

# Exploring Angles in a Circle

10.1

**MathLinks 9, pages 378–385**

## Suggested Timing

50–60 minutes

## Materials

- compass or circular geoboard with elastic bands
- protractor
- ruler
- compass
- coloured pencils or markers
- other materials for designing a piece of art

## Blackline Masters

Master 2 Communication Peer Evaluation  
 Master 22 Circular Geoboard  
 BLM 10–3 Chapter 10 Warm-Up  
 BLM 10–5 Section 10.1 Extra Practice  
 BLM 10–6 Section 10.1 Math Link

## Mathematical Processes

- Communication (C)
- Connections (CN)
- Mental Mathematics and Estimation (ME)
- Problem Solving (PS)
- Reasoning (R)
- Technology (T)
- Visualization (V)

## Specific Outcomes

- SS1** Solve problems and justify the solution strategy using circle properties including:
- the perpendicular from the centre of a circle to a chord bisects the chord
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
  - the inscribed angles subtended by the same arc are congruent
  - a tangent to a circle is perpendicular to the radius at the point of tangency.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 6, 8, 10, 11, 13, 15, Math Link
Typical	#1–3, 5–8, 11, 13, 15, 16, Math Link
Extension/Enrichment	#2, 7, 9, 13, 15, 18–22, Math Link

## Planning Notes

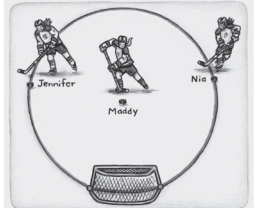
Have students complete the warm-up questions on **BLM 10–3 Chapter 10 Warm-Up** to reinforce material learned in previous sections.

## 10.1 Exploring Angles in a Circle

**FOCUS ON...**  
 After this lesson, you will be able to...

- describe a relationship between inscribed angles in a circle
- relate the inscribed angle and central angle subtended by the same arc

Maddy, Jennifer, and Nia are each about to shoot at the empty net. If they are each equally accurate with their shot, who do you think is most likely to score?



**Materials**

- compass or circular geoboard with elastic bands
- protractor
- ruler

**chord**

- a line segment with both endpoints on a circle

**central angle**

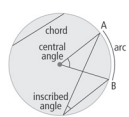
- an angle formed by two radii of a circle

**inscribed angle**

- an angle formed by two chords that share a common endpoint

**arc (of a circle)**

- a portion of the circumference



**Explore Relationships Between Angles in a Circle**

- Construct a large circle and label its centre C. Construct a **chord** AB and a **central angle**  $\angle BCA$ . Measure  $\angle BCA$ .
- Create the **inscribed angle**  $\angle BDA$ . What is the measure of  $\angle BDA$ ?
- How do the measures of  $\angle BCA$  and  $\angle BDA$  compare?
- Create a second inscribed angle  $\angle BEA$ . What is the measure of  $\angle BEA$ ?
- Choose another point on the circle between D and E. Create one more inscribed angle that has its arms touching the endpoints of the **arc** AB. What is the measure of this inscribed angle?
- Repeat steps 1 to 5 for a different sized circle, and a different sized chord AB.

**Reflect and Check**

- What is the relationship between a central angle and an inscribed angle that stands on the same arc?
- What is the relationship between all the inscribed angles that stand on the same arc?

- Predict which hockey player in the opening paragraph is most likely to score on the empty net. Explain.

## Explore Relationships Between Angles in a Circle

Students develop the relationship between inscribed angles and a central angle containing the same arc. You may wish to provide students with **Master 22 Circular Geoboard** if circular geoboards are not available.

**Method 1** Using geoboards, have students work in pairs. As you circulate, ensure that students are locating the correct angles to measure. Ensure that students are carefully measuring the angles with a protractor. Watch whether students are recording the measures of the angles. Encourage them to draw the diagram and record the measures on the diagram or in a table under the diagram.

**Method 2** Using a photocopy of a geoboard, have students work in pairs. Students will need to draw in the chord AB. As you circulate, ensure that students are drawing and measuring inscribed and central angles properly with a ruler and protractor. Watch whether students are recording the measures of the angles. Encourage them to record the measures on the diagram or in a table under the diagram.

**Method 3** Students could use the Web Link on page 379 to construct and measure the required angles in this Explore. Watch whether students are recording the measures of the angles. Encourage them to draw the diagram and record the measures on the diagram or in a table under the diagram.

As a class, go over the Reflect and Check questions (#7 and 8) to ensure that the correct relationship between inscribed angles and the central angle has been determined.

**Meeting Student Needs**

- Discuss with students their experiences with hockey, and with taking shots on net. Ask them why they think taking a shot from the side is more difficult.

**ELL**

- Some students may benefit from a discussion of the terms *shoot at the net* and *construct*.

**Answers**

**Explore Relationships Between Angles in a Circle**

- 3. The measure of  $\angle BDA$  is one-half the measure of  $\angle BCA$ .  
Or, the measure of  $\angle BCA$  is twice the measure of  $\angle BDA$ .
- 7. a) The central angle's measurement is twice the measurement of the inscribed angle.  
Or, the measurement of the inscribed angle is one-half the measurement of the central angle.  
b) All inscribed angles subtended by the same arc have equal measurements.
- 8. Example: Maddy is closer to the net and has a larger range of shot.

Assessment	Supporting Learning
<b>Assessment as Learning</b>	
<p><b>Reflect and Check</b> Listen as students discuss what they discovered during the Explore. Check that they discuss the relationship between central and inscribed angles in #7 and 8. Ensure that students are using the terminology appropriately.</p>	<ul style="list-style-type: none"> <li>• Pair students to compare their measures of inscribed and central angles. Have them agree on the relationship, and write it in their own words.</li> </ul>

### Link the Ideas

You can use properties related to angles in a circle to solve problems.

#### Inscribed Angles

The inscribed angles *subtended* by the same arc are congruent.

#### Central and Inscribed Angles

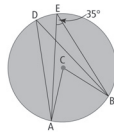
The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc.

#### Example 1: Determine Angle Measures in a Circle

Point C is the centre of the circle.  $\angle AEB = 35^\circ$

a) What is the measure of  $\angle ADB$ ?  
Justify your answer.

b) What is the measure of  $\angle ACB$ ?  
Justify your answer.



#### Literacy Link

An angle that subtends an arc or a chord is an angle that "stands on" or is formed by the endpoints of the arc or chord.

#### Web Link

You may wish to explore these geometric properties on a geoboard or on a computer. Go to [www.mathlinks9.ca](http://www.mathlinks9.ca) and follow the links.

#### Solution

a) The inscribed angles,  $\angle ADB$  and  $\angle AEB$ , are equal because they are subtended by the same arc, AB.

Therefore,  $\angle ADB = 35^\circ$ .

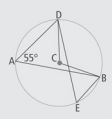
b) The central angle  $\angle ACB$  is subtended by the same arc AB as the inscribed angle  $\angle AEB$ . A central angle is twice the measure of an inscribed angle that is subtended by the same arc.

$$\begin{aligned}\angle ACB &= 2\angle AEB \\ &= 2 \times 35^\circ \\ &= 70^\circ\end{aligned}$$

Therefore,  $\angle ACB = 70^\circ$ .

#### Show You Know

Point C is the centre of the circle.  $\angle DAB = 55^\circ$ . What are the measures of angles  $\angle DEB$  and  $\angle DCB$ ? Justify your answers.



## Link the Ideas

### Example 1

The first example revisits the findings from the Explore. This chapter provides students with many opportunities to justify their answers. As students construct their knowledge of circle geometry, they will need to articulate their thinking. Ensure that students are using appropriate terminology for these angles. Ask:

- Is the angle inscribed or central?
- How do you know?

Make sure that students are aware that an arc with endpoints A and B is indicated by  $\widehat{AB}$ .

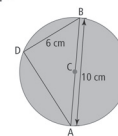
Some students may benefit from actually constructing the circle and accurately drawing  $\angle AEB = 35^\circ$ .

When discussing the solution provided, ask students what the term *subtended* means.

Have students complete the Show You Know. Again, some students may benefit from actually constructing the diagram with an accurate measurement of  $\angle DAB = 55^\circ$ . If students construct this diagram with a protractor, ask students why everyone gets the same answer for the measure of  $\angle DEB$  and  $\angle DCB$  even though they are placed at different positions on the circle from their classmates.

### Example 2: Use Central and Inscribed Angles to Recognize Relationships

Point C is the centre of the circle.  
diameter  $AB = 10$  cm  
chord  $BD = 6$  cm



- a) What is the measure of  $\angle ADB$ ?  
Explain your reasoning.  
b) What is the length of the chord AD?  
Justify your answer.

#### Solution

a) The diameter AB divides the circle into two semicircles. Since AB is a straight line, the central angle  $\angle ACB$  is  $180^\circ$ . Then,  $\angle ADB$  must be half of  $180^\circ$  because it is an inscribed angle that is subtended by the same arc, AB. The measure of  $\angle ADB$  is  $90^\circ$ .

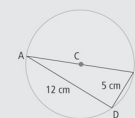
b) Since  $\angle ADB = 90^\circ$ ,  $\triangle ABD$  is a right triangle. The Pythagorean relationship can be used to find the length of AD.

$$\begin{aligned}AD^2 + BD^2 &= AB^2 \\ AD^2 + 6^2 &= 10^2 \\ AD^2 + 36 &= 100 \\ AD^2 &= 64 \\ AD &= \sqrt{64} \\ AD &= 8\end{aligned}$$

Therefore,  $AD = 8$  cm.

#### Show You Know

Point C is the centre of the circle. AB is the diameter.  
chord  $AD = 12$  cm  
chord  $BD = 5$  cm



- a) What is the measure of  $\angle ADB$ ? Explain your reasoning.  
b) What is the length of the diameter AB?

## Example 2

When discussing the Example, you may wish to have students create the visual and label what they know.

Consider asking questions such as:

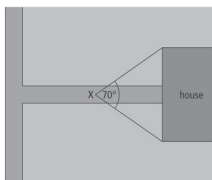
- What does AB form? How do you know?
- What is the angle at C?
- How is this angle related to the angle at D?
- What is the angle at D?
- How can you use this information to help you calculate chord AD?

Some students will struggle with this example because they will not recognize  $\angle ACB$  as a central angle and a straight angle with a measure of  $180^\circ$ . Consequently, students will not recognize  $\angle ADB$  as an inscribed angle containing the same arc as  $\angle ACB$ . Some students may benefit from solving part a) by measuring  $\angle ADB$  with a protractor, and then reading through the solution provided.

Make sure that students are clear that the length of chord AD can be found only because  $\triangle ABD$  is right-angled. Some students may wish to explore further the possibility of  $AD = -8$  as a possible answer. Since  $(-8) \times (-8) = 64$ , discuss the fact that AD represents a length measurement; therefore, it is a positive quantity.

**Example 3: Use Central and Inscribed Angles to Solve Problems**

Jamie works for a realtor. One of his jobs is to photograph houses that are for sale. He photographed a house two months ago using a camera lens that has a  $70^\circ$  field of view. He has returned to the house to update the photo, but he has forgotten his original lens. Today he only has a telephoto lens with a  $35^\circ$  field of view.



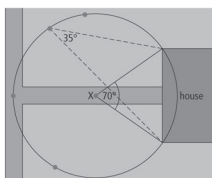
From what location(s) could Jamie photograph the house, with the telephoto lens, so that the entire house still fills the width of the picture. Explain your choices.

**Solution**

Draw a circle with the centre located at the vertex of the  $70^\circ$  angle. Use one arm of the angle as the radius of the circle. Construct any number of different inscribed angles that each contains the front of the house. Any of these points are locations from which Jamie could take the photo. The measure of each of these inscribed angles will be half the measure of the central angle.

$$70^\circ \div 2 = 35^\circ$$

Each inscribed angle will measure  $35^\circ$ , which corresponds to the field of view for Jamie's telephoto lens. Depending on access, and whether there are any trees or a garden in the way, any point on the major arc that is outside of the house will work.



**Strategies**  
Draw a Diagram  
Identify all Possibilities

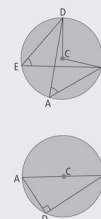
**Literacy Link**  
A major arc is more than a semicircle. A minor arc is less than a semicircle.

**Show You Know**

A flashlight has a field of view measuring  $25^\circ$ , and a camera has a field of view measuring  $50^\circ$ . How can you position the camera and flashlight so that the camera will capture the same area as the flashlight illuminates?

**Key Ideas**

- Inscribed angles subtended by the same arc of a circle are equal.  $\angle DEB = \angle DAB$
- A central angle is twice the measure of an inscribed angle subtended by the same arc.  $\angle DCB = 2\angle DAB$
- An inscribed angle is one half the measure of a central angle subtended by the same arc.  $\angle DAB = \frac{1}{2}\angle DCB$
- When the inscribed angle is subtended by a diameter of the circle, the inscribed angle is equal to  $90^\circ$ .

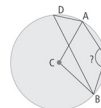


**Literacy Link**  
The identical markings at the inscribed angles  $\angle DEB$  and  $\angle DAB$  indicate that the measures of these angles are equal.

**Check Your Understanding**

**Communicate the Ideas**

- In the diagram,  $\angle BDA$  measures half of  $\angle BCA$ . Does the rule for inscribed angles hold true for  $\angle BEA$ ? Explain your reasoning.

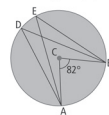


- Manny constructed a circle using a compass. He used a straight edge to draw a diameter. Then, he constructed an inscribed angle that shared endpoints with the diameter. What is the measure of the inscribed angle he constructed? How do you know?

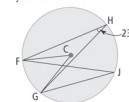
**Practise**

For help with #3 to #5, refer to Example 1 on page 379.

- What are the measures of  $\angle ADB$  and  $\angle AEB$ ? Justify your answers.



- What is the measure of  $\angle FJG$ ? Explain your reasoning.
- What is the measure of  $\angle FCG$ ? Justify your answer.



This question explores an important corollary of the relationship between inscribed and central angles containing the same arc: All inscribed angles that contain a diameter of a circle are right angles.

Have students complete the Show You Know. Ask students what type of triangle  $\triangle ABD$  is and how they know. Again, some students may benefit from constructing the diagram and solving the problem with a protractor and ruler.

**Example 3**

This example provides an applied situation where these circle properties are used. The original diagram does not have a circle. You may wish to present this problem on the board (out of view of the solution) and ask students to think about how they could create a  $35^\circ$  angle with only a compass and a ruler. Discuss this scenario with the class before going over the provided solution. Ideally, students could try their strategies at their desk before proceeding.

For the Show You Know, encourage students to include a diagram in their explanation. Ask students for realistic applications of this question. You may wish to expand more on the science of camera lenses (and/or the human eye) at this point of the lesson.

**Literacy Link** Direct students' attention to the Literacy Link on page 381. Discuss the difference between major and minor arcs, in relation to semicircles.

**Key Ideas**

The Key Ideas summarize the relationships between inscribed angles and the central angle that contain a common arc. Have students draw and label these angles in a circle. Have students verify these relations by measuring the angles with a protractor and labelling these angles. Ask students how the fourth point relates to the second point. (It is a special corollary or sub-case where the diameter is a straight angle.)

**Literacy Link** Review the Literacy Link on page 382 with students. Discuss some of the ways that congruent angles are labelled: single arcs, double arcs, dots, Xs.

### Meeting Student Needs

- Some students may benefit from doing the examples as a full-class activity, and completing the Show You Know work in small groups.
- Some students may benefit from expanding their list of terms by discussing the following: *twice the measure*, *realtor*, and *illuminate*. You may also wish to discuss the different types of camera lenses, such as wide angle, regular, and telephoto.
- As mentioned earlier, some students may benefit from constructing the diagrams in the worked examples in order to verify the angle relationships. They may also benefit from using different colours to outline and label different lines that make angles. Encourage the same constructions for the Show You Know questions.

### Gifted and Enrichment

- Consider having students create their own worked example, by creating one similar to Example 3, and share their results by explaining their example to classmates.

### Common Errors

- Some students may double the measure of the central angle to determine the inscribed angle instead of halving the value of the central angle.
- R<sub>x</sub>** Review the difference between central and inscribed angles, using diagrams to visually show the impossibility of an inscribed angle being larger than a central angle that contains a common chord/arc.
- Some students may have difficulty seeing a diameter as a straight angle.
- R<sub>x</sub>** Review the concept of a straight angle and that a diameter represents a central angle that is a straight angle.



## Answers

### Example 1: Show You Know

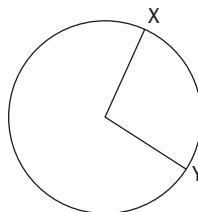
The measure of  $\angle DEB$  is  $55^\circ$ .  
 $\angle DEB$  and  $\angle DAB$  are inscribed angles subtended by the same arc,  $DB$ .  
The measure of  $\angle DCB$  is  $110^\circ$ .  $\angle DCB$  is a central angle subtended by the same arc,  $DB$ , as the inscribed angle,  $\angle DEB$ .

### Example 2: Show You Know

- a) The measure of  $\angle ADB$  is  $90^\circ$ . Since  $AB$  is a diameter, the measure of the central angle  $\angle ACB$  is  $180^\circ$ .  $\angle ADB$  is half of  $180^\circ$  because it is an inscribed angle subtending the same arc,  $AB$ .
- b) The measure of segment  $AB$  is 13 cm.

### Example 3: Show You Know

The camera can be positioned at any point of the major arc,  $XY$ .

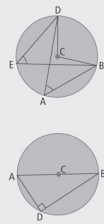


Assessment	Supporting Learning
<b>Assessment for Learning</b>	
<p><b>Example 1</b> Have students do the Show You Know related to Example 1.</p>	<ul style="list-style-type: none"> <li>• Encourage students to verbalize their thinking.</li> <li>• You may wish to have students work with a partner.</li> <li>• Encourage students to create the diagram in their notebook and label angles as they are determined.</li> <li>• Some students may benefit from identifying angles with the proper terms of <i>inscribed angles</i> and <i>central angles</i>.</li> <li>• Encourage the use of the Foldable to help them determine the measurements.</li> </ul>
<p><b>Example 2</b> Have students do the Show You Know related to Example 2.</p>	<ul style="list-style-type: none"> <li>• Encourage students to verbalize their thinking.</li> <li>• You may wish to have students work with a partner.</li> <li>• Review the Pythagorean relationship. Check that students know that the relationship only applies to right triangles.</li> <li>• Some students may benefit from identifying the different parts with the proper terms of <i>right angle</i>, <i>diameter</i>, and <i>chord</i>.</li> <li>• Encourage the use of the Foldable to help them determine the right measurements. Have them include any terminology that they feel may further assist them in their Foldable. Encourage them to develop their own definitions.</li> </ul>
<p><b>Example 3</b> Have students do the Show You Know related to Example 3.</p>	<ul style="list-style-type: none"> <li>• Encourage students to verbalize their thinking.</li> <li>• You may wish to have students work with a partner.</li> </ul>



### Key Ideas

- Inscribed angles subtended by the same arc of a circle are equal.  $\angle DEB = \angle DAB$
- A central angle is twice the measure of an inscribed angle subtended by the same arc.  $\angle DCB = 2\angle DAB$
- An inscribed angle is one half the measure of a central angle subtended by the same arc.  $\angle DAB = \frac{1}{2}\angle DCB$
- When the inscribed angle is subtended by a diameter of the circle, the inscribed angle is equal to  $90^\circ$ .

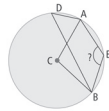


**Literacy Link**  
The identical markings at the inscribed angles  $\angle DEB$  and  $\angle DAB$  indicate that the measures of these angles are equal.

### Check Your Understanding

#### Communicate the Ideas

- In the diagram,  $\angle BDA$  measures half of  $\angle BCA$ . Does the rule for inscribed angles hold true for  $\angle BEA$ ? Explain your reasoning.

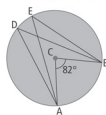


- Manny constructed a circle using a compass. He used a straight edge to draw a diameter. Then, he constructed an inscribed angle that shared endpoints with the diameter. What is the measure of the inscribed angle he constructed? How do you know?

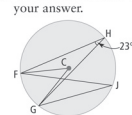
#### Practise

For help with #3 to #5, refer to Example 1 on page 379.

- What are the measures of  $\angle ADB$  and  $\angle AEB$ ? Justify your answers.



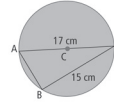
- What is the measure of  $\angle FJG$ ? Explain your reasoning.
  - What is the measure of  $\angle FCG$ ? Justify your answer.



- Draw a circle with a central angle that measures  $60^\circ$ . Draw and label the measure of two inscribed angles that are subtended by the same arc as the central angle.

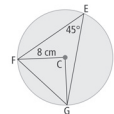
For help with #6 and #7, refer to Example 2 on page 380.

- Point C is the centre of the circle. diameter  $AD = 17$  cm chord  $BD = 15$  cm



- What is the measure of  $\angle ABD$ ? Explain.
- What is the length of the chord  $AB$ ?

- The circle has centre C and a radius of 8 cm.  $\angle FEG = 45^\circ$ .



- What is the measure of  $\angle FCG$ ?
- What is the length of the chord  $FG$ ? Express your answer to the nearest tenth of a centimetre.

For help with #8 and #9, refer to Example 3 on page 381.

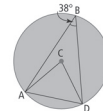
- After a power outage, Jacob helps his mother by shining a flashlight beam at the breaker panel while she locates the tripped breakers. His flashlight projects light through an angle of  $15^\circ$ , while his mother's flashlight projects light through an angle of  $30^\circ$ . Use a diagram to show a good place for Jacob to stand so that his flashlight will illuminate the same area of the breaker panel as his mother's flashlight does.

- For a high school drama production, three spotlights are positioned on an arc at the back of the theatre, just above the audience. Each spotlight projects light through an angle of  $22^\circ$  and fills the rectangular front of the stage. Use a diagram to identify an ideal location to take a photo of the performance using a camera with a lens that has a field of view of  $44^\circ$ .



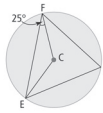
#### Apply

- In the diagram, C is the centre of the circle and  $\angle ABD = 38^\circ$ . For each of the following questions, justify your answer.



- What is the measure of  $\angle ACD$ ?
- What type of triangle is  $\triangle ACD$ ?
- What is the measure of  $\angle CAD$ ?

- Point C is the centre of the circle and  $\angle CFE = 25^\circ$ . Justify each of your answers to the following questions.



- What is the measure of  $\angle ECF$ ?
- What is the measure of  $\angle EGF$ ?

## Check Your Understanding

### Communicate the Ideas

The first question asks students to demonstrate their understanding of the relationship between central and inscribed angles. If protractors are available, have students measure the angles to verify that the first statement made in the question is correct. Students may have difficulty seeing that the reflex angle for  $\angle ACB$  must be considered.

Have students try both questions and discuss their answers with a classmate before discussing as a whole class.

### Practise

Most problems in this section contain diagrams. Some students may need to create the diagram in their notebook in order to label angles and chord lengths as they work through a specific question. For the three questions that do not include diagrams, students are requested to make a diagram. Reinforce with students that they carefully draw and label these diagrams.

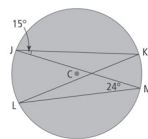
## Apply

Students should find these problems to be reasonable in difficulty after completing the Practise questions. Before working on #10 and 11, some students may benefit from a general discussion about the different types of triangles and their properties. In #13, the centre of the circle is not labelled and does not factor into the question. Students may need to be reminded that since radii of a circle are equal, then triangles that consist of two radii will be isosceles triangles. In #17, students may need to be encouraged to draw a diagram.

## Extend

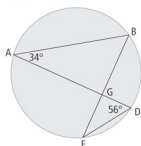
In #20, students will need to create and solve simple algebraic equations. Some students may first need to reactivate some of their algebraic skills.

12. If  $\angle KJM = 15^\circ$ ,  $\angle JML = 24^\circ$ , and point C is at the centre of the circle, what is the measure of each of the following angles?



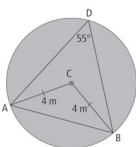
- a)  $\angle KLM$       b)  $\angle JKL$   
c)  $\angle JCL$       d)  $\angle KCM$

13. In the diagram,  $\angle BAD = 34^\circ$  and  $\angle ADE = 56^\circ$ .



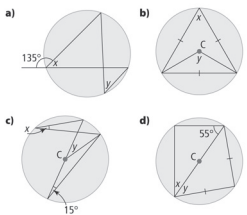
- a) What is the measure of  $\angle ABE$ ?  
b) What is the measure of  $\angle AGB$ ?  
c) What type of triangle is  $\triangle ABG$ ?  
d) What is the measure of  $\angle DGE$ ?

14. After looking at the diagram of the circle, Amanda decides to use the Pythagorean relationship to calculate the length of chord AB. Will this method work? Explain.



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15. Find the unknown angle measures,  $x$  and  $y$ , in each diagram. Where C is labelled, it is the centre of the circle.

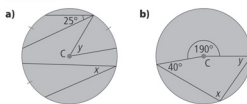


16. Design a geometry question involving a given central angle for which the answer is an inscribed angle measuring  $30^\circ$ . Include a diagram with your question.

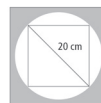
17. A circle with centre C has a diameter AB. The inscribed angle  $\angle ADE$  measures  $14^\circ$ . What are the measures of  $\angle ACE$  and  $\angle ABE$ ? Draw a diagram.

#### Extend

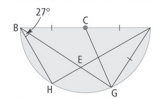
18. Find the unknown angle measures,  $x$  and  $y$ , in each diagram, given that C is the centre of the circle.



19. A hole has a diameter of 20 cm. What is the maximum side length of a square that will fit into the hole?



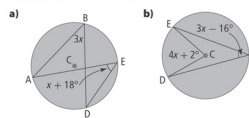
21. In the semicircle,  $\angle HBE = 27^\circ$ . C is on the diameter and is the midpoint of AB.



Determine the measure of each angle, justifying your work mathematically.

- a)  $\angle BHA$   
b)  $\angle BEH$   
c)  $\angle AEG$   
d)  $\angle ACG$   
e)  $\angle BCG$

20. For each of the following diagrams, calculate the value of  $x$ .



#### Math Link

- a) Design a piece of art using one circle and any number of inscribed and central angles.  
b) Describe how the angles and line segments in your design are related.

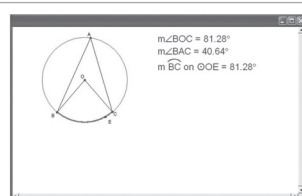
#### Tech Link

##### Inscribed and Central Angles

In this activity, you will use dynamic geometry software to explore inscribed and central angles in a circle. To use this activity, go to [www.mathlinks9.ca](http://www.mathlinks9.ca) and follow the links.

##### Explore

- a) What is the measure of the central angle?  
b) What is the measure of the inscribed angle?  
c) What is the measure of the minor arc BC?
- Drag point A around the circle. What happens to the measure of the two angles  $\angle BOC$  and  $\angle BAC$ ? Why does this happen?
- Drag either point B or point C around the circle. Record at least four measurements of the inscribed angle and the central angle from different locations on the circle.
- Describe any relationships between the central angle  $\angle BOC$  and the inscribed angle  $\angle BAC$  subtended by the same arc.



$\angle BAC$	$\angle BOC$

10.1 Exploring Angles in a Circle • MHR 385

### Math Link

This Math Link provides students with an opportunity to create a piece of art with the properties that they have been studying in circle geometry. It is recommended that students complete this Math Link as it may be the precursor to the art that some students will design in the Math Link: Wrap It Up! at the end of the chapter. Ensure that students are incorporating central and inscribed angles in their design.

### Meeting Student Needs

- It may be important for some students to use manipulatives for as long as they need in order to understand the difference between a central and inscribed angle. Manipulatives would include geoboard, protractor, compass, and ruler.
- Students may need not only a refresher on how to use a protractor but may require individual help.
- Having students draw in the chord that joins the two endpoints of the arc when it is not shown may be useful in identifying the central and inscribed angles.
- Provide **BLM 10–5 Section 10.1 Extra Practice** to students who would benefit from more practice.

### Gifted and Enrichment

- Challenge students to decide which parts of a question can be solved more than one way, and have them explain their thinking.

### Common Errors

- Some students may struggle with the reflex angle for  $\angle ACB$  in #1.
- R<sub>x</sub>** Provide students with opportunities to practise identifying and measuring reflex angles.
- Some students may not be able to visualize what Manny is doing in #2.
- R<sub>x</sub>** Students will benefit from constructing the diagram that Manny creates.
- Some students may struggle with properties of isosceles and equilateral triangles.
- R<sub>x</sub>** Help students to recall properties of special triangles.



## Answers

### Communicate the Ideas

1. Yes, the measure of  $\angle BEA$  is one half the measure of the reflex angle  $\angle ACB$  or one half ( $360^\circ - \text{measure of } \angle ACB$ ). Example: An inscribed angle is one half the measure of a central angle subtended by the same arc.  $\angle BEA$  and  $\angle BCA$  are both subtended by the major arc  $AB$  (the larger arc).

2. The inscribed angle is  $90^\circ$  because the inscribed angle is subtended by the diameter of the circle.

Assessment	Supporting Learning
<b>Assessment as Learning</b>	
<p><b>Communicate the Ideas</b> Have all students complete #1 and 2.</p>	<ul style="list-style-type: none"> <li>• Encourage students to verbalize their thinking.</li> <li>• You may wish to have students work with a partner.</li> <li>• Some students may benefit from referring back to Examples 1 and 2 to help with the questions.</li> <li>• The use of their Foldable should be encouraged.</li> <li>• You may wish to have students use <b>Master 2 Communication Peer Evaluation</b> to assess each other's answers to one or both of these questions.</li> </ul>
<b>Assessment for Learning</b>	
<p><b>Practise</b> Have students do #3, 5, 6, 8, 10, 11, and 13. Students who have no problems with these questions can go on to the remaining Apply questions.</p>	<ul style="list-style-type: none"> <li>• Students who struggle with #3 and 6 may need to have more guided help with these questions. Then, use #4 and 7 to assess whether these students have mastered the ideas.</li> <li>• For #5 and 6, some students may benefit from using a ruler and protractor to verify their thinking, or from more guided help with these questions. They may benefit from reviewing Examples 2 and 3. Use #4 and 7 to assess students' mastering of the ideas.</li> <li>• For the remaining questions, have students verbalize the angles they see. Have them identify the inscribed and central angles.</li> <li>• For these questions, some students may benefit from creating the diagram in their notebook so that it focuses only on the questions and removes the extra angles within the circle.</li> </ul>
<p><b>Math Link</b> The Math Link on page 385 is intended to help students work toward the chapter problem wrap-up titled Math Link: Wrap It Up! on page 407.</p>	<ul style="list-style-type: none"> <li>• This is a good open-ended exercise to check whether students are developing an understanding of the relationship between central and inscribed angles.</li> <li>• Students who need help getting started could use <b>BLM 10–6 Section 10.1 Math Link</b>, which provides scaffolding.</li> </ul>
<b>Assessment as Learning</b>	
<p><b>Literacy Link (page 375)</b> Help students to recall the terms introduced in this section by adding the new terms to their web.</p>	<ul style="list-style-type: none"> <li>• Have students use these new terms, for example, by describing to one another how they created the inscribed angle.</li> </ul>
<p><b>Math Learning Log</b> Have student respond to the following prompt: • Jenny says that a central angle cannot be larger than <math>180^\circ</math>. Explain why Jenny is not correct. Use a diagram in your explanation with an inscribed angle that contains the same arc as the central angle.</p>	<ul style="list-style-type: none"> <li>• Encourage students to add definitions from this section to their Foldable. Advise them also to record notes, examples, and Key Ideas.</li> <li>• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with.</li> </ul>