11.3 Probability in Society

***MathLinks 9*, pages 430–439**

Suggested Timing
80–100 minutes

Materials
- magazines, newspapers, or computer with Internet access

Blackline Masters
Master 2 Communication Peer Evaluation
BLM 11–3 Chapter 11 Warm-Up
BLM 11–10 Section 11.3 Extra Practice
BLM 11–11 Section 11.3 Math Link

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☐ Visualization (V)

Specific Outcomes
SP4 Demonstrate an understanding of the role of probability in society.
SP3 Develop and implement a project plan for the collection, display and analysis of data by:
- formulating a question for investigation
- choosing a data collection method that includes social considerations
- selecting a population or a sample
- collecting the data
- displaying the collected data in an appropriate manner
- drawing conclusions to answer the question.

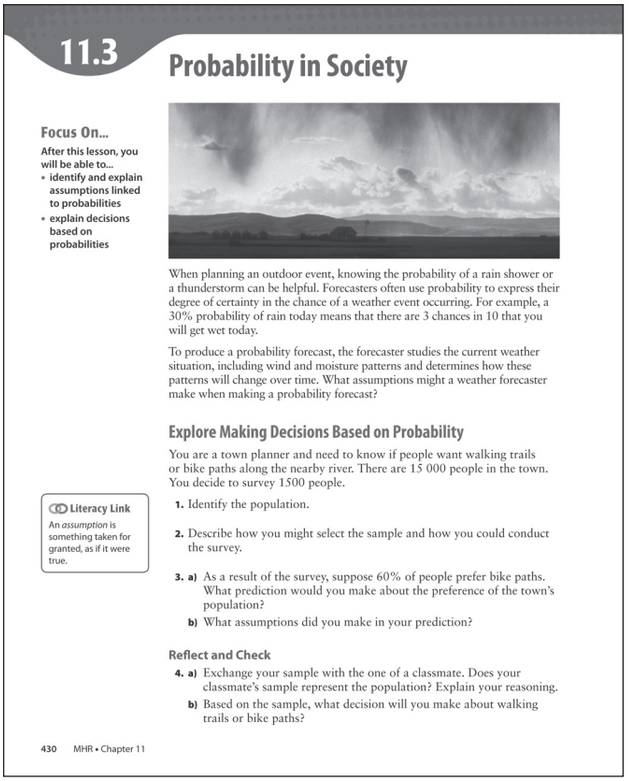
Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6, 9–12, 14, Math Link
Typical	#1–3, 4 *or* 5, 6 *or* 7, 8–12, 14, Math Link
Extension/Enrichment	#1–3, 13, 15–18, Math Link

Planning Notes

Have students complete the warm-up questions on **BLM 11–3 Chapter 11 Warm-Up** to reinforce material learned in previous sections.

In this section, students use what they learned in sections 11.1 and 11.2 to explore making decisions based on probability. They use a sample to make predictions about the population, and explain the reasonableness of the predictions. In order to do this, students identify any assumptions that were made. Next, they consider how to avoid making false predictions. Finally, they make decisions based on probability. You may need to reactivate students' understanding of how to determine probability and the difference between theoretical and experimental probabilities. Students also need to know how to calculate percents of a number and round numbers appropriately.

As a class, review the opening text and visual. Have students discuss the opening question about assumptions that a weather forecaster might make. For instance, in the case of showers, it is unlikely that it will rain in all parts of a specific region, whereas a steady rain is likely to be more widespread. You might prompt students to discuss some limitations of probability forecasts. For instance, probability forecasts cannot be used to predict exactly where, when, and how much it will rain. A 90% probability of rain tomorrow does not mean that it will rain during 90% of the day. This probability does indicate that there is a great possibility of a measurable amount of rain falling.



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Focus On...
After this lesson, you will be able to...
- identify and explain assumptions linked to probabilities
- explain decisions based on probabilities

When planning an outdoor event, knowing the probability of a rain shower or a thunderstorm can be helpful. Forecasters often use probability to express their degree of certainty in the chance of a weather event occurring. For example, a 30% probability of rain today means that there are 3 chances in 10 that you will get wet today.

To produce a probability forecast, the forecaster studies the current weather situation, including wind and moisture patterns and determines how these patterns will change over time. What assumptions might a weather forecaster make when making a probability forecast?

Explore Making Decisions Based on Probability

You are a town planner and need to know if people want walking trails or bike paths along the nearby river. There are 15 000 people in the town. You decide to survey 1500 people.

Literacy Link
An *assumption* is something taken for granted, as if it were true.

1. Identify the population.
2. Describe how you might select the sample and how you could conduct the survey.
3. a) As a result of the survey, suppose 60% of people prefer bike paths. What prediction would you make about the preference of the town's population?
   b) What assumptions did you make in your prediction?

Reflect and Check

4. a) Exchange your sample with the one of a classmate. Does your classmate's sample represent the population? Explain your reasoning.
   b) Based on the sample, what decision will you make about walking trails or bike paths?

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Prompt students to conclude that it is important to consider assumptions when making weather predictions.

Explore Making Decisions Based on Probability

In this Explore, students make a prediction, list assumptions related to the prediction, and make a decision based on their sample.

Method 1 Have students work individually to complete #1 to 3. For #4a), explain that a sample that represents the population has the same profile as the population. Have them exchange their sample with that of a classmate and give feedback about the representativeness of the sample, before answering #4b). Ask students to explain how considering whether the sample is representative of the population affects their decision for #4b).

As a class, have students discuss the assumptions they made in their predictions. Have students justify the decision they made based on their sample.

Method 2 Have students work with a partner to complete #1 to 3. Have them exchange their sample with the one of another pair of students and give each other feedback.

Literacy Link Direct students to the Literacy Link on page 430 that explains an *assumption*. You might ask what assumptions they might consider for a class survey about favourite TV progams.

Meeting Student Needs

- Consider working through the Explore as a whole class.

ELL

- Clearly define and give examples of the terms *prediction* and *assumption*. Explain that a prediction is a statement or a claim that a specific event will occur in the future. For example, you might predict that it will snow in December. Explain that an assumption is something taken for granted, as if it were true. For example, you might assume that people who stay indoors during a storm would stay warm and dry.
- Teach the following terms in context: *thunderstorm*, *degree of certainty*, *weather event*, *probability forecast*, and *moisture patterns*.

Answers

Explore Making Decisions Based on Probability

1. The people in the town are the population.

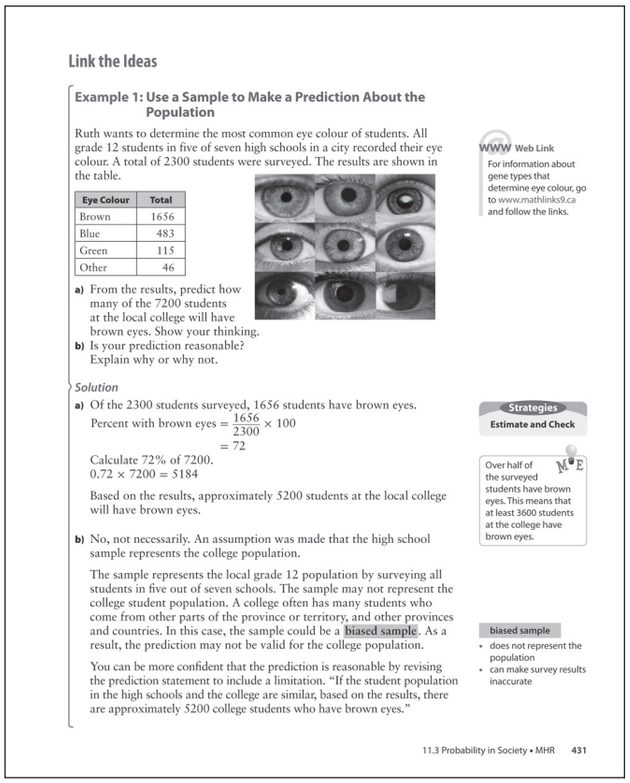
2. Example: Use a systematic sample and ask every tenth person in the phone book.

3. a) Example: Since 9000 out of 1500 people prefer bike paths, I predict that the town's population would prefer bike paths.

b) Examples:
- I assumed that the survey accurately predicted the preferences of the town's population.
- I assumed that the sample represented the town's population.

4. b) Example: Based on the sample, I would build a combination of walking and biking trails.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Check Listen as students discuss what they discovered during the Explore.	• As a class, have students present the different samples they have chosen and justify the reasons for their choice. Attempt to have students agree about which samples represent the population. The class discussion will help some students come to a better understanding. • Some students may benefit from using the class discussion as a springboard to revise their own response.



Link the Ideas

Example 1: Use a Sample to Make a Prediction About the Population

Ruth wants to determine the most common eye colour of students. All grade 12 students in five of seven high schools in a city recorded their eye colour. A total of 2300 students were surveyed. The results are shown in the table.

Eye Colour	Total
Brown	1656
Blue	483
Green	115
Other	46

a) From the results, predict how many of the 7200 students at the local college will have brown eyes. Show your thinking.
b) Is your prediction reasonable? Explain why or why not.

Solution

a) Of the 2300 students surveyed, 1656 students have brown eyes.
Percent with brown eyes $= \frac{1656}{2300} \times 100$
$= 72$
Calculate 72% of 7200.
$0.72 \times 7200 = 5184$
Based on the results, approximately 5200 students at the local college will have brown eyes.

b) No, not necessarily. An assumption was made that the high school sample represents the college population.
The sample represents the local grade 12 population by surveying all students in five out of seven schools. The sample may not represent the college student population. A college often has many students who come from other parts of the province or territory, and other provinces and countries. In this case, the sample could be a biased sample. As a result, the prediction may not be valid for the college population.
You can be more confident that the prediction is reasonable by revising the prediction statement to include a limitation. "If the student population in the high schools and the college are similar, based on the results, there are approximately 5200 college students who have brown eyes."

WWW Web Link
For information about gene types that determine eye colour, go to www.mathlinks9.ca and follow the links.

Strategies
Estimate and Check

Over half of the surveyed students have brown eyes. This means that at least 3600 students at the college have brown eyes.

biased sample
- does not represent the population
- can make survey results inaccurate

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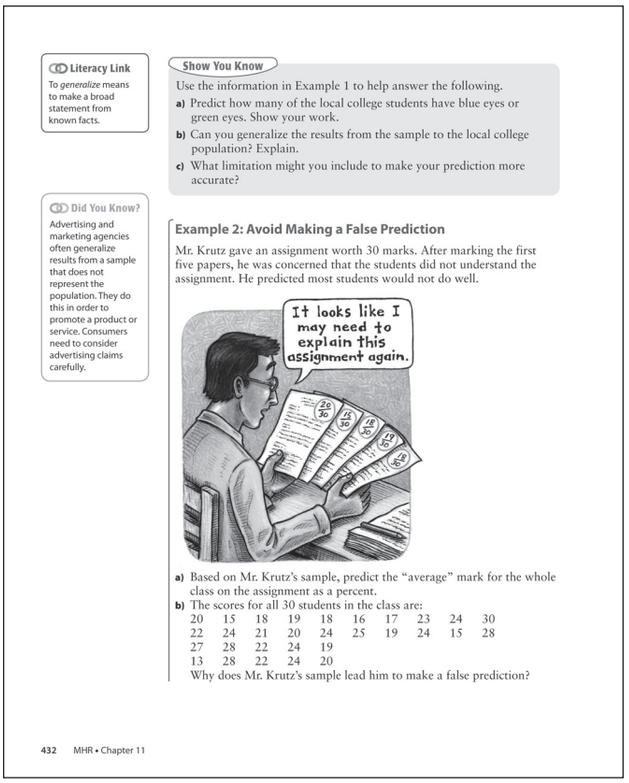

Literacy Link
To *generalize* means to make a broad statement from known facts.

Show You Know
Use the information in Example 1 to help answer the following.
a) Predict how many of the local college students have blue eyes or green eyes. Show your work.
b) Can you generalize the results from the sample to the local college population? Explain.
c) What limitation might you include to make your prediction more accurate?

Did You Know?
Advertising and marketing agencies often generalize results from a sample that does not represent the population. They do this in order to promote a product or service. Consumers need to consider advertising claims carefully.

Example 2: Avoid Making a False Prediction

Mr. Krutz gave an assignment worth 30 marks. After marking the first five papers, he was concerned that the students did not understand the assignment. He predicted most students would not do well.

a) Based on Mr. Krutz's sample, predict the "average" mark for the whole class on the assignment as a percent.
b) The scores for all 30 students in the class are:
20 15 18 19 18 16 17 23 24 30
22 24 21 20 24 25 19 24 15 28
27 28 22 24 19
13 28 22 24 20
Why does Mr. Krutz's sample lead him to make a false prediction?

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Link the Ideas

Example 1: Use a Sample to Make a Prediction About the Population

Example 1 illustrates the importance of stating limitations in a prediction statement in order to make a more reasonable prediction. Students are introduced to the term *biased sample*, which can result in inaccurate survey results and an invalid prediction.

Before walking through Example 1, you may wish to briefly discuss dominant and recessive genes for eye colour. The related Web Link in the student resource on page 431 may be helpful.

For solution part b), ensure that students understand how the sample is biased by asking how the college population is different from the high school sample.

You might develop a scenario familiar to students such as surveying teens about readership of fashion and mechanics magazines. Since all the boys were absent, only girls were surveyed. The prediction based on the biased sample concluded that teens prefer fashion magazines to mechanics magazines. Ask:

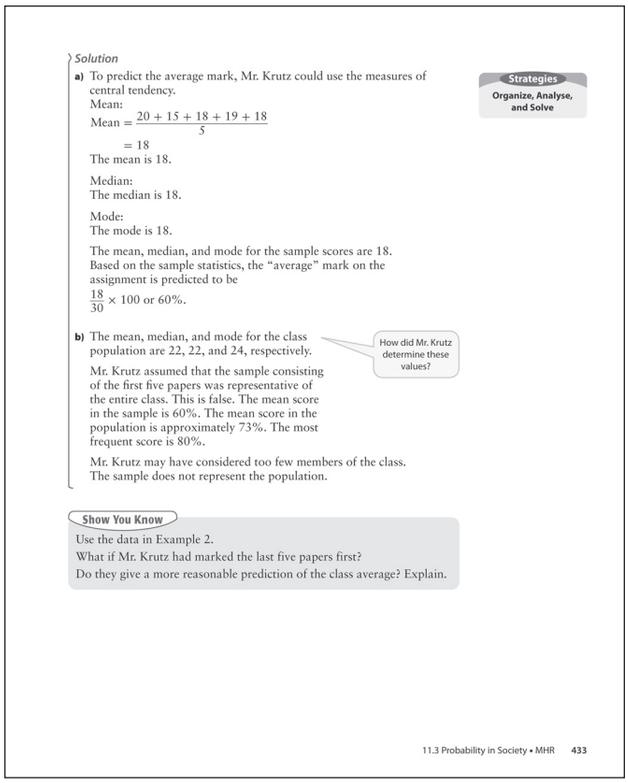
- Why is the prediction not valid?
- What assumptions were made?
- What limitation could you include in the prediction statement to make the prediction more reasonable?

You might discuss different types of biased samples. For example:

- excluding certain groups from a sample. For example, a survey of college students to determine young adults' favourite types of music is a biased sample if it does not include young adults not at college. (Young adults may be working or enrolled in other educational and training programs.)
- under representing or over representing certain groups. For example, a survey of seniors at a community fitness centre to determine the overall level of fitness of seniors is a biased sample if it does not include seniors in other settings. (Seniors at a fitness centre may be more fit than those at home or in institutional settings.)

Literacy Link Direct students to the term *generalize* on page 432. Point out how the term is used in part a) of the Show You Know.

Lead in to the Did You Know? about how marketers use biased samples to promote their product or service. Use an example such as surveying teens entering a heavy metal concert about their favourite type of music. The next day, an ad in the newspaper claims that teens' favourite type of music is heavy



Solution

a) To predict the average mark, Mr. Krutz could use the measures of central tendency.

Mean:

$$\text{Mean} = \frac{20 + 15 + 18 + 19 + 18}{5}$$
$$= 18$$

The mean is 18.

Median:
The median is 18.

Mode:
The mode is 18.

The mean, median, and mode for the sample scores are 18. Based on the sample statistics, the "average" mark on the assignment is predicted to be

$\frac{18}{30} \times 100$ or 60%.

Strategies
Organize, Analyse, and Solve

b) The mean, median, and mode for the class population are 22, 22, and 24, respectively.

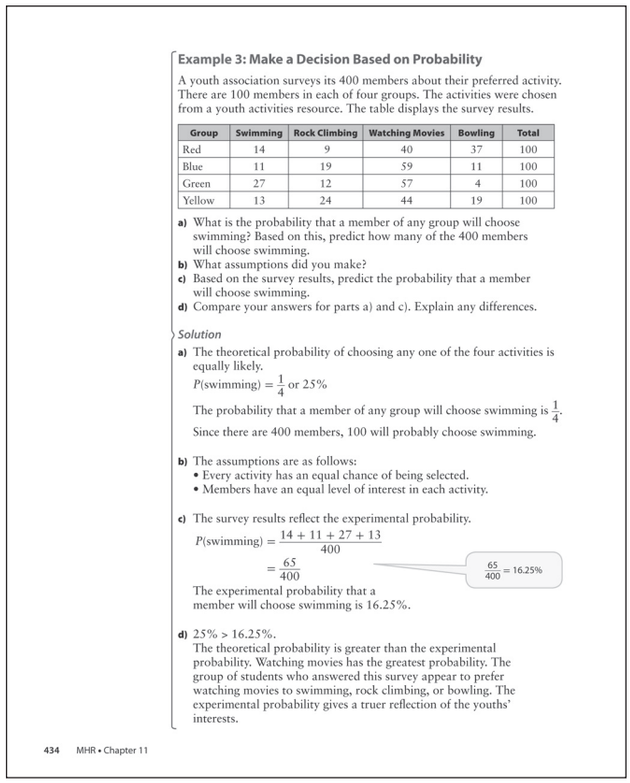
How did Mr. Krutz determine these values?

Mr. Krutz assumed that the sample consisting of the first five papers was representative of the entire class. This is false. The mean score in the sample is 60%. The mean score in the population is approximately 73%. The most frequent score is 80%.

Mr. Krutz may have considered too few members of the class. The sample does not represent the population.

Show You Know

Use the data in Example 2.
What if Mr. Krutz had marked the last five papers first?
Do they give a more reasonable prediction of the class average? Explain.



Example 3: Make a Decision Based on Probability

A youth association surveys its 400 members about their preferred activity. There are 100 members in each of four groups. The activities were chosen from a youth activities resource. The table displays the survey results.

Group	Swimming	Rock Climbing	Watching Movies	Bowling	Total
Red	14	9	40	37	100
Blue	11	19	59	11	100
Green	27	12	57	4	100
Yellow	13	24	44	19	100

a) What is the probability that a member of any group will choose swimming? Based on this, predict how many of the 400 members will choose swimming.
b) What assumptions did you make?
c) Based on the survey results, predict the probability that a member will choose swimming.
d) Compare your answers for parts a) and c). Explain any differences.

Solution

a) The theoretical probability of choosing any one of the four activities is equally likely.

$P(\text{swimming}) = \frac{1}{4}$ or 25%

The probability that a member of any group will choose swimming is $\frac{1}{4}$.

Since there are 400 members, 100 will probably choose swimming.

b) The assumptions are as follows:
- Every activity has an equal chance of being selected.
- Members have an equal level of interest in each activity.

c) The survey results reflect the experimental probability.

$$P(\text{swimming}) = \frac{14 + 11 + 27 + 13}{400}$$
$$= \frac{65}{400}$$

$\frac{65}{400} = 16.25\%$

The experimental probability that a member will choose swimming is 16.25%.

d) 25% > 16.25%.
The theoretical probability is greater than the experimental probability. Watching movies has the greatest probability. The group of students who answered this survey appear to prefer watching movies to swimming, rock climbing, or bowling. The experimental probability gives a truer reflection of the youths' interests.

metal. Ask students how this is a biased sample. You might ask students to discuss examples of advertising claims that they are familiar with.

Have students complete the Show You Know individually. Have them exchange their response with a classmate. Tell students to read their partner's prediction carefully and check that any limitations are included. As a class, have students justify their prediction.

Example 2: Avoid Making a False Prediction

Example 2 demonstrates using measures of central tendency to make a prediction.

As a class, walk through Example 2. For part a), you might have students recall the meaning of mean, median, and mode, and how to calculate each measure of central tendency. You might have students who need practice do the calculations for mean, median, and mode in their notebook.

For part b), you might have students work with a partner to check the values for mean, median, and mode. Then, discuss how by chance the first five papers that Mr. Krutz graded scored lower than the class average, and therefore do not represent the class results. A sample of five students may be too small to generalize results.

Have students work with a partner to complete the Show You Know. Students use the same data to compare a second sample to the class results. As a class, discuss the results. Emphasize that even though by chance the second sample represents the population more closely than the first sample in Example 2, it is a very small sample and does not represent the population.

Example 3: Make a Decision Based on Probability

Example 3 demonstrates using experimental probability and theoretical probability to make decisions based on probability.

As a class, walk through Example 3. To help students recall experimental probability and theoretical probability, you might have them define and identify examples of experimental probability from the survey. (Experimental probability is the probability of an event occurring based on experimental results. The experimental probability of swimming is $\frac{65}{400} = 16.25\%$.) Have students define and identify examples of theoretical probability. (Theoretical probability is the calculated probability of an event occurring. Every activity has an equal chance

Show You Know

Refer to the information in Example 3.

a) Based on the survey results, what is the probability that a member will choose watching movies? bowling?

b) If you were the youth coordinator planning the activities, how would you determine the favourite activity? Explain your reasoning.

Key Ideas

- A biased sample can make survey results inaccurate.
- When a sample represents the population, you can generalize the results to the population.
- You can use experimental probability and theoretical probability to help make decisions based on probability.

A biased sample may result when a large percent of individuals in a sample refuse to participate in a survey.

Check Your Understanding

Communicate the Ideas

1. Kelly is confused about the difference between a sample that represents the population and one that does not. Use an example to help explain the difference to him.

2. Use the cartoon to explain how a sample might result in a false prediction.

3. How might you use experimental probability and theoretical probability to help make a decision about what flavours of ice cream to offer at a sport tournament?

Practise

For help with #4 and #5, refer to Example 1 on page 431.

4. A light bulb factory samples light bulbs as they come off the assembly line. A random sample shows that 1 bulb out of every 20 is defective. In a run of 1380 bulbs, the quality manager predicts that 69 bulbs will be defective. What assumptions did the quality manager make in his prediction?

5. A toothpick factory samples every 100th toothpick for damage. The sample shows a 0.17% probability of damage. How many toothpicks would you predict to be damaged in the daily production of 2.4 million toothpicks? Include any assumptions you made in your prediction.

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of being chosen. The theoretical probability of swimming is 25%.) Ensure that students can calculate experimental probability and theoretical probability. You might use an alternative set of questions to reinforce their understanding. Ask:

- What is the probability that a member will choose rock climbing? (25%)
- Predict how many of the 400 members will choose rock climbing? (100)
- What is the experimental probability that a member will choose rock climbing? (16%)
- Compare the theoretical and experimental probability. (25% > 16%)
- Which gives a more accurate picture of the youths' interest? (experimental probability)
  Point out how the experimental probability and theoretical probability vary.

You might ask students to discuss whether sample size could affect the experimental probability and theoretical probability.

Have students work with a partner to complete the Show You Know. Students use the same data to find the probability that members will choose watching movies and bowling. As a class, discuss part b).

Key Ideas

The Key Ideas summarize the importance of using samples that represent the population in order to make reasonable predictions, and of using experimental probability and theoretical probability to make decisions based on probability.

Discuss the bias that occurs when a large percent of people refuse to participate in a survey. For example, people may refuse to disclose their taxable income. Ask how the biased sample, in this case, might skew the results.

Have students define and give an example of a biased sample in their Foldable. Encourage them to define and include an example of experimental probability and theoretical probability, and store their notes in the pocket of their Foldable.

Meeting Student Needs

- Before Example 1, some students may benefit from recalling how to convert fractions to a percent and calculate the percent of a number.
- For Example 1, depending on the ethnic make up of your class, discuss whether the prediction is valid for the class.
- Before Example 2, some students may need help to review *mean*, *median*, and *mode*, and how to calculate each measure. You might provide a sample set of numbers and have them practise calculating these measures.
- Before Example 3, some students may benefit from reactivating their knowledge and skills with experimental probability and theoretical probability. Prompt students to recall that
  $\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$.
  Help them recall that theoretical probability is the calculated probability of an event occurring. Experimental probability is the probability of an event occurring based on experimental results.
- Consider working through the examples as a whole class and having students work in pairs or small groups to complete the Show You Knows. Have students work individually to complete an additional Show You Know related to each example.

ELL

- For Example 1, ensure that students understand the meaning of *limitation* as a restriction or a boundary.
- For Example 2, ensure that students understand the terms *mean*, *median*, and *mode*. Have students add unfamiliar terms along with a sample calculation to their dictionary.

Gifted and Enrichment

- Challenge students to use the information in the Web Link on page 431 in the student resource to help them research gene types that determine eye colour. Invite them to make an oral report of their findings to the class.

Answers

Example 1: Show You Know

a) $\frac{598}{2300} = 26\%$. 26% of 7200 is 1872. 1872 college students have blue or green eyes.

b) Example: If the high school sample represents the college population, the results from the high school survey can be generalized to the college population.

c) Example: You may include the limitation that the prediction is correct if the student population in the high school and the local college are similar.

Example 2: Show You Know

Example: The mean is 21.4. The median is 22. There is no mode. The mean and the median of the last five papers give a closer prediction of the class average.

Example 3: Show You Know

a) $P(\text{movies}) = 50\%$; $P(\text{bowling}) = 17.75\%$

b) Example: Use the experimental probability because it reflects students' preferences. You cannot assume that all students will like the activities equally well.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Show You Know related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from coaching about converting fractions to a percent and calculating the percent of a number. Discuss rounding to the nearest whole number when numbers of people are involved.
Example 2 Have students do the Show You Know related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may find it helpful to review the calculations for mean, median, and mode. Have students define mean, median, and mode, write the steps for each calculation, and include a sample calculation for each measure. In part a), for mean, students may say to calculate the average of all the scores $\left(\frac{20 + 15 + 18 + 19 + 18}{5}\right)$. For median, they may say to arrange the values from least to greatest and then select the middle value (15, 18, **18**, 19, 20). For mode, they may say to select the value that appears most often (15, **18**, **18**, 19, 20). Have them store their notes in the pocket of their Foldable. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.
Example 3 Have students do the Show You Know related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Coach students to recall what they know about the difference between experimental probability and theoretical probability. Clarify any misunderstandings. Then, have them explain how to calculate each probability. They may benefit from writing a definition and recording an example calculation of each probability in their Foldable. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.

Show You Know

Refer to the information in Example 3.

a) Based on the survey results, what is the probability that a member will choose watching movies? bowling?

b) If you were the youth coordinator planning the activities, how would you determine the favourite activity? Explain your reasoning.

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Practise

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For help with #6 and #7, refer to Example 2 on pages 432-433.

6. A cafeteria supervisor asked three students who are vegetarians about their preference for a lunch menu. All three chose garden salad, tomato soup, and garlic bread. The supervisor plans to serve their menu choice the next day, thinking that it will sell well.

a) Did the sample lead the supervisor to make a false prediction? Explain.

b) If the prediction is false, explain how you might make a more accurate one.

7. A manufacturer makes the following claim about the mass of its health bars.

Each bar has a mass of at least 50 g.

Erika and Brett weighed ten health bars to check the claim. Three bars had a mass less than 50 g and one bar had a mass of exactly 50 g. The students predicted that 30% of the health bars made by the company would not meet the claim.

a) Did the sample lead the students to make a false prediction? Explain.

b) If the prediction is false, explain how you might make a more accurate one.

For help with #8 and #9, refer to Example 3 on page 434.

8. Greenville, a town with 4000 people, is having an election for mayor. A reporter polled 40 people and found 53% chose Candidate A, 23% chose Candidate B, and the rest chose Candidate C.

a) How many people polled chose Candidate C?

b) What is the theoretical probability that a voter will choose Candidate A? What assumptions did you make?

c) Compare the experimental and theoretical probabilities of Candidate A winning.

d) The reporter predicts that Candidate A will win the election. Do you agree with his prediction? Explain your reasoning.

9. A movie rental company has five types of movies. They are drama, comedy, horror, action, and science fiction/fantasy movies.

a) What is the theoretical probability that a person will choose a comedy?

b) What assumptions did you make?

c) The table displays the movie preferences from a random survey of 50 customers. Predict the probability that a customer will choose a comedy movie.

Movie Type	Responses
Drama	15
Comedy	10
Horror	12
Action	11
Science fiction/Fantasy	2

d) Compare your answers for parts a) and c). Explain any differences.

e) About how many rentals out of a total of 2000 movies will be drama movies?

Apply

10. Miya received the following scores from ten judges in a skating competition. Skating performances are given a score out of 10.

Judge	1	2	3	4	5	6	7	8	9	10
Score	8.5	6	6.5	6.5	6.5	7	6	6.5	4.5	7

a) Calculate Miya's mean score based on all ten judges.

b) Use the first three judges' scores as a sample. Calculate the mean.

c) Use the last three judges' scores as a sample. Calculate the mean.

d) Compare the mean from each sample to the mean for all judges. Are the samples a good predictor for Miya's overall score? Explain.

Check Your Understanding

Communicate the Ideas

These questions provide an opportunity for students to explain their understanding of representative samples, false predictions, and using experimental probability and theoretical probability to make decisions. Have students complete #1 to 3 individually, and then discuss their answers with a classmate.

For #1, students develop an example to explain the difference between a sample that represents the population and one that does not. Require students to develop an original example.

For #2, students use a cartoon to explain how a sample might result in a false prediction. You might encourage them to create a different cartoon that leads to a false prediction.

For #3, students use a given scenario and explain how to use experimental probability and theoretical probability to make a decision. You might prompt students to develop sample data for several flavours of ice cream and use the data to help with their explanation.

Practise

Note that #4 and 5, #6 and 7, and #8 and 9 are pairs of similar questions. Consider giving students a choice to do one question from each pair initially to demonstrate their understanding.

Consider using the responses to #9 as an opportunity to reinforce what students will do in the Math Link for this section. Check that they can identify assumptions and compare and explain differences between experimental probability and theoretical probability. Discussing responses as a class may help students to address similar issues in their research project.

Apply

Consider using the responses to #11, 12, and 14 as an opportunity to reinforce what students will do in the Math Link for this section. For #11, have students discuss the biased sample and the false prediction. For #12 and 14, have students compare the experimental probability and the theoretical probability. Discussing responses as a class may help students to address similar issues in their research project.

For #13, students may find the information about risk factors and factors affecting car insurance rates in the Web Link on page 437 in the student resource helpful.

11. Jack wants to know the weekly earnings of grade 9 students who work part-time in the summer. He surveyed five grade 9 students. Here are the results: $75, $120, $45, $250, and $85.

a) Is this a biased sample? Explain your reasoning.

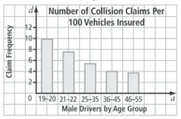
b) Jack concluded that grade 9 students earn an average of $115 per week. Do you agree with his conclusion? Explain.

12. Colin read an article that claims that more girls are born than boys. Colin predicted that a couple has a 50% chance of having a boy. He tested the prediction by tossing a coin 100 times for each of 10 trials. Here are the results.

Trial	Boys	Girls
1	40	60
2	43	57
3	50	50
4	46	54
5	48	52
6	58	42
7	50	50
8	49	51
9	50	50
10	53	47

Do these experimental results confirm Colin's prediction or the article's claim? Show your thinking.

13. The graph shows the number of collision claims per 100 vehicles insured for male drivers in Alberta, in 2004.



Many insurance companies charge drivers under the age of 25 higher insurance premiums based on the probability of accidents. Find information about car insurance costs based on the probability of collision. Paste the article into your notebook.

a) In the article, what are the assumptions associated with each probability? Is each probability accurate? Explain.

b) In your opinion, is there a bias against young drivers? Explain your reasoning.

c) "Decisions about car insurance costs are based on a combination of experimental probability, theoretical probability, and biased judgment." Do you agree or disagree with this statement? Explain your reasoning.

WWW Web Link

For information about road safety, risk factors, and factors affecting car insurance rates, go to www.mathlinks9.ca and follow the links.

14. Cathy and John are waiting for the bus. John predicts that one of the next five vehicles to pass the bus stop will be a minivan. Cathy predicts two of the next five vehicles will be minivans. John made his prediction based on five types of vehicles on the road: cars, sport utility vehicles (SUVs), buses, minivans, and trucks.

a) How do you think John and Cathy made their predictions?

b) John and Cathy decided to test their predictions by conducting a survey. They observed vehicles passing the bus stop for 1 h at the same time each day for five days. The table shows their results.

Type	Day 1	Day 2	Day 3	Day 4	Day 5
Car	30	28	25	27	25
SUV	20	25	18	22	18
Bus	4	4	3	4	4
Minivan	8	10	12	9	7
Truck	10	7	11	8	10

From the results, whose prediction was more accurate? Explain.

c) In one day, 800 vehicles passed the bus stop. Based on the survey results, predict the number of trucks.

15. The Jackson family is celebrating the coming birth of triplets. They currently have two boys. Mrs. Jackson is hoping for three girls.

a) What is the theoretical probability that she will have three girls? Show your thinking.

b) The Jacksons used a random number generator to simulate the situation. They decide that 1 indicates a girl and 0 indicates a boy. The table shows the results for ten trials. What is the experimental probability of three girls?

Trial	Experimental Results		
1	0	0	0
2	0	1	1
3	0	1	0
4	1	0	1
5	1	0	1
6	1	1	0
7	1	0	0
8	1	1	0
9	0	0	0
10	0	1	0

c) Compare the experimental probability and the theoretical probability.

d) The boys predict that their mother will have three more boys. Do you agree with their prediction? Justify your answer.

e) What assumptions did you make?

Extend

16. A random sample of 160 students out of 2100 participants in a summer youth program responded to a survey.

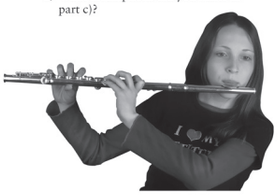
	Yes	No
Do you play a musical instrument?	40	120
Do you play on a sports team?	25	135

a) Does the sample represent the population? Explain.

b) Based on the data, what is the probability that a participant, chosen at random, will play a musical instrument?

c) Of the 160 students interviewed, 20 students played a musical instrument and were on a sports team. Predict how many of the 2100 youth group members do not play a musical instrument or play on a sports team.

d) What assumptions did you make in part c)?



17. Search magazines, newspapers, or the Internet. Look for an article that uses probability to make a prediction for a population.

a) Identify the assumptions associated with the probability.

b) Explain the limitations of each assumption.

c) In your opinion, is the prediction accurate? Justify your answer.

18. Search magazines, newspapers, or the Internet for an issue of personal interest.

a) Take a stand on the issue. Write an argument that includes a probability statement to support your stand. Use methods of your choice to display your data.

b) Take the opposite stand. Using the same data, or new data, write an argument that includes a probability statement to support your new stand.

c) Present your arguments to a classmate. Have your classmate point out the strengths and weaknesses of both arguments.

Extend

For #17 and 18, students need access to magazines, newspapers, or the Internet.

For #17, students look for an article that uses probability to make predictions. Students may find the Web Link on TR page 588 about updated science news helpful.

For #18, students research an issue of personal interest and take a stand on the issue. They develop an argument that includes a probability statement for and against their stand. This question provides students with the opportunity to apply their learning in section 11.3 by completing a mini-research project and may be helpful preparation for what they will do for their research project in the Math Link. Have students exchange their arguments with a classmate and provide feedback about the strengths and weaknesses of each argument.

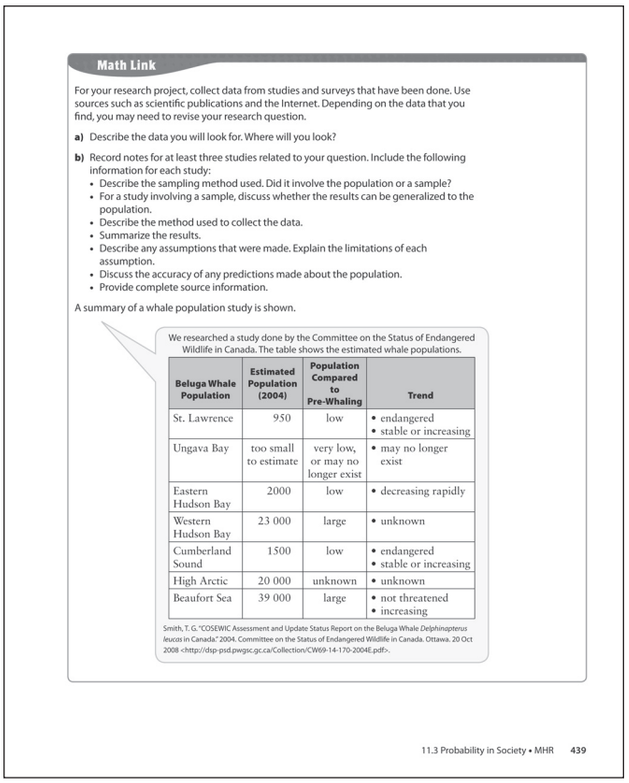
Literacy Link Direct students to their concept map, and have them complete a definition for *biased sample*. They should connect the oval with the definition to the oval titled sample. Consider having students attach an additional oval to biased sample and use it to summarize their own example.

Math Link

The Math Link provides students with an opportunity to apply their understanding of representative samples, assumptions linked to probabilities, and predictions based on probabilities. In this Math Link, students research and record notes for at least three studies related to their research question. They describe the sampling method used, discuss whether the results can be generalized to the population, summarize the results, describe any assumptions, discuss the accuracy of any predictions, and provide complete source information. Encourage students to record their notes in the booklet related to the project in their Foldable.

As a class, read the Math Link and direct them to the example of the study about beluga whale populations. Note that the example shows only research results and source information, and no analysis. Point out the format for recording source information that is shown below the table. Encourage students to record the complete source information for each research study as soon as they select the study, to avoid losing it later.

Provide students with access to magazines, scientific publications, or the Internet. As students work, circulate and coach students as needed to assist them with analysing the research studies they find.



Math Link

For your research project, collect data from studies and surveys that have been done. Use sources such as scientific publications and the Internet. Depending on the data that you find, you may need to revise your research question.

a) Describe the data you will look for. Where will you look?

b) Record notes for at least three studies related to your question. Include the following information for each study:
- Describe the sampling method used. Did it involve the population or a sample?
- For a study involving a sample, discuss whether the results can be generalized to the population.
- Describe the method used to collect the data.
- Summarize the results.
- Describe any assumptions that were made. Explain the limitations of each assumption.
- Discuss the accuracy of any predictions made about the population.
- Provide complete source information.

A summary of a whale population study is shown.

We researched a study done by the Committee on the Status of Endangered Wildlife in Canada. The table shows the estimated whale populations.

Beluga Whale Population	Estimated Population (2004)	Population Compared to Pre-Whaling	Trend
St. Lawrence	950	low	• endangered • stable or increasing
Ungava Bay	too small to estimate	very low, or may no longer exist	• may no longer exist
Eastern Hudson Bay	2000	low	• decreasing rapidly
Western Hudson Bay	23 000	large	• unknown
Cumberland Sound	1500	low	• endangered • stable or increasing
High Arctic	20 000	unknown	• unknown
Beaufort Sea	39 000	large	• not threatened • increasing

Smith, T. G. "COSEWIC Assessment and Update Status Report on the Beluga Whale *Delphinapterus leucas* in Canada." 2004. Committee on the Status of Endangered Wildlife in Canada. Ottawa. 20 Oct 2008 <http://dsp-psd.pwgsc.gc.ca/Collection/CW69-14-170-2004E.pdf>.

11.3 Probability in Society • MHR 439

Meeting Student Needs

- Consider assigning fewer questions to students who need help with text-dense questions. Alternatively, provide students with more time.
- For #15, you might remind students that a random number generator on a computer or calculator can generate a large number of outcomes for a simulation. If computers are available, have students use the random number generator described in the Web Link on this TR page to give them results for a number of trials. They might generate new results for #15 and then answer the questions using the new results.
- Provide **BLM 11-10 Section 11.3 Extra Practice** to students who would benefit from more practice.

ELL

- For #1 and 3, consider allowing English language learners to discuss their ideas in their first language, and then develop their response in English. This offers them the opportunity to activate their knowledge using familiar language. Afterward, it may be easier for them to ask for the missing vocabulary to express their thinking in English.
- For #3, you might limit the number of choices of flavours for some students.
- For #8, clarify the terms *election*, *mayor*, *polled*, and *candidate*.
- Clarify the meaning of any terms that are unfamiliar to students.
- For the Math Link, ensure students understand that *pre-whaling population* refers to the population before commercial harvesting of whales began.

Gifted and Enrichment

- For #13, you might have students research what an actuary does in terms of analysing data to determine the probability of being involved in a car accident and the costs that result from an accident. They may find the related Web Link on this TR page useful.
- Have students discuss the following questions:
  – Why hold an election if a poll is sufficient?
  – How might the effect of bias be magnified in a small sample?
- Have students explore sample size. You might ask how many respondents are needed for a survey. Students may conclude that generally they should sample a larger percent of a small population and a smaller percent such as 10% of a large population. The related Web Link on this TR page that discusses error and level of confidence may be of interest.
- Have students evaluate the claim made on an ad or a commercial for a lottery such as Lotto 649 by researching the probability of holding a winning ticket. They may find the related Web Link on this TR page helpful.

Web Link

For information about what actuaries do, go to www.mathlinks9.ca and follow the links.

For a random number generator, go to www.mathlinks9.ca and follow the links.

For articles about updated science news, go to www.mathlinks9.ca and follow the links.

For an interactive site that provides information about sample size and a random sample calculator, go to www.mathlinks9.ca and follow the links.

For an analysis of the chances of holding a winning ticket for Lotto 649, go to www.mathlinks9.ca and follow the links.

Answers

Communicate the Ideas

1. Example: If the school cafeteria wants to find out which flavour of juice to carry, the population is the students in the school. A sample that represents the population is a random sample of all students in the school. A sample that does not represent the population is students who do not buy juice in the cafeteria.

Answers

Communicate the Ideas

2. Example: The student conducting the survey assumed that one person's response could be generalized to the class population. This led to a false prediction.

3. Example: If four flavours are offered, the theoretical probability would suggest that each flavour has a 25% chance of being chosen. If a survey is taken, the experimental probability may differ from the theoretical probability.

Assessment	Supporting Learning
Assessment *as* Learning	
Communicate the Ideas Have all students complete #1 to 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • For #1, some students may find it helpful to record the differences between a sample that is representative and one that is not, in their Foldable. • For #2, prompt students to explain why the sample is biased and how that might result in a false prediction. • For #3, ensure that students can verbalize the difference between experimental probability and theoretical probability. Clarify any misunderstandings before coaching students through the question. Prompt students to identify what information they need to determine each probability. Students may find reviewing Example 3 helpful. Ask which probability they think will be a better predictor of sales of different flavours of ice cream. • You may wish to have students use **Master 2 Communication Peer Evaluation** to assess each other's responses to #1 and 3.
Assessment *for* Learning	
Practise and Apply Have students do #4, 6, 9 to 12, and 14. Students who have no problems with these questions can go on to the remaining Apply questions.	• Students who need assistance with #4 may benefit from additional coaching with Example 1. Coach them through #4 by prompting students to convert $\frac{1}{20}$ to a decimal and a percent. Ask how this value can help them answer the question. Have them explain their thinking and then try #5 on their own. • Students who need assistance with #6 may benefit from additional coaching with Example 2. Coach them through #6 by asking: Was the sample biased? If so, how was it biased? How did the sample lead to a false prediction? Have them explain their thinking and then try #7 on their own. • Students who need assistance with #9 may benefit from additional coaching with Example 3. Coach them through corrections to their answers for #9, and then have them try #8 on their own. • Students who need assistance with #10 to 12, and 14 may benefit from additional coaching with the related examples. Have students explain their thinking and clarify any misunderstandings. These questions provide evidence of understanding of the concepts and skills addressed in section 11.3. Ensure that students are successful with these questions before moving on to the Math Link, where they will apply these concepts and skills.
Math Link The Math Link on page 439 is intended to help students work toward the chapter problem wrap-up titled Wrap It Up! on page 443.	• It is recommended that all students complete the Math Link. • Assist students who may have difficulty finding three studies for their question. They may have assumed that each study must use the same sampling method. Clarify that this is not the case. • Students who experience a difficult time in selecting their question should work with a partner or be provided dialogue time with the teacher. • Students who need help getting started could use **BLM 11–11 Section 11.3 Math Link**, which provides scaffolding.
Assessment *as* Learning	
Literacy Link By the end of section 11.3, have students complete a definition for *biased sample*.	• Some students may benefit from first recording the definition in their Foldable. Then, have them summarize the definition in their own words and organize it on the concept map. • Some students may benefit from attaching an additional oval and using it to summarize their own example of a biased sample.
Math Learning Log Have students complete the following statements: • I know that a sample represents the population when … • Before I complete my research project, I still need help to understand …	• Encourage students to refer to the notes in their Foldable. This may assist them in identifying false predictions. • Depending on students' learning styles, have them provide oral or written answers. • Encourage students to use the What I Need to Work On section of their Foldable to note what they continue to have difficulties with.