# 10.3 Tangents to a Circle

***MathLinks 9*, pages 394–403**

## Suggested Timing
50–60 minutes

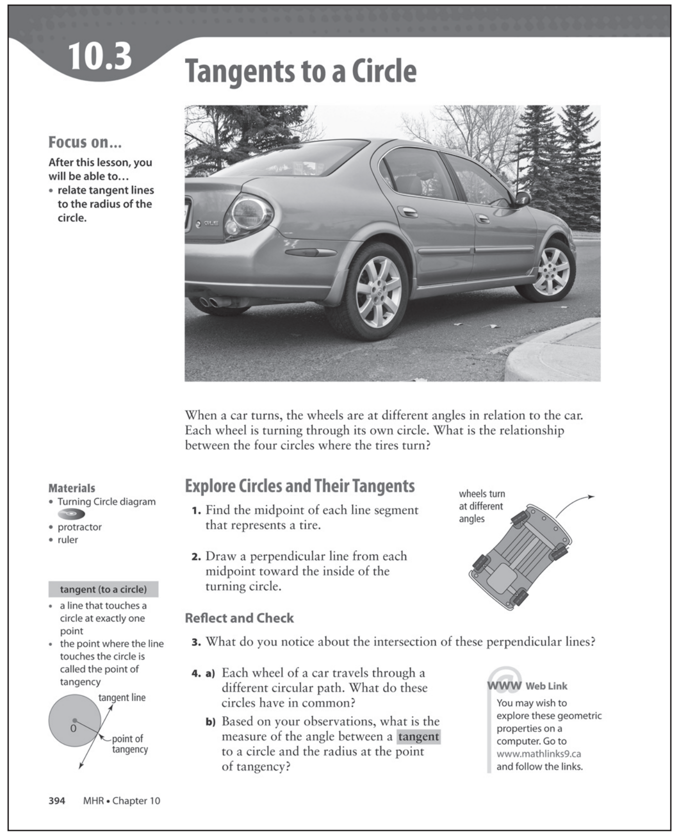
## Materials
- Turning Circle diagram
- protractor
- ruler
- grid paper
- compass
- coloured pencils or markers
- other materials for designing a piece of art or logo

## Blackline Masters
Master 2 Communication Peer Evaluation
Master 8 Centimetre Grid Paper
Master 9 0.5 Centimetre Grid Paper
BLM 10–3 Chapter 10 Warm-Up
BLM 10–9 Turning Circle Diagram
BLM 10–10 Section 10.3 Extra Practice
BLM 10–11 Section 10.3 Math Link

## Mathematical Processes
- ✓ Communication (C)
- ✓ Connections (CN)
- ☐ Mental Mathematics and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ☐ Technology (T)
- ✓ Visualization (V)

## Specific Outcomes
**SS1** Solve problems and justify the solution strategy using circle properties including:
- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- a tangent to a circle is perpendicular to the radius at the point of tangency.

| Category | Question Numbers |
|---|---|
| Essential (minimum questions to cover the outcomes) | #1–3, 6–8, 11, Math Link |
| Typical | #1–3, 6–8, 11, 13, 15, Math Link |
| Extension/Enrichment | #2, 8, 12, 14, 16–22, Math Link |



10.3 Tangents to a Circle

Focus on…
After this lesson, you will be able to…
- relate tangent lines to the radius of the circle.

When a car turns, the wheels are at different angles in relation to the car. Each wheel is turning through its own circle. What is the relationship between the four circles where the tires turn?

Explore Circles and Their Tangents

Materials
- Turning Circle diagram
- protractor
- ruler

1. Find the midpoint of each line segment that represents a tire.
2. Draw a perpendicular line from each midpoint toward the inside of the turning circle.

wheels turn at different angles

Reflect and Check

3. What do you notice about the intersection of these perpendicular lines?
4. a) Each wheel of a car travels through a different circular path. What do these circles have in common?
b) Based on your observations, what is the measure of the angle between a tangent to a circle and the radius at the point of tangency?

tangent (to a circle)
- a line that touches a circle at exactly one point
- the point where the line touches the circle is called the point of tangency

tangent line, O, point of tangency

Web Link
You may wish to explore these geometric properties on a computer. Go to www.mathlinks9.ca and follow the links.

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## Planning Notes

Have students complete the warm-up questions on **BLM 10–3 Chapter 10 Warm-Up** to reinforce material learned in previous sections.

## Explore Circles and Their Tangents

Students may benefit from a demonstration on the overhead of a toy car travelling through a semicircle. Help students visualize the pathway that each wheel will make through the semicircle.

**Method 1** Have students work individually with **BLM 10–9 Turning Circle Diagram** to find the midpoints of the tire segments and to construct the four perpendicular bisectors. Encourage students to compare their pictures with a classmate. Ask:
- Is there an intersection point of the bisectors?
- What does the intersection point represent?

**Method 2** Have students work in pairs on the activity. After students complete their diagram, discuss the results as a class.

### Gifted and Enrichment

- Consider having students explain mathematically how different parts of a solid, such as a CD, can travel at different speeds at the same time. For example, the outside edge of the CD moves faster than the inner part of the CD.

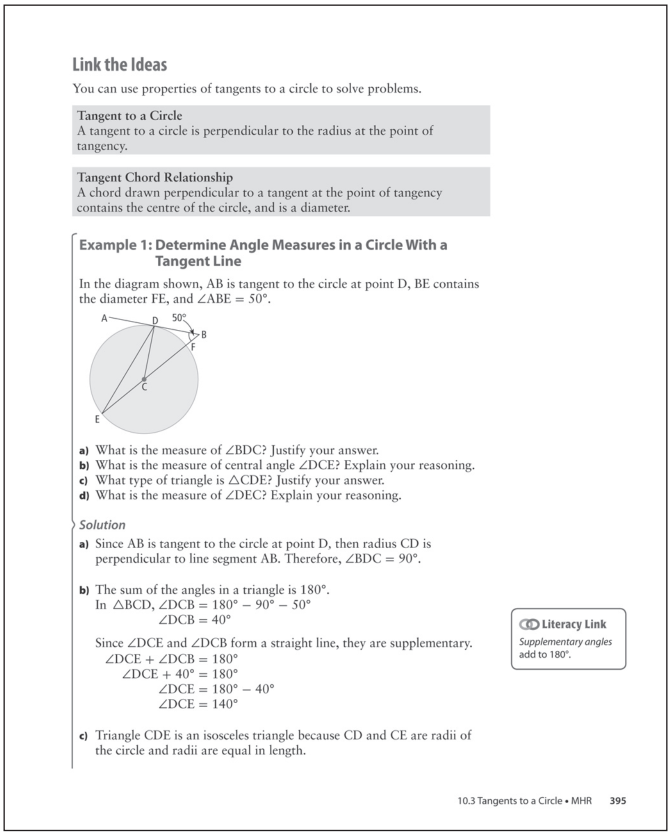
## Answers

**Explore Circles and Their Tangents**

**3. a)** Example: All these lines intersect at the same point.
**b)** Centre of the circle

**4. a)** Example: The circles all have the same centre. **b)** 90°

| Assessment | Supporting Learning |
|---|---|
| **Assessment *as* Learning** | |
| **Reflect and Check**<br>Listen as students discuss what they discovered during the Explore. Help them identify that each wheel is turning through a different circle. Each of these circles is concentric (shares the same centre). | • Help students visualize the line segment that is going to represent the tangent to the circle; it runs along the edge of the tire that is inside with respect to the turning circle.<br>• Students may find it visually helpful to draw the concentric circles for the tires in order to see the placement of a tangent line. This will help them see where the tangent line should be placed on a tire. |



## Link the Ideas

You can use properties of tangents to a circle to solve problems.

**Tangent to a Circle**
A tangent to a circle is perpendicular to the radius at the point of tangency.

**Tangent Chord Relationship**
A chord drawn perpendicular to a tangent at the point of tangency contains the centre of the circle, and is a diameter.

### Example 1: Determine Angle Measures in a Circle With a Tangent Line

In the diagram shown, AB is tangent to the circle at point D, BE contains the diameter FE, and ∠ABE = 50°.

**a)** What is the measure of ∠BDC? Justify your answer.
**b)** What is the measure of central angle ∠DCE? Explain your reasoning.
**c)** What type of triangle is △CDE? Justify your answer.
**d)** What is the measure of ∠DEC? Explain your reasoning.

*Solution*

**a)** Since AB is tangent to the circle at point D, then radius CD is perpendicular to line segment AB. Therefore, ∠BDC = 90°.

**b)** The sum of the angles in a triangle is 180°.
In △BCD, ∠DCB = 180° − 90° − 50°
∠DCB = 40°

Since ∠DCE and ∠DCB form a straight line, they are supplementary.
∠DCE + ∠DCB = 180°
∠DCE + 40° = 180°
∠DCE = 180° − 40°
∠DCE = 140°

**c)** Triangle CDE is an isosceles triangle because CD and CE are radii of the circle and radii are equal in length.

**Literacy Link**
*Supplementary angles* add to 180°.

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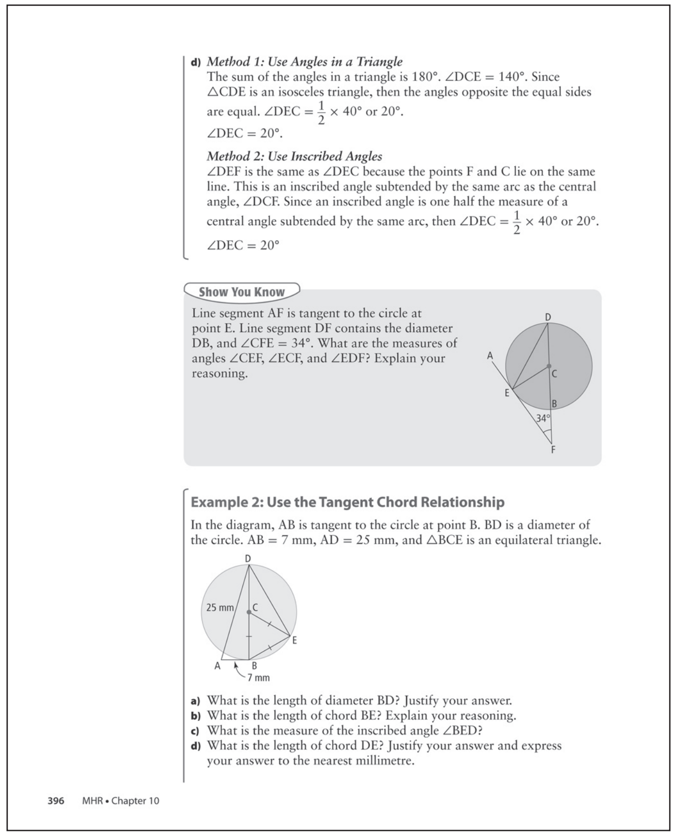

**d)** ***Method 1: Use Angles in a Triangle***
The sum of the angles in a triangle is 180°. ∠DCE = 140°. Since △CDE is an isosceles triangle, then the angles opposite the equal sides are equal. ∠DEC = $\frac{1}{2}$ × 40° or 20°.
∠DEC = 20°.

***Method 2: Use Inscribed Angles***
∠DEF is the same as ∠DEC because the points F and C lie on the same line. This is an inscribed angle subtended by the same arc as the central angle, ∠DCF. Since an inscribed angle is one half the measure of a central angle subtended by the same arc, then ∠DEC = $\frac{1}{2}$ × 40° or 20°.
∠DEC = 20°

**Show You Know**

Line segment AF is tangent to the circle at point E. Line segment DF contains the diameter DB, and ∠CFE = 34°. What are the measures of angles ∠CEF, ∠ECF, and ∠EDF? Explain your reasoning.

### Example 2: Use the Tangent Chord Relationship

In the diagram, AB is tangent to the circle at point B. BD is a diameter of the circle. AB = 7 mm, AD = 25 mm, and △BCE is an equilateral triangle.

**a)** What is the length of diameter BD? Justify your answer.
**b)** What is the length of chord BE? Explain your reasoning.
**c)** What is the measure of the inscribed angle ∠BED?
**d)** What is the length of chord DE? Justify your answer and express your answer to the nearest millimetre.

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# Link the Ideas

## Example 1

Example 1 illustrates the process of applying the tangent radius property of circles. Encourage students to draw the diagram in their notebook and label angles with their measures as they are determined. In part b), students will first need to determine ∠BCD using the fact that the sum of the angles in a triangle equals 180°. Then, ∠DCE can be determined as a supplementary angle to ∠BCD. Ask students about the relationships between these angles as you circulate.

For the Show You Know, encourage students to find the angles in the order requested in the question. For the last angle, students will need to identify that △CDE is an isosceles triangle. Ask students what type of triangle △CDE is. This can be determined at any time during the question by identifying that CD and CE are both radii and are, therefore, equal in length.

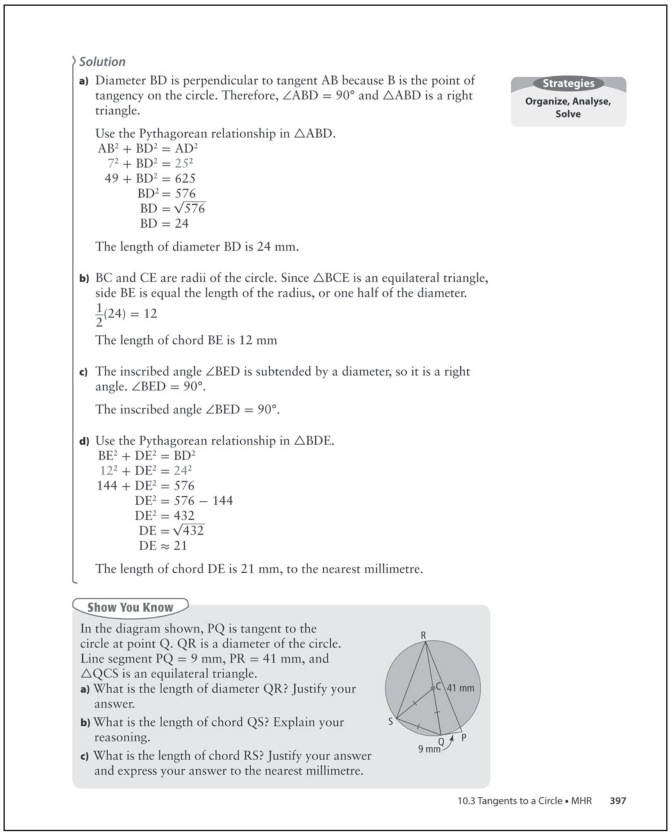
**Literacy Link** Make sure students read and understand the Literacy Link on page 395 about supplementary angles. Encourage students to think of examples of supplementary angles.

## Example 2

In this example, students use the tangent chord relationship to establish that a triangle in the diagram is right-angled. With this knowledge, the application of the Pythagorean relationship is used to determine a missing dimension. Finally, this information is used to apply the Pythagorean relationship to find a missing dimension of a different right triangle.

The question contains a number of different triangles and a quadrilateral. In part c), some students may not realize that ∠BED is an inscribed angle that contains a diameter.

Encourage students to work through the Show You Know in pairs. Ask students about the special triangles that exist in this question: two right triangles and one equilateral triangle.



*Solution*

**a)** Diameter BD is perpendicular to tangent AB because B is the point of tangency on the circle. Therefore, $\angle ABD = 90°$ and $\triangle ABD$ is a right triangle.

Use the Pythagorean relationship in $\triangle ABD$.

$$AB^2 + BD^2 = AD^2$$
$$7^2 + BD^2 = 25^2$$
$$49 + BD^2 = 625$$
$$BD^2 = 576$$
$$BD = \sqrt{576}$$
$$BD = 24$$

The length of diameter BD is 24 mm.

**b)** BC and CE are radii of the circle. Since $\triangle BCE$ is an equilateral triangle, side BE is equal the length of the radius, or one half of the diameter.

$$\frac{1}{2}(24) = 12$$

The length of chord BE is 12 mm

**c)** The inscribed angle $\angle BED$ is subtended by a diameter, so it is a right angle. $\angle BED = 90°$.

The inscribed angle $\angle BED = 90°$.

**d)** Use the Pythagorean relationship in $\triangle BDE$.

$$BE^2 + DE^2 = BD^2$$
$$12^2 + DE^2 = 24^2$$
$$144 + DE^2 = 576$$
$$DE^2 = 576 - 144$$
$$DE^2 = 432$$
$$DE = \sqrt{432}$$
$$DE \approx 21$$

The length of chord DE is 21 mm, to the nearest millimetre.

**Strategies**
Organize, Analyse, Solve

**Show You Know**

In the diagram shown, PQ is tangent to the circle at point Q. QR is a diameter of the circle. Line segment PQ = 9 mm, PR = 41 mm, and $\triangle QCS$ is an equilateral triangle.

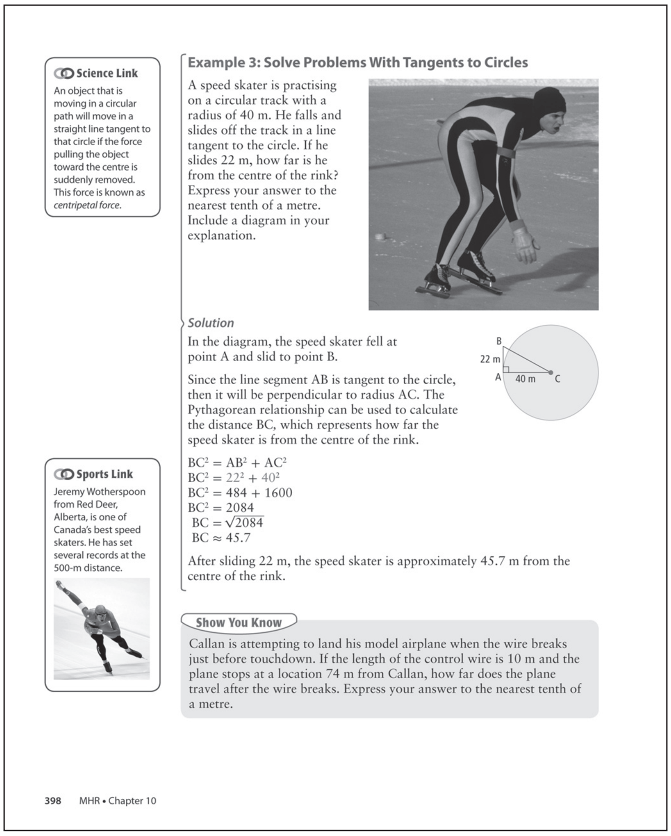
**a)** What is the length of diameter QR? Justify your answer.

**b)** What is the length of chord QS? Explain your reasoning.

**c)** What is the length of chord RS? Justify your answer and express your answer to the nearest millimetre.







**Example 3: Solve Problems With Tangents to Circles**

A speed skater is practising on a circular track with a radius of 40 m. He falls and slides off the track in a line tangent to the circle. If he slides 22 m, how far is he from the centre of the rink? Express your answer to the nearest tenth of a metre. Include a diagram in your explanation.

*Solution*

In the diagram, the speed skater fell at point A and slid to point B.

Since the line segment AB is tangent to the circle, then it will be perpendicular to radius AC. The Pythagorean relationship can be used to calculate the distance BC, which represents how far the speed skater is from the centre of the rink.



$$BC^2 = AB^2 + AC^2$$
$$BC^2 = 22^2 + 40^2$$
$$BC^2 = 484 + 1600$$
$$BC^2 = 2084$$
$$BC = \sqrt{2084}$$
$$BC \approx 45.7$$

After sliding 22 m, the speed skater is approximately 45.7 m from the centre of the rink.

**Science Link**
An object that is moving in a circular path will move in a straight line tangent to that circle if the force pulling the object toward the centre is suddenly removed. This force is known as *centripetal force*.

**Sports Link**
Jeremy Wotherspoon from Red Deer, Alberta, is one of Canada's best speed skaters. He has set several records at the 500-m distance.

**Show You Know**

Callan is attempting to land his model airplane when the wire breaks just before touchdown. If the length of the control wire is 10 m and the plane stops at a location 74 m from Callan, how far does the plane travel after the wire breaks. Express your answer to the nearest tenth of a metre.



## Example 3

Point out to students that when the skater begins to slide, he slides in a line tangent to the circle through which he is skating. The Pythagorean relationship is used to find the missing hypotenuse. The dimensions are larger so students should take care to make sure that their answer is reasonable.

Direct students' attention to the Science Link. You may wish to develop the scientific concept of centripetal force with students to help them establish the relationship between the skater's pathway during the slide with the radius of the circle. For more information about centripetal force, see the Web Link that follows.

Students may be interested in reading the Sports Link about speed skater Jeremy Wotherspoon. For more information about him, refer to the Web Link that follows.

## Key Ideas

The diagram shows the simplicity of the relationship. Ensure that students understand the term *tangent* through examples and non-examples.

### *Meeting Student Needs*

- Some students may benefit from doing the examples as a full-class activity, or completing the Show You Know work in small groups. You might also have students create the diagrams, and label what they know before completing the problems.
- Some students may benefit by working in groups to discuss what they know about angles and lines, and relate that to Examples 1 and 2.

**Web Link**

For more information about centripetal force, go to www.mathlinks9.ca and follow the links.

For more information about speed skater Jeremy Wotherspoon, go to www.mathlinks9.ca and follow the links.

## Answers

**Example 1: Show You Know**

∠CEF = 90°, since AF is tangent to the circle at point E, then the radius EC is perpendicular to the segment AF.
∠ECF = 56°.
Since ΔCEF is a right triangle,
∠ECF = 180° − 90° − 34°
= 56°
∠EDF = 28°
Since ∠EDF is an inscribed angle subtending the same arc as the central angle ∠ECF, its measure is one-half the measure of ∠ECF which is 56°.

**Example 2: Show You Know**

**a)** ΔABD is a right triangle. By using the Pythagorean relationship,

$$9^2 + BD^2 = 41^2$$
$$81 + BD^2 = 1681$$
$$BD^2 = 1600$$
$$BD = \sqrt{1600}$$
$$BD = 40$$

**b)** BC is a radius, so its measurement is half the diameter BD which is 40 mm.
40 ÷ 2 = 20.
The radius is 20 mm.

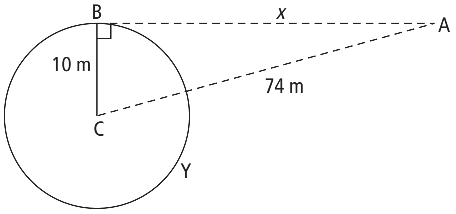
**c)** ∠DEB is 90° since ∠DEB is an inscribed angle subtending a diameter. Therefore, triangle DEB is a right triangle. By using the Pythagorean relationship,

$$20^2 + DE^2 = 40^2$$
$$400 + DE^2 = 1600$$
$$DE^2 = 1200$$
$$DE = \sqrt{1200}$$
$$DE \approx 34.6$$

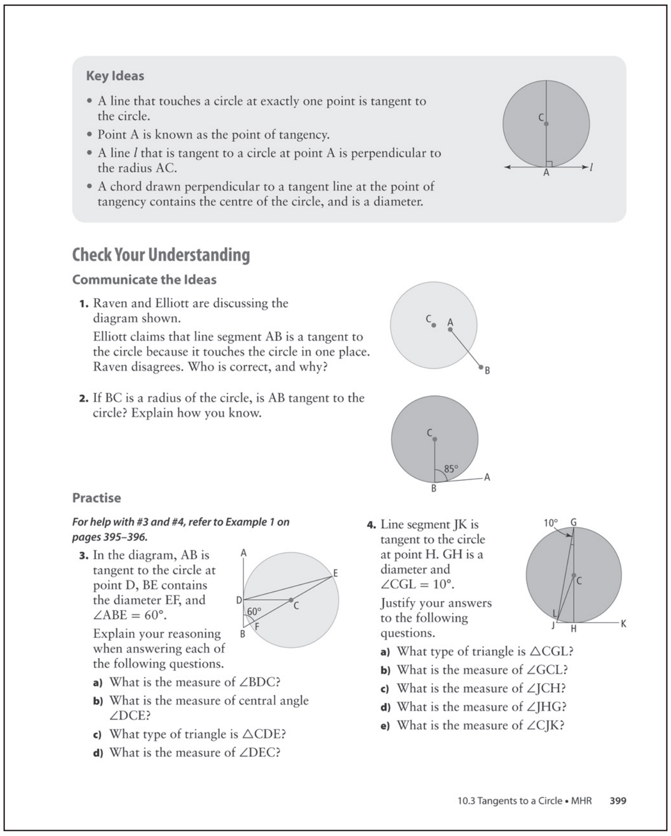
**Example 3: Show You Know**

$$742 = 10^2 + x^2$$
$$742 - 10^2 = x^2$$
$$5476 - 100 = x^2$$
$$5376 = x^2$$
$$73.3 \text{ m} = x$$

The plane travels 73.3 m after the wire breaks.




| Assessment | Supporting Learning |
|---|---|
| **Assessment *for* Learning** | |
| **Example 1**<br>Have students do the Show You Know related to Example 1. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• This example is good for checking the concept of the tangent radius relationship. Make sure that students are moving logically through their determination of each angle and that they are providing appropriate reasoning for their answers. Encourage multiple strategies for solutions.<br>• Some students may benefit from discussing what they know about tangents and radii. Have them verbally describe the process to find the missing angles in order to clarify any misunderstandings. |
| **Example 2**<br>Have students do the Show You Know related to Example 2. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• Students may find it helpful to redraw the triangle separate from the circle, to minimize confusion.<br>• Ensure that students are able to make the necessary connection to determine the right angle. Review the rules and Example 2 as needed. |
| **Example 3**<br>Have students do the Show You Know related to Example 3. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• Students may find it helpful to draw and label a right triangle outside of the diagram.<br>• A class discussion about movement and momentum may be appropriate in the solution to this question. |



### Key Ideas

- A line that touches a circle at exactly one point is tangent to the circle.
- Point A is known as the point of tangency.
- A line *l* that is tangent to a circle at point A is perpendicular to the radius AC.
- A chord drawn perpendicular to a tangent line at the point of tangency contains the centre of the circle, and is a diameter.

### Check Your Understanding

#### Communicate the Ideas

**1.** Raven and Elliott are discussing the diagram shown.

Elliott claims that line segment AB is a tangent to the circle because it touches the circle in one place. Raven disagrees. Who is correct, and why?

**2.** If BC is a radius of the circle, is AB tangent to the circle? Explain how you know.

#### Practise

*For help with #3 and #4, refer to Example 1 on pages 395–396.*

**3.** In the diagram, AB is tangent to the circle at point D, BE contains the diameter EF, and ∠ABE = 60°.

Explain your reasoning when answering each of the following questions.

**a)** What is the measure of ∠BDC?

**b)** What is the measure of central angle ∠DCE?

**c)** What type of triangle is △CDE?

**d)** What is the measure of ∠DEC?

**4.** Line segment JK is tangent to the circle at point H. GH is a diameter and ∠CGL = 10°.

Justify your answers to the following questions.

**a)** What type of triangle is △CGL?

**b)** What is the measure of ∠GCL?

**c)** What is the measure of ∠JCH?

**d)** What is the measure of ∠JHG?

**e)** What is the measure of ∠CJK?

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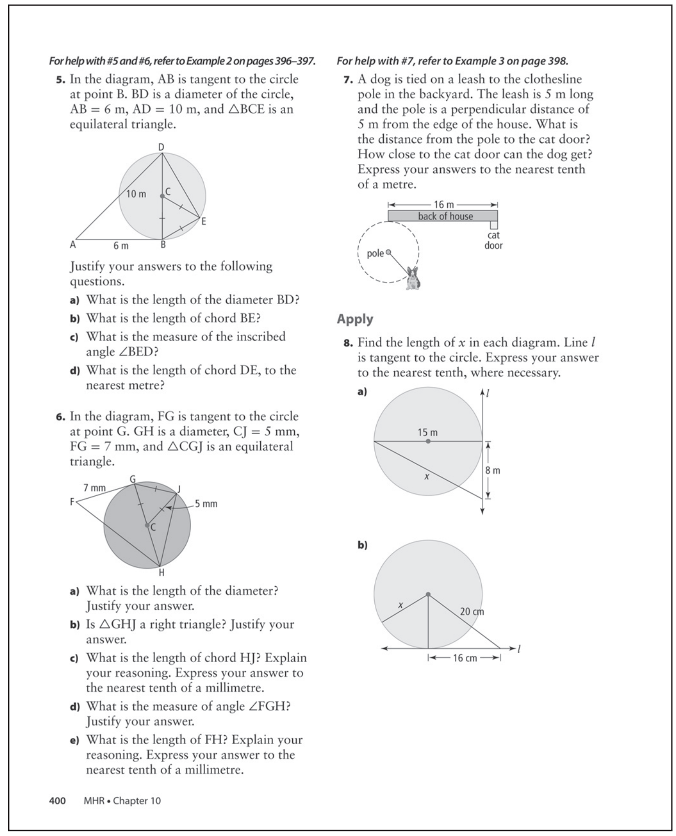

*For help with #5 and #6, refer to Example 2 on pages 396–397.*

**5.** In the diagram, AB is tangent to the circle at point B. BD is a diameter of the circle, AB = 6 m, AD = 10 m, and △BCE is an equilateral triangle.

Justify your answers to the following questions.

**a)** What is the length of the diameter BD?

**b)** What is the length of chord BE?

**c)** What is the measure of the inscribed angle ∠BED?

**d)** What is the length of chord DE, to the nearest metre?

**6.** In the diagram, FG is tangent to the circle at point G. GH is a diameter, CJ = 5 mm, FG = 7 mm, and △CGJ is an equilateral triangle.

**a)** What is the length of the diameter? Justify your answer.

**b)** Is △GHJ a right triangle? Justify your answer.

**c)** What is the length of chord HJ? Explain your reasoning. Express your answer to the nearest tenth of a millimetre.

**d)** What is the measure of angle ∠FGH? Justify your answer.

**e)** What is the length of FH? Explain your reasoning. Express your answer to the nearest tenth of a millimetre.

*For help with #7, refer to Example 3 on page 398.*

**7.** A dog is tied on a leash to the clothesline pole in the backyard. The leash is 5 m long and the pole is a perpendicular distance of 5 m from the edge of the house. What is the distance from the pole to the cat door? How close to the cat door can the dog get? Express your answers to the nearest tenth of a metre.

#### Apply

**8.** Find the length of *x* in each diagram. Line *l* is tangent to the circle. Express your answer to the nearest tenth, where necessary.

**a)**

**b)**

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## Check Your Understanding

### Communicate the Ideas

Question #1 provides a great springboard through a non-example for discussing the concept of a tangent line. In question #2, students need to understand that since ∠ABC is not right-angled, AB is not tangent to the radius. If you could zoom in on the region around point B, you would see that the line AB actually intersects the circle in two spots. Students could try this construction with a fine-tip pencil to verify.

### Practise

All of the Practise questions match closely with the Example questions. Students should be encouraged to copy the diagram and label angles and special triangles as they are determined.

As students begin working on #3, you may wish to ask questions such as:

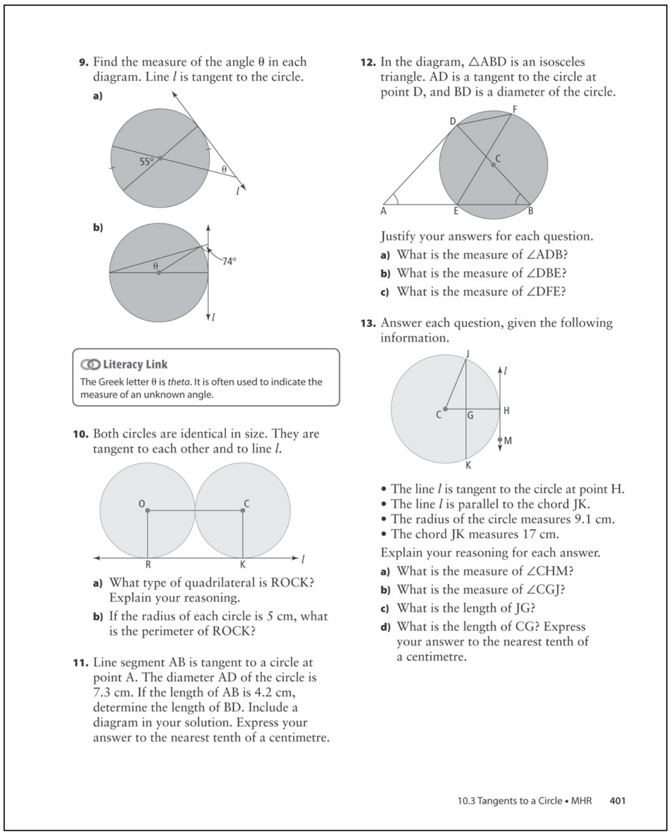
- How can line segments AB and DC help you determine the measure of ∠BDC?
- What is the measure of ∠DBC?
- What methods can you use to determine the measure of the central angle?
- What is the measure of the central angle?
- Mark the radii of the circle. What do these marks suggest about △CDE?
- How can you use the type of triangle and the measure of ∠CDE to help you determine the size of ∠DEC?
- If FCE is a diameter, what is the measure of ∠FCE?
- How can you use this information to check that you have the correct sizes for ∠DCE and ∠DCB?

### Apply

These questions include new ways of applying the tangent radius property. There are not many steps required for each question, so students should succeed in this section without much support. Encourage students to create diagrams for questions that do not include diagrams (e.g., #11). Also, encourage students to copy in their notebooks the diagrams from the question so that they can label angles and dimensions as they are discovered.

In #9, reactivate student knowledge of complementary angles and the characteristics of triangles.

In #10, have students mark the radii and any angles that they know. Ask how they know the measure of these angles.



9. Find the measure of the angle θ in each diagram. Line $l$ is tangent to the circle.

a)

b)



Literacy Link

The Greek letter θ is *theta*. It is often used to indicate the measure of an unknown angle.

10. Both circles are identical in size. They are tangent to each other and to line $l$.



a) What type of quadrilateral is ROCK? Explain your reasoning.

b) If the radius of each circle is 5 cm, what is the perimeter of ROCK?

11. Line segment AB is tangent to a circle at point A. The diameter AD of the circle is 7.3 cm. If the length of AB is 4.2 cm, determine the length of BD. Include a diagram in your solution. Express your answer to the nearest tenth of a centimetre.

12. In the diagram, △ABD is an isosceles triangle. AD is a tangent to the circle at point D, and BD is a diameter of the circle.



Justify your answers for each question.

a) What is the measure of ∠ADB?

b) What is the measure of ∠DBE?

c) What is the measure of ∠DFE?

13. Answer each question, given the following information.



- The line $l$ is tangent to the circle at point H.
- The line $l$ is parallel to the chord JK.
- The radius of the circle measures 9.1 cm.
- The chord JK measures 17 cm.

Explain your reasoning for each answer.

a) What is the measure of ∠CHM?

b) What is the measure of ∠CGJ?

c) What is the length of JG?

d) What is the length of CG? Express your answer to the nearest tenth of a centimetre.

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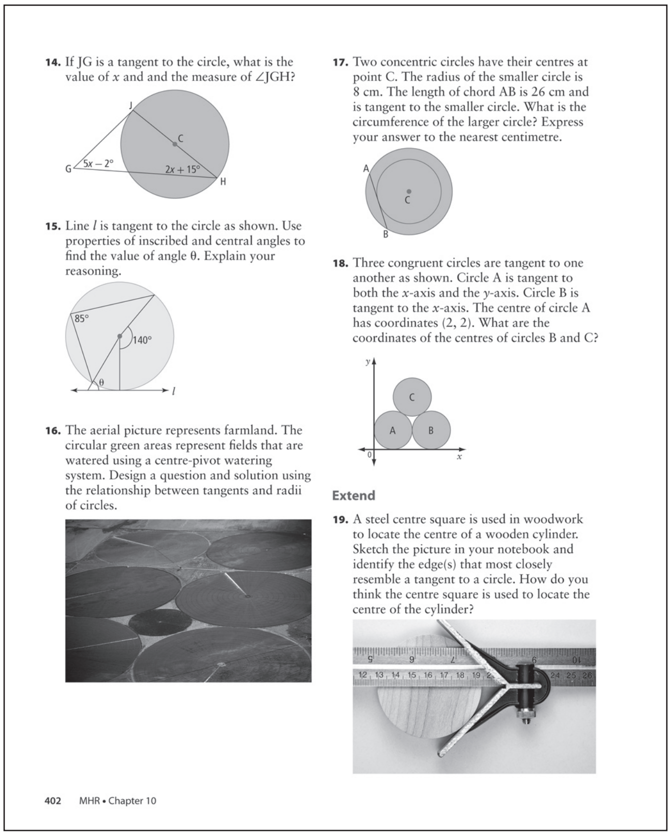

14. If JG is a tangent to the circle, what is the value of $x$ and and the measure of ∠JGH?



15. Line $l$ is tangent to the circle as shown. Use properties of inscribed and central angles to find the value of angle θ. Explain your reasoning.



16. The aerial picture represents farmland. The circular green areas represent fields that are watered using a centre-pivot watering system. Design a question and solution using the relationship between tangents and radii of circles.

17. Two concentric circles have their centres at point C. The radius of the smaller circle is 8 cm. The length of chord AB is 26 cm and is tangent to the smaller circle. What is the circumference of the larger circle? Express your answer to the nearest centimetre.



18. Three congruent circles are tangent to one another as shown. Circle A is tangent to both the $x$-axis and the $y$-axis. Circle B is tangent to the $x$-axis. The centre of circle A has coordinates (2, 2). What are the coordinates of the centres of circles B and C?



Extend

19. A steel centre square is used in woodwork to locate the centre of a wooden cylinder. Sketch the picture in your notebook and identify the edge(s) that most closely resemble a tangent to a circle. How do you think the centre square is used to locate the centre of the cylinder?

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In #13, have students sketch and add all known labels to this diagram. As they calculate each answer, have them label the answer on the diagram. Ask them to notice how the labels help them with subsequent parts of the question.

In #15, have students sketch and label what they know about this diagram.

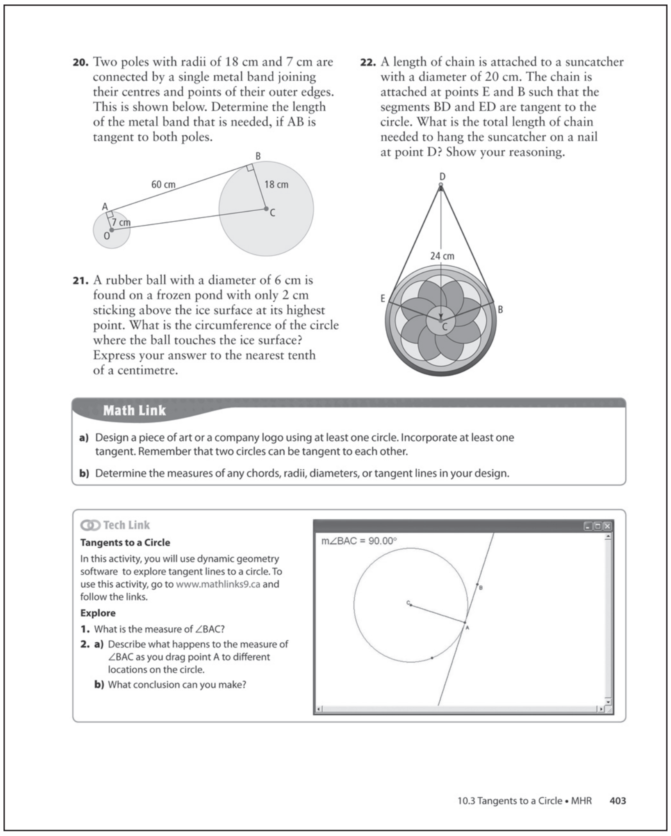
In #17, students may need to be reminded what the term *concentric* means.

## Extend

Most of the Extend questions include a diagram. The exceptions are #19 and 22. For #19, students are required to draw their own diagram. For #22, it may help students in their thinking to draw a diagram.

## Math Link

This Math Link may be incorporated into the Math Link: Wrap It Up! at the end of the chapter. With students, read through the Math Link: Wrap It Up! before starting the current Math Link.



20. Two poles with radii of 18 cm and 7 cm are connected by a single metal band joining their centres and points of their outer edges. This is shown below. Determine the length of the metal band that is needed, if AB is tangent to both poles.



21. A rubber ball with a diameter of 6 cm is found on a frozen pond with only 2 cm sticking above the ice surface at its highest point. What is the circumference of the circle where the ball touches the ice surface? Express your answer to the nearest tenth of a centimetre.

22. A length of chain is attached to a suncatcher with a diameter of 20 cm. The chain is attached at points E and B such that the segments BD and ED are tangent to the circle. What is the total length of chain needed to hang the suncatcher on a nail at point D? Show your reasoning.



Math Link

a) Design a piece of art or a company logo using at least one circle. Incorporate at least one tangent. Remember that two circles can be tangent to each other.

b) Determine the measures of any chords, radii, diameters, or tangent lines in your design.

Tech Link

**Tangents to a Circle**

In this activity, you will use dynamic geometry software to explore tangent lines to a circle. To use this activity, go to www.mathlinks9.ca and follow the links.

**Explore**

1. What is the measure of ∠BAC?

2. a) Describe what happens to the measure of ∠BAC as you drag point A to different locations on the circle.

b) What conclusion can you make?



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## *Meeting Student Needs*

- Provide **BLM 10–10 Section 10.3 Extra Practice** to students who would benefit from more practice.

### ELL

- Some students may benefit from discussing the following terms: *clothesline*, *identical in size*, *aerial picture*, and *farmland*.

## *Common Errors*

- Some students may not be clear on the concept of a tangent line.

$R_x$ Redefine *tangent* and *point of tangency* using examples and non-examples.

## Answers

**Communicate the Ideas**

1. Raven is correct. Example: Segment AB is not a tangent to the circle because it touches the circle in more than on place.
2. Example: Segment AB is not 90°. A line segment tangent to a circle must be perpendicular to the radius at the point of tangency.

| Assessment | Supporting Learning |
|---|---|
| **Assessment *as* Learning** | |
| **Communicate the Ideas**<br>Have all students complete #1 and 2. As a class, have students share their responses to #2. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• As a class, have students share their responses to #2. Some students may benefit from seeing the diagram constructed on an overhead, and seeing that the segment AB actually crosses the circle in two unique places. |
| **Assessment *for* Learning** | |
| **Practise**<br>Have students do #3, 6–8, and 11. Students who have no problems with these questions can go on to the remaining Apply questions. | • Have the students review the examples and refer to their Foldable for guidelines and terminology for #3, 6, and 7.<br>• Redrawing and labelling the triangles away from the circle may assist some students in solving #11.<br>• Some students may benefit from a review of congruent angles and sides for questions such as #8. |
| **Math Link**<br>The Math Link on page 403 is intended to help students work toward the chapter problem wrap-up titled Math Link: Wrap It Up! on page 407. | • Students who need help getting started could use **BLM 10–11 Section 10.3 Math Link**, which provides scaffolding.<br>• Some students may find it helpful to refer back to the collection of circular logos used in a previous Math Link.<br>• You may wish to provide **Master 8 Centimetre Grid Paper** and **Master 9 0.5 Centimetre Grid Paper** to students for drawing their designs. |
| **Assessment *as* Learning** | |
| **Literacy Link (page 375)**<br>Help students to recall the terms introduced in this section by adding the new terms to their web. | • Students should be encouraged to transfer terminology from their Foldable to the appropriate place on the web. This process could be a valuable assessment-as-learning tool. |
| **Math Learning Log**<br>Have students respond to the following question:<br>• Given two tangent circles, how can you construct a rectangle that has two vertices at the centres of the two circles? Explain your reasoning and include a diagram. | • Encourage students to add definitions from this section to their Foldable. Also, advise them to record notes, examples, and Key Ideas.<br>• Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with. |