another pair and discuss how to solve the Show You Know.

- What different methods were used?
- Which one(s) were more efficient?

Key Ideas

The Key Ideas reiterate the two main concepts of this section:

- The idea that multi-step inequalities can be solved using a method similar to that used to solve multi-step equations, as long as the inequality sign is reversed when necessary.
- Multi-step inequalities can be used in modelling situations such as those involving comparison of quantities.

Ensure that students are clear on both these concepts. Have students prepare their own summary of the Key Ideas for this section in their Foldable.

Meeting Student Needs

- Point out in Example 2 that the inequality is written first with words and then changed to algebraic expressions in a step-by-step fashion. This method will likely help students as they work on problems of their own later in the section. Some students might increase their success rate by starting with a word statement showing what is being compared on each side of an inequality and then writing an expression for each side.
- Help students explore various ways of remembering when an inequality sign needs to be reversed. Continue to encourage students to verify solutions.
- Some students will benefit from modelling the inequalities they write using concrete objects, pictures, or diagrams.

ELL

- Ensure students understand the term *commission*.

Answers

Example 1: Show You Know

a) $x > 6$

b) $x < -2$

c) $x \leq 4$

Example 2: Show You Know

a) $15h < 8h + 40$

b) Option B will be less expensive for any job requiring less than 5.71 h.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Many students might benefit from the use of an overhead with algebra tiles or other concrete materials to model the steps in solving multi-step inequalities.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students use concrete materials or diagrams to model situations as they try to develop inequalities to solve problems.

Check Your Understanding

Communicate the Ideas

1. Describe the similarities and differences between the process for solving a multi-step linear equation and a multi-step linear inequality. Discuss your answer with a classmate.

2. Consider the inequality $3x + 10 > 5x + 22$. Lindsay started to solve the inequality by subtracting $5x$ from both sides. Victoria told her to start by subtracting $3x$ from both sides.

a) Use Lindsay's approach to solve the inequality.

b) Use Victoria's approach to solve the inequality.

c) Are the solutions the same? Explain.

d) Explain why you think Victoria gave her advice. Is her reasoning helpful in solving the inequality? Explain.

e) Which method of solving the inequality do you prefer? Explain why.

Practise

For help with #3 to #7, refer to Example 1 on pages 361–363.

3. Solve each inequality and verify the solution.

a) $5x - 19 < 36$

b) $27 + 2x > -13$

c) $3 \leq \frac{x}{5} - 7$

4. Determine the solution of each inequality.

a) $-5y + 92 \geq 40$

b) $2.2 > 10.6 + 4y$

c) $\frac{y}{-6} - 2 < 16$

d) $\frac{3}{2}x + 6 \leq 10\frac{4}{5}$

5. a) Verify that $x \geq 8$ is the correct solution to the inequality $3x + 11 \geq 35$.

b) Verify that $x < -3$ is the correct solution to the inequality $24 - 5x > 39$.

6. Solve each inequality and verify the solution.

a) $7x < 2x + 30$

b) $10x - 22 \geq 8x$

c) $-12x + 10 > 19 - 4x$

d) $\frac{1}{2}(x + 5) > 22$

7. Determine each solution.

a) $-2y > 8y - 20$

b) $9y - 17 \leq 8 + 6.5y$

c) $3.4 - 1.3y < 0.5y - 2.2$

d) $\frac{3}{4}y - 1 \geq -\frac{1}{4}(1 - 2y)$

For help with #8 and #9, refer to Example 2 on pages 363–364.

8. For each situation
- choose a variable and explain what it represents
- write an inequality

a) A basketball team wants to buy new jerseys. Uniforms R Us charges $50 per jersey. Jerseys Unlimited charges $40 per jersey plus $80 for a logo design. How many jerseys does the team need to buy for Jerseys Unlimited to be the better option?

b) Ann uses her cell phone to send text messages. The monthly charge for text messaging is currently $15 plus $0.05 per message sent. The company is offering a new plan that costs a flat rate of $0.12 per text message. How many text messages does Ann need to send in order for the new plan to be the better option?

9. John is considering two paper delivery jobs. The *Advance* will pay $10 plus $0.05 for each paper delivered daily, and the *Times* will pay $15 plus $0.04 for each paper delivered daily. How many papers delivered each day would make the *Advance* the better offer?

a) Write an inequality to model the problem.

b) Solve the inequality and interpret the solution.

WWW Web Link

To learn how to solve inequalities using a graphing calculator, go to www.mathlinks9.ca and follow the links.

Apply

10. Kim is comparing the rates at two car rental companies for a one-day rental. She wants to determine how many kilometres she would need to drive for ABC Rentals to be the better rental option.

ABC Rentals	It's a Deal Rentals
$23 per day plus $0.14 per kilometre	$55 per day

a) Estimate the number of kilometres that would make ABC Rentals the better option.

b) Represent the situation using an inequality.

c) Solve the inequality and interpret the solution.

d) Compare the solution with your estimate.

11. Kevin is comparing job offers at two stores. Dollar Deal offers $8/h plus 10% commission. Great Discounts offers $18/h with no commission. What do Kevin's weekly sales need to be in order for Dollar Deal to pay more? Assume that he works an 8-h day five days per week.

12. The student council is considering two different companies to print the school's yearbooks. Great Graphics charges $250 plus $12.25 per book. Print Express charges $900 plus $9.50 per book. How many orders for yearbooks would make Print Express the better option?

13. Greenway Golf Course offers two plans for paying for buckets of balls at the driving range. How many buckets of balls used per month make the members' plan the better deal?

14. Molly has a business making candles. Her business costs are $200 plus $0.70 per candle made. She charges her customers $3.50 for each candle. If she sells all of the candles she makes, how many candles sold would allow her to make a profit?

Check Your Understanding

Communicate the Ideas

Have students work on their own or with a partner to complete the questions. The two questions ask students to reflect on their understanding of solving multi-step inequalities. To give students a chance to assess their own learning, have them compare answers for these questions with their partners, in a small group, or as a class.

Question #2 is intended to help students realize that when solving a multi-step inequality, one method of solving might require the sign to be reversed while another might not.

Practise

The Practise questions give students an opportunity to check their ability to solve multi-step inequalities as well as to model real-world situations using multi-step inequalities. If graphing calculators are available, you may wish to have students use the Web Link on page 366 and solve some of these questions using a graphing calculator.

Apply

These questions give students a variety of contexts in which they need to model situations involving comparison of quantities using inequalities.

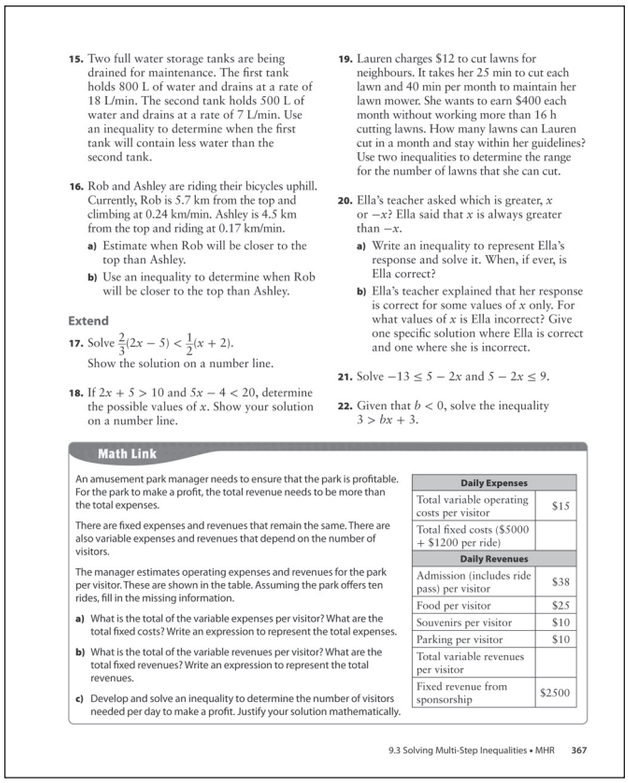
Questions #10 to 13 involve comparing two different options and deciding when one of the options is preferable.

In #14 and 15, because the quantities being compared are decreasing over time, the inequalities involved will have a negative variable term on each side. In solving these inequalities, students may have to reverse the sign. This will depend on the method they use.

Extend

In #18, students need to solve two given inequalities and use both solutions to specify a range of values. This concept is extended in #19. In each question, students need to create two inequalities of their own to solve the problem.

Note that students can give answers to #18 and 19 as a combination of inequalities.



15. Two full water storage tanks are being drained for maintenance. The first tank holds 800 L of water and drains at a rate of 18 L/min. The second tank holds 500 L of water and drains at a rate of 7 L/min. Use an inequality to determine when the first tank will contain less water than the second tank.

16. Rob and Ashley are riding their bicycles uphill. Currently, Rob is 5.7 km from the top and climbing at 0.24 km/min. Ashley is 4.5 km from the top and riding at 0.17 km/min.

a) Estimate when Rob will be closer to the top than Ashley.

b) Use an inequality to determine when Rob will be closer to the top than Ashley.

Extend

17. Solve $\frac{2}{3}(2x - 5) < \frac{1}{2}(x + 2)$. Show the solution on a number line.

18. If $2x + 5 > 10$ and $5x - 4 < 20$, determine the possible values of x. Show your solution on a number line.

19. Lauren charges $12 to cut lawns for neighbours. It takes her 25 min to cut each lawn and 40 min per month to maintain her lawn mower. She wants to earn $400 each month without working more than 16 h cutting lawns. How many lawns can Lauren cut in a month and stay within her guidelines? Use two inequalities to determine the range for the number of lawns that she can cut.

20. Ella's teacher asked which is greater, x or $-x$? Ella said that x is always greater than $-x$.

a) Write an inequality to represent Ella's response and solve it. When, if ever, is Ella correct?

b) Ella's teacher explained that her response is correct for some values of x only. For what values of x is Ella incorrect? Give one specific solution where Ella is correct and one where she is incorrect.

21. Solve $-13 \leq 5 - 2x$ and $5 - 2x \leq 9$.

22. Given that $b < 0$, solve the inequality $3 > bx + 3$.

Math Link

An amusement park manager needs to ensure that the park is profitable. For the park to make a profit, the total revenue needs to be more than the total expenses.

There are fixed expenses and revenues that remain the same. There are also variable expenses and revenues that depend on the number of visitors.

The manager estimates operating expenses and revenues for the park per visitor. These are shown in the table. Assuming the park offers ten rides, fill in the missing information.

a) What is the total of the variable expenses per visitor? What are the total fixed costs? Write an expression to represent the total expenses.

b) What is the total of the variable revenues per visitor? What are the total fixed revenues? Write an expression to represent the total revenues.

c) Develop and solve an inequality to determine the number of visitors needed per day to make a profit. Justify your solution mathematically.

Daily Expenses	
Total variable operating costs per visitor	$15
Total fixed costs ($5000 + $1200 per ride)	
Daily Revenues	
Admission (includes ride pass) per visitor	$38
Food per visitor	$25
Souvenirs per visitor	$10
Parking per visitor	$10
Total variable revenues per visitor	
Fixed revenue from sponsorship	$2500

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The problem in #20 would be a good one to ask all students, even if only to investigate by Guess and Check. Students who do analyse the question using inequalities might find it challenging, even though the inequalities they write look quite simple.

Question #21 asks students to isolate the variable in an inequality requiring several steps to solve, including reversing the inequality sign.

Math Link

The Math Link provides students with an opportunity to apply the use of inequalities to model an open-ended, real-world situation. Have students read the Math Link directions, and identify any words they do not know. Circulate and ensure that students understand the meaning of the words *revenue* and *expenses*. The sequence of questions is intended to help students develop a multi-step inequality to model the situation one step at a time. Encourage students to develop an inequality with a word expression on each side as a starting point (e.g., Revenue > Expenses, or Revenue – Expenses > 0).

Meeting Student Needs

- Provide **BLM 9–10 Section 9.3 Extra Practice** to students who would benefit from more practice.
- Some students may need assistance providing an example of a solution to a multi-step inequality that requires a reversal of the sign. You may wish to have some of these students use an example from the student resource.

Gifted and Enrichment

- Challenge the students to complete all of the Extend questions.
- Students may want to investigate stock market prices and pick stocks to apply an inequality to in order to see if they can make a profit over time. The Vancouver-based market is a good choice for this because it is very volatile and students can see big changes in the value of their portfolio. You can begin the process by having them select $1000 worth of stock shares. Each day they can see if the value of their portfolio is *greater than or equal to* or *less than or equal to* $1000.

Common Errors

- Some students may have difficulty translating a problem situation given verbally into an algebraic inequality.

R_x Encourage students to use concrete materials, diagrams and visual models, and/or inequalities involving word expressions.

- Some students may use information from a problem situation incorrectly, resulting in an incorrect inequality for that situation.

R_x Encourage students to compare their inequalities with a classmate's before applying them to solve the related problem.

- Some students may forget to reverse the inequality sign when multiplying or dividing by a negative number.

R_x Have students use a balance model while solving inequalities where division or multiplication by a negative number is required.

Answers

Communicate the Ideas

1. Example: The same operations are used to isolate the variable including the use of the distributive property. The main difference is that when multiplying or dividing both sides of an inequality by a negative coefficient the direction of the inequality is reversed.

2. a) $x < -6$

b) $-6 > x$

c) Yes, the solutions are the same. Example: The variable is isolated on the left side of the inequality using Lindsay's approach whereas the variable is isolated on the right side of the inequality using Victoria's approach.

d) Example: By subtracting $3x$ from both sides, the coefficient of the variable term remains positive; this will ensure that the direction of the inequality does not have to be reversed when the coefficient is divided out.

e) Example: Victoria's method is preferred for the reason given in part d).

Math Link

a) Variable costs: \$15 per visitor; Fixed costs: \$17 000; Total expenses: $15v + 17000$

b) Variable revenues: \$83 per visitor; Fixed revenues: \$2500; Total revenues: $83v + 2500$

c) $83v + 2500 > 15v + 17\,000$; The park requires at least 214 visitors each day to make a profit.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 and 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to show steps carefully when completing #2. • When working on #2, students will benefit from writing the two methods side by side so that a comparison between the methods is more convenient.
Assessment *for* Learning	
Practise Have students do #3, 6, 8, and 11. Students who have no problems with these questions can go on to do additional Apply questions.	• Encourage students who need coaching with #3 and 6 to refer back to Example 1. They may also benefit from summarizing the discussion held by the class for #2. Have students verbalize the process first and clarify any misunderstandings. Assign selected questions from #4 and 7 to check for understanding. • Students who need coaching with #8 will benefit from reviewing Example 2. Refer students to the Problem Solving section found in the front of the student resource on page xv in order to clarify what needs to be done and how to approach it.
Math Link The Math Link on page 367 is intended to help students work toward the chapter problem wrap-up titled MathLink: Wrap It Up! on page 371.	• Students might benefit by beginning with an inequality that has a word expression on each side. • Students may benefit from referring to their terminology lists included in their Foldable. • Students might benefit from coaching to set up their inequality in order to solve for the number of visitors. Encourage students to solve the problem in more than one way. • Students who need help getting started could use **BLM 9–11 Section 9.3 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Math Learning Log Have students complete the following: • Write a sample problem that involves a situation requiring a multi-step inequality to model and solve. Assess how effective your example is in illustrating the concept. Have students respond to the following prompts: • The easiest part about inequalities is … because … and... • The most challenging part about inequalities is...	• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue having difficulties with. • Encourage students to discuss the responses to their log entries. Any areas students are concerned about should be addressed before moving on.