10.2 Exploring Chord Properties

***MathLinks 9*, pages 386–393**

Suggested Timing
50–60 minutes

Materials
- compass
- tracing paper
- ruler
- coloured pencils or markers
- other materials for creating a mandala

Blackline Masters
Master 2 Communication Peer Evaluation
BLM 10–3 Chapter 10 Warm-Up
BLM 10–7 Section 10.2 Extra Practice
BLM 10–8 Section 10.2 Math Link

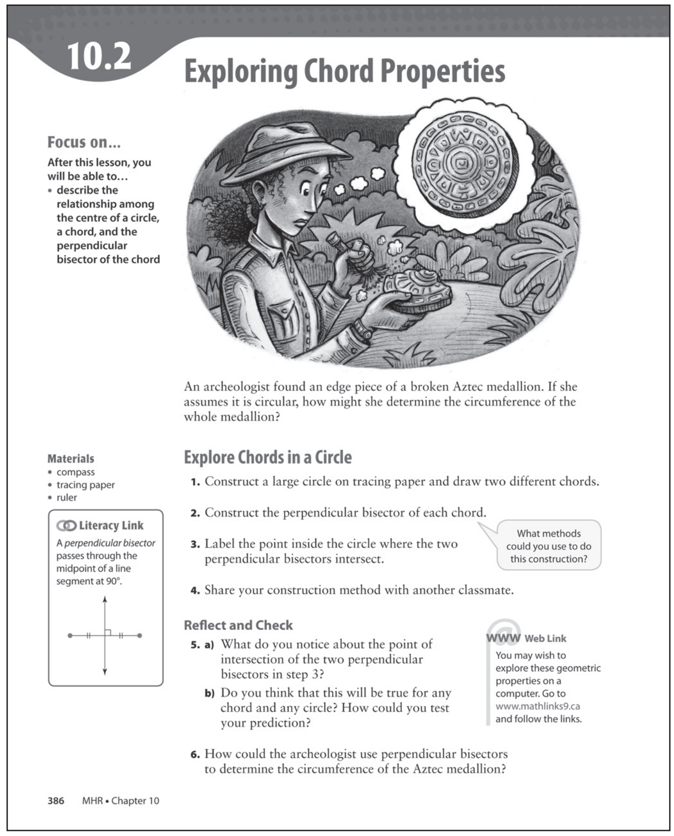
Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
SS1 Solve problems and justify the solution strategy using circle properties including:
- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- a tangent to a circle is perpendicular to the radius at the point of tangency.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6, 9, 11, 12, Math Link
Typical	#1–3, 4 or 5, 6, 7 or 8, 11, 12, 14, 16, Math Link
Extension/Enrichment	#3, 9, 16–21, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 10–3 Chapter 10 Warm-Up** to reinforce material learned in previous sections.



10.2 Exploring Chord Properties

Focus on...
After this lesson, you will be able to...
- describe the relationship among the centre of a circle, a chord, and the perpendicular bisector of the chord

An archeologist found an edge piece of a broken Aztec medallion. If she assumes it is circular, how might she determine the circumference of the whole medallion?

Materials
- compass
- tracing paper
- ruler

Literacy Link
A *perpendicular bisector* passes through the midpoint of a line segment at 90°.

Explore Chords in a Circle

1. Construct a large circle on tracing paper and draw two different chords.
2. Construct the perpendicular bisector of each chord.
3. Label the point inside the circle where the two perpendicular bisectors intersect.

What methods could you use to do this construction?

4. Share your construction method with another classmate.

Reflect and Check

5. a) What do you notice about the point of intersection of the two perpendicular bisectors in step 3?
   b) Do you think that this will be true for any chord and any circle? How could you test your prediction?
6. How could the archeologist use perpendicular bisectors to determine the circumference of the Aztec medallion?

WWW Web Link
You may wish to explore these geometric properties on a computer. Go to www.mathlinks9.ca and follow the links.

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Explore Chords in a Circle

In this exploration, students learn how to find the centre of a circle using perpendicular bisectors of chords. This activity provides a constructivist approach to developing the relationship between a chord, its perpendicular bisector, and the centre of a circle.

Method 1 By using tracing paper, students can construct the perpendicular bisectors by simple paper folding without the need for a protractor. Also, students can confirm that they have found the centre of the circle by using paper folding to construct two unique diameters and then finding their intersection. Have students work in groups of two and make sure that they compare their results with another pair of students.

Method 2 Without tracing paper, have students use a ruler to locate the midpoint of the chord and then construct a perpendicular bisector of the chord using a protractor. Again, have students work in pairs and compare their answers with another pair of students.

Ideally, students will have the opportunity to select either method listed above if they have access to all necessary equipment. As you circulate, identify students who are not sure how to get started. Ask them if they remember constructing perpendicular bisectors from last year, and try to draw out the procedure from their prior knowledge. Ask students who accomplish the task quickly to draw a third chord and perpendicular bisector to confirm that the bisector intersects with the centre as well.

Have students complete the Reflect and Check questions in their notebook. As a class, discuss students' response to #6 regarding the Aztec medallion. Ask:

- Why would an archaeologist want to determine the circumference?
- Is there is a minimum-sized piece of the medallion that would be required to use this method?
- How could this exercise be simulated in the math class?

Meeting Student Needs

- It is important to provide time for students to complete the Explore as this will provide students with a visual of a perpendicular bisector.
- Invite students to research designing arm bands for fancy costumes. Have them relate their research to using perpendicular bisectors to determine circumference. They can find information at the Web Link below.

ELL

- Some students may benefit from discussing the following terms that relate to the Explore: *archaeologist* and *medallion*.
- If several students share a common first language, consider having them work through the questions in small groups.

Gifted and Enrichment

- Challenge students to investigate the role of chords in the development of sine values in trigonometry, and write about how chords and trigonometry are related.

WWW Web Link

For more information on arm bands for fancy dress costumes, go to www.mathlinks9.ca and follow the links.

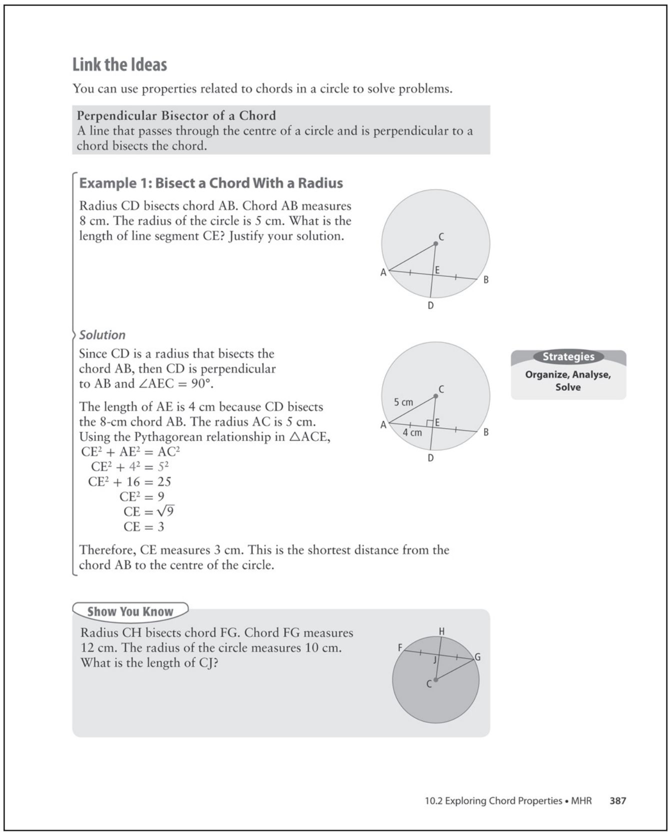
Answers

Explore Chords in a Circle

5. a) The point is the centre of the circle.
b) Yes. Example: You could construct more examples, and compare your constructions with other students` constructions.

6. Example: Use perpendicular bisectors to find centre of the circle. Use an endpoint of a chord to find the radius.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore.	• Encourage students to draw additional chords and their perpendicular bisectors to confirm their findings. • For #6, focus students on the fact that they need to determine the length of the radius in order to determine the circumference. • Some students may benefit from discussing alternative methods. For example, have students who folded paper to find the perpendicular bisectors discuss their method with students who used protractors.



Link the Ideas

You can use properties related to chords in a circle to solve problems.

Perpendicular Bisector of a Chord
A line that passes through the centre of a circle and is perpendicular to a chord bisects the chord.

Example 1: Bisect a Chord With a Radius

Radius CD bisects chord AB. Chord AB measures 8 cm. The radius of the circle is 5 cm. What is the length of line segment CE? Justify your solution.

Solution

Since CD is a radius that bisects the chord AB, then CD is perpendicular to AB and ∠AEC = 90°.

The length of AE is 4 cm because CD bisects the 8-cm chord AB. The radius AC is 5 cm. Using the Pythagorean relationship in △ACE,

$CE^2 + AE^2 = AC^2$
$CE^2 + 4^2 = 5^2$
$CE^2 + 16 = 25$
$CE^2 = 9$
$CE = \sqrt{9}$
$CE = 3$

Therefore, CE measures 3 cm. This is the shortest distance from the chord AB to the centre of the circle.

Strategies
Organize, Analyse, Solve

Show You Know

Radius CH bisects chord FG. Chord FG measures 12 cm. The radius of the circle measures 10 cm. What is the length of CJ?

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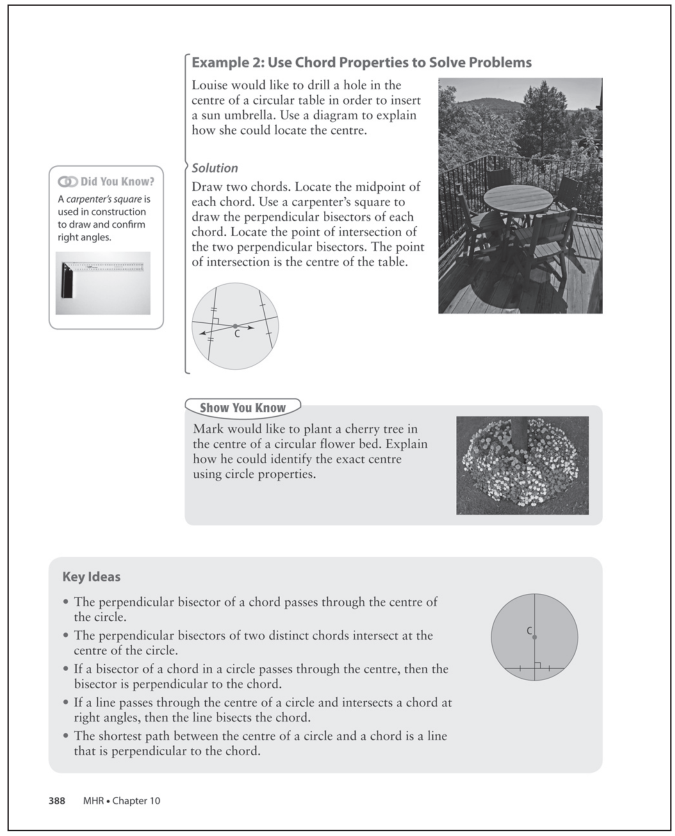

Example 2: Use Chord Properties to Solve Problems

Louise would like to drill a hole in the centre of a circular table in order to insert a sun umbrella. Use a diagram to explain how she could locate the centre.

Solution

Draw two chords. Locate the midpoint of each chord. Use a carpenter's square to draw the perpendicular bisectors of each chord. Locate the point of intersection of the two perpendicular bisectors. The point of intersection is the centre of the table.

Did You Know?
A *carpenter's square* is used in construction to draw and confirm right angles.

Show You Know

Mark would like to plant a cherry tree in the centre of a circular flower bed. Explain how he could identify the exact centre using circle properties.

Key Ideas

- The perpendicular bisector of a chord passes through the centre of the circle.
- The perpendicular bisectors of two distinct chords intersect at the centre of the circle.
- If a bisector of a chord in a circle passes through the centre, then the bisector is perpendicular to the chord.
- If a line passes through the centre of a circle and intersects a chord at right angles, then the line bisects the chord.
- The shortest path between the centre of a circle and a chord is a line that is perpendicular to the chord.

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Link the Ideas

Example 1

This example focuses on the fact that a radius that bisects a chord must be perpendicular to the chord. Students would benefit from constructing this circle with a radius of 5 cm and then drawing a chord that is exactly 8 cm in length. Then, have students determine the midpoint of the chord and draw a radius through the midpoint. Students can then verify with a protractor that the radius is perpendicular to the chord. The focus then shifts to the right triangle that exists. Have students find the missing side of the triangle with the Pythagorean relation. Are students using the relation correctly and identifying the missing side as a leg of the triangle and not the hypotenuse? If students constructed the diagram, have them verify the length of the missing leg with a ruler. Have students try the Show You Know using the Pythagorean relation. As you circulate, ask students why the Pythagorean relation can be used if there is not a square symbol on the diagram indicating that ∠CJG is right-angled. Before starting the Show You Know, you may wish to have students first create the visual and label what they know, and then complete the question.

Example 2

This example is similar to the Explore in terms of identifying the centre of a circle from two chords' perpendicular bisectors. Discuss the question with the class and ask students to provide possible solutions in the context of the actual table. If the Explore was completed, some students will suggest using a piece of tracing paper that is big enough to cover the table. Explore with students some more practical methods. In this case, a tool such as a carpenter's square would be more appropriate for identifying the midpoint of a chord (drawn in pencil) and constructing the perpendicular bisector.

Have students try the Show You Know in their notebook. As a class, have several students share their written responses.

Key Ideas

Try to focus students on the fact that if any two of the following three conditions are in place, then the third condition is true for a given line and a given chord in a circle:

- the line bisects the chord
- the line passes through the centre of the circle
- the line is perpendicular to the chord

Meeting Student Needs

- Some students may benefit from doing the examples as a full-class activity, or completing the Show You Know work in small groups.
- Some students may benefit from discussing the following term that relates to Example 2: *sun umbrella*.
- A revisit of, and careful guidance through, the Pythagorean relationship is recommended.
- Direct students' attention to the Did You Know? Students may benefit from seeing a carpenter's square. Borrow a carpenter's square from a shop teacher to assist students in visualizing how Example 2 could be solved using this measuring tool.

Common Errors

- Some students may apply the Pythagorean relation incorrectly.

R$_x$ Provide students with two questions related to the Pythagorean relation where students must find, first, a missing leg in a right triangle given the length of the other leg and the hypotenuse and, second, the hypotenuse given the length of both legs.

Answers

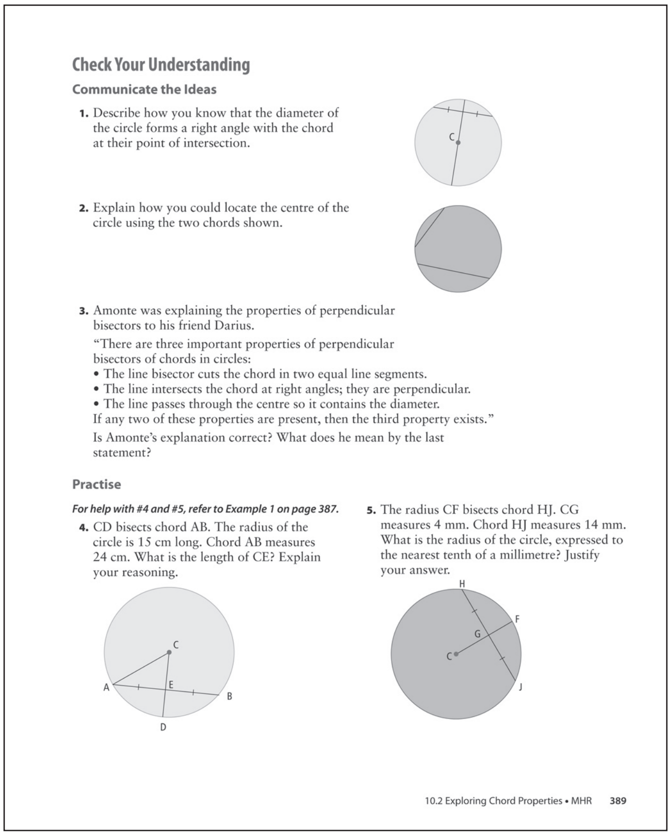
Example 1: Show You Know

8 cm

Example 2: Show You Know

Example: Draw two chords. Find the midpoint of each chord. Use a carpenter's square to locate the perpendicular bisectors of each chord. Locate the point of intersection of the two perpendicular bisectors. This point is the centre of the flower bed.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Check that students are applying the Pythagorean relationship correctly. Ask students specifically how they know that △CJG is a right triangle.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Check that students are using realistic procedures to locate the centre of the flower garden. Tracing paper will not work in this case because of the size of the circle. • Some students may benefit from having an extra large piece of paper available to use to explore possible strategies. Drawing chords, folding the chords over on one another, and then locating their midpoint will provide two lines that cross and locate the centre of the circle. • Bringing in carpenter squares so that students can work with them is a valuable learning opportunity for the class. Groups of students can explain or model their thinking to other groups or the rest of the class.



Check Your Understanding

Communicate the Ideas

1. Describe how you know that the diameter of the circle forms a right angle with the chord at their point of intersection.

2. Explain how you could locate the centre of the circle using the two chords shown.

3. Amonte was explaining the properties of perpendicular bisectors to his friend Darius.

   "There are three important properties of perpendicular bisectors of chords in circles:
   - The line bisector cuts the chord in two equal line segments.
   - The line intersects the chord at right angles; they are perpendicular.
   - The line passes through the centre so it contains the diameter.

   If any two of these properties are present, then the third property exists."

   Is Amonte's explanation correct? What does he mean by the last statement?

Practise

For help with #4 and #5, refer to Example 1 on page 387.

4. CD bisects chord AB. The radius of the circle is 15 cm long. Chord AB measures 24 cm. What is the length of CE? Explain your reasoning.

5. The radius CF bisects chord HJ. CG measures 4 mm. Chord HJ measures 14 mm. What is the radius of the circle, expressed to the nearest tenth of a millimetre? Justify your answer.

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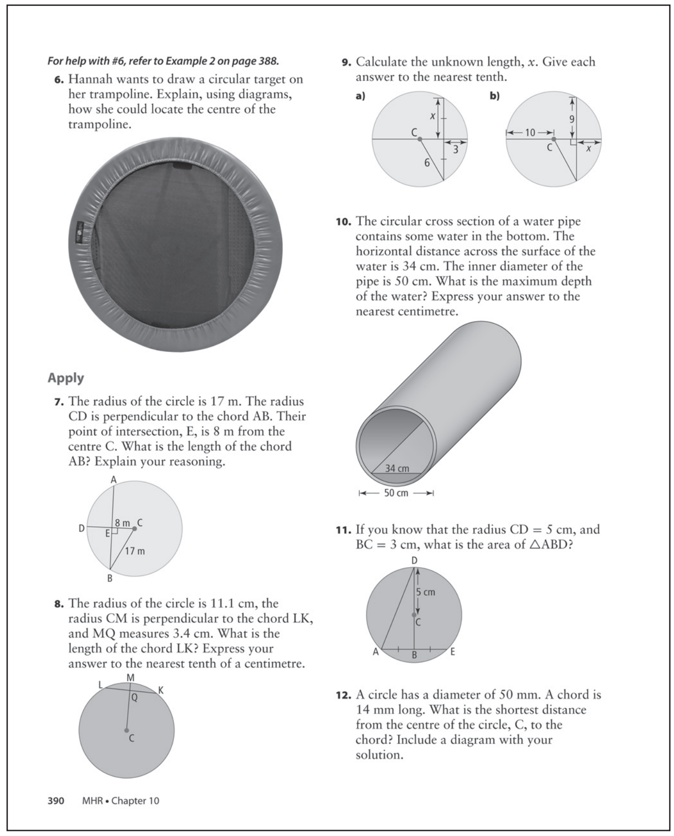

For help with #6, refer to Example 2 on page 388.

6. Hannah wants to draw a circular target on her trampoline. Explain, using diagrams, how she could locate the centre of the trampoline.

Apply

7. The radius of the circle is 17 m. The radius CD is perpendicular to the chord AB. Their point of intersection, E, is 8 m from the centre C. What is the length of the chord AB? Explain your reasoning.

8. The radius of the circle is 11.1 cm, the radius CM is perpendicular to the chord LK, and MQ measures 3.4 cm. What is the length of the chord LK? Express your answer to the nearest tenth of a centimetre.

9. Calculate the unknown length, x. Give each answer to the nearest tenth.

   a)

   b)

10. The circular cross section of a water pipe contains some water in the bottom. The horizontal distance across the surface of the water is 34 cm. The inner diameter of the pipe is 50 cm. What is the maximum depth of the water? Express your answer to the nearest centimetre.

11. If you know that the radius CD = 5 cm, and BC = 3 cm, what is the area of △ABD?

12. A circle has a diameter of 50 mm. A chord is 14 mm long. What is the shortest distance from the centre of the circle, C, to the chord? Include a diagram with your solution.

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Check Your Understanding

Communicate the Ideas

For #1, ensure that students realize they are given the information that the diameter is passing through the midpoint of the chord.

For #2, encourage students to include a diagram in their explanation even though this is not requested in the question. Students may wish to trace the circle into their notebooks and guess where they think the centre is. They can then take measurements and explain how to accurately find the centre.

Question #3 is a good entry point to discuss the teaching notes for the Key Ideas listed above.

Practise

Question #6 could be difficult for students given the scale of the circle. Some students may need some hints of how a measuring tape or piece of wood (e.g., 2 × 4) could be used to construct the chords.

Apply

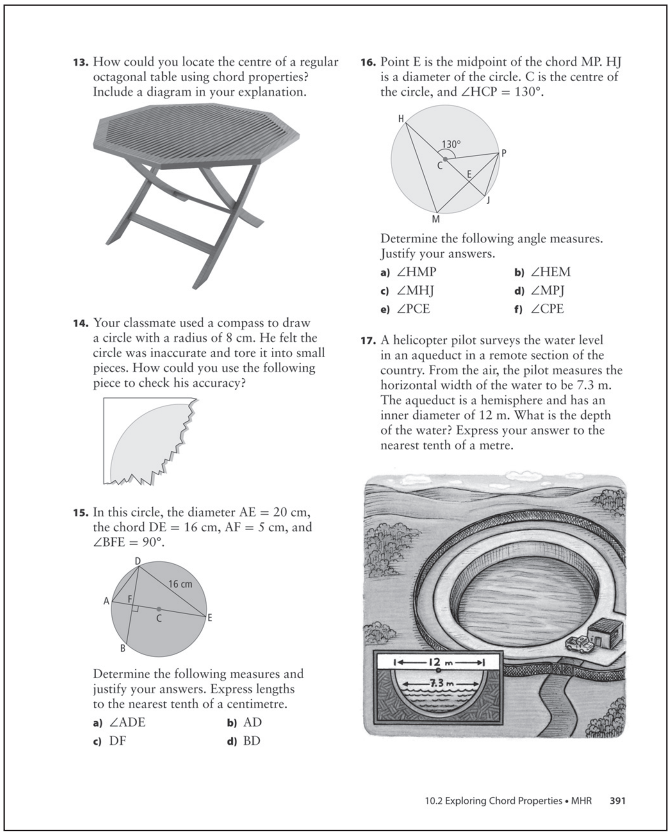
In question #8, the dimensions provided are not included in the diagram. Ensure that students are labelling the proper line segments with the correct dimensions.

In #9, some students may need guidance about which line segment is intended by x.

In #13, students may need assistance in visualizing the regular octagon inside a circle where each side of the table represents a chord of the circle.

For #14, you may wish to have students first create the visual and label what they know, and then complete the question.

In #18, Gavyn's mistake is a common one with the Pythagorean relationship.



13. How could you locate the centre of a regular octagonal table using chord properties? Include a diagram in your explanation.

14. Your classmate used a compass to draw a circle with a radius of 8 cm. He felt the circle was inaccurate and tore it into small pieces. How could you use the following piece to check his accuracy?

15. In this circle, the diameter AE = 20 cm, the chord DE = 16 cm, AF = 5 cm, and ∠BFE = 90°.

Determine the following measures and justify your answers. Express lengths to the nearest tenth of a centimetre.

a) ∠ADE **b)** AD
c) DF **d)** BD

16. Point E is the midpoint of the chord MP. HJ is a diameter of the circle. C is the centre of the circle, and ∠HCP = 130°.

Determine the following angle measures. Justify your answers.

a) ∠HMP **b)** ∠HEM
c) ∠MHJ **d)** ∠MPJ
e) ∠PCE **f)** ∠CPE

17. A helicopter pilot surveys the water level in an aqueduct in a remote section of the country. From the air, the pilot measures the horizontal width of the water to be 7.3 m. The aqueduct is a hemisphere and has an inner diameter of 12 m. What is the depth of the water? Express your answer to the nearest tenth of a metre.

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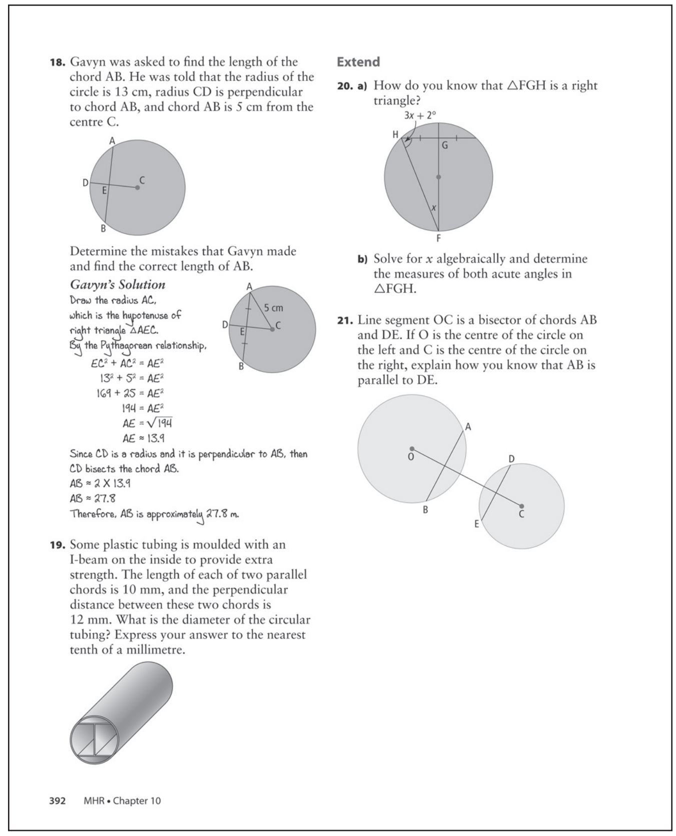

18. Gavyn was asked to find the length of the chord AB. He was told that the radius of the circle is 13 cm, radius CD is perpendicular to chord AB, and chord AB is 5 cm from the centre C.

Determine the mistakes that Gavyn made and find the correct length of AB.

Gavyn's Solution

Draw the radius AC, which is the hypotenuse of right triangle △AEC.
By the Pythagorean relationship,
$EC^2 + AC^2 = AE^2$
$13^2 + 5^2 = AE^2$
$169 + 25 = AE^2$
$194 = AE^2$
$AE = \sqrt{194}$
$AE \approx 13.9$
Since CD is a radius and it is perpendicular to AB, then CD bisects the chord AB.
AB ≈ 2 X 13.9
AB ≈ 27.8
Therefore, AB is approximately 27.8 m.

19. Some plastic tubing is moulded with an I-beam on the inside to provide extra strength. The length of each of two parallel chords is 10 mm, and the perpendicular distance between these two chords is 12 mm. What is the diameter of the circular tubing? Express your answer to the nearest tenth of a millimetre.

Extend

20. a) How do you know that △FGH is a right triangle?

b) Solve for x algebraically and determine the measures of both acute angles in △FGH.

21. Line segment OC is a bisector of chords AB and DE. If O is the centre of the circle on the left and C is the centre of the circle on the right, explain how you know that AB is parallel to DE.

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Extend

Question #21 provides students with a opportunity to use their understanding of circle properties to prove that segments AB and DE must always be parallel.

Math Link

This Math Link is important for students who are going to create a piece of art in the Math Link: Wrap It Up! at the end of the chapter. There are connections between math, art, and culture that may be explored here.

Meeting Student Needs

- Some students may benefit from discussing the following terms that relate to the Check Your Understanding questions: *distinct chords*, *aqueduct*, *moulded*, *I-beam*, and *mandala*.
- Provide **BLM 10–7 Section 10.2 Extra Practice** to students who would benefit from more practice.
- You may wish to reactivate student knowledge about how to calculate the area of a triangle. As a class, discuss different methods that can be used.

Math Link

The North American Plains Indians and Tibetan Buddhists create mandalas. A mandala is a piece of art framed within a circle. The design draws the viewer's eyes to the centre of the circle. Mandalas have spiritual significance for their creators. The photo shows a Buddhist monk using coloured sand to create a mandala.

a) Refer to the portion of the sand mandala shown in the picture. Design a mandala with a similar pattern but your own design. For example, you could create a mandala to celebrate the work of a famous mathematician. Your design should show only part of the mandala.

b) If you want to display your mandala, you will need to know how much room the entire design will take up. What is a reasonable estimate for the circumference of your mandala? Explain your reasoning.

c) How do you think the monks ensure symmetry in their mandalas? How could you use your knowledge of circle properties to help you?

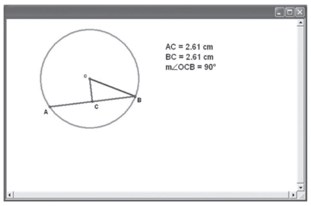
WWW Web Link

For more information about sand mandalas, go to www.mathlinks9.ca and follow the links.

Tech Link

Perpendicular Lines to a Chord

In this activity, you will use dynamic geometry software to explore perpendicular lines from the centre of a circle to a chord. To use this activity, go to www.mathlinks9.ca and follow the links.



Explore

1. a) What is the measure of ∠OCB?
   b) What is the measure of line segment AC?
   c) What is the measure of line segment BC?
2. Drag point A to another location on the circle.
   a) Describe what happens to the measure of ∠OCB when you drag point A to a different location on the circle.
   b) What happens to the measures of the line segments AC and BC? Explain.
3. Drag point B around the circle.
   a) What effect does this have on the measure of ∠OCB?
   b) What effect does this have on the lengths of line segment AC and line segment BC?
4. What conclusions can you make about ∠OCB, the angle formed by the segment from the centre of the circle to the midpoint of the chord?
5. What conclusions can you make about the relationship between line segment AC and line segment BC?

Answers

Communicate the Ideas

1. If a bisector of a chord passes through the centre of the circle, then the bisector is perpendicular or meets at right angles to the chord.
2. Locate and draw the perpendicular bisectors of each chord. The point of intersection is the centre of the circle.
3. Example: Yes, the third statement would not be true unless the first two statements are true.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1–3. Check their responses for conceptual understanding.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to use multiple approaches for #1 and 2. Some students may benefit from sharing these strategies as a list on the board. Encourage students to write down their approaches in their Foldable for future reference. • Some students may benefit from discussing #3, which is a higher-level thinking question. Referring back to the Key Ideas may facilitate the discussion. • You may want to provide students with **Master 2 Communication Peer Evaluation** to assess each other's responses to one or more of these questions.
Assessment *for* Learning	
Practise Have students do #4, 6, 9, 11, and 12. Students who have no problems with these questions can go on to the rest of the Apply questions.	• Some students will need assistance with appropriate materials to construct two chords in #6. • You may wish to have students work with a partner. • Some students may benefit from reviewing the respective examples for #4 (Example 1) and 6 (Example 2). It may also be beneficial to complete the actual construction before solving the problem. • Have students verbalize their process to assist in linking their learning and understanding to the solution. • Have students draw and label their diagrams for #10 and 12 before attempting to solve them.
Math Link The Math Link on page 393 is intended to help students work toward the chapter problem wrap-up titled Math Link: Wrap It Up! on page 407.	• Encourage students to view mandalas online in order to get an appreciation of their aesthetic value. • You may wish to save some images of mandalas and present them in class. Students may benefit from discussing them as a class, and reviewing the patterns and characteristics. • Encourage students to verbalize their thinking. • Students who need help getting started could use **BLM 10–8 Section 10.2 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Literacy Link (page 375) Help students to recall the terms introduced in this section by adding the new terms to their web.	• When introducing the term *perpendicular bisector* for students, it might help if students note that *bi* means two and *sector* sounds like section. • Discuss the meaning of the terms *hypotenuse* and *radius*, as well as *chord*, *perpendicular*, and *bisector*.
Math Learning Log Have students complete the following statements to identify their thinking: • The process I understand best to draw a perpendicular bisector is … because… • The part I understand best about the Pythagorean relationship is…	• Encourage students to add definitions from this section to their Foldable. Advise them also to record notes, examples, and Key Ideas. • Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with.