

CHAPTER
5

Linear Systems

Get Set

Answer these questions to check your understanding of the Get Ready concepts on pages 196–197 of the *Foundations of Mathematics 10* textbook.

Algebraic Expressions

1. Collect like terms to simplify each expression.

a) $6c + 4 + 1 - 4c$

b) $-5x + 3 + 2x - 12$

c) $3x - 5y + 11 + 4y - 3x - 1$

Manipulate and Solve Equations

2. Rearrange each equation to isolate x .

a) $3x - 9y = 21$

b) $5x - 15y - 20 = 0$

c) $3x + 4y = 18$

d) $4y = 2x + 5$

3. Solve each equation for x when $y = 3$.

a) $x = y + 11$

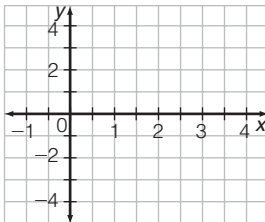
b) $y = 5x - 2$

c) $2x + 3y = 15$

Graph Linear Relations

4. Graph the following linear relation. State the slope and y -intercept.

$$y = 3x - 4$$



slope: _____

y -intercept: _____

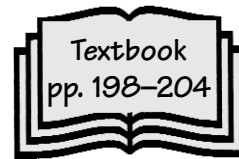
Translate Words to Algebra

5. Write an algebraic equation that models each situation. Define all variables used.

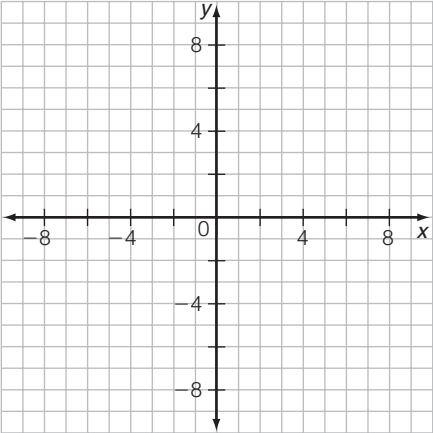
a) The sum of Leah's age and Joan's age is 29.

b) The total cost of renting a piece of equipment includes a flat fee of \$25 and an hourly fee of \$10/h.

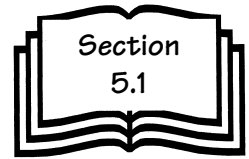
5.1

Solve Linear Systems
by Graphing

Warm-Up

| | |
|---|--|
| <p>1. Number Operations</p> <p>Evaluate.</p> <p>a) $\frac{2}{3}(3) + \frac{1}{2}$</p> <p>b) $\frac{2(-5) + 3(2)}{2}$</p> <p>c) Rewrite part b) as the sum of two fractions each having 2 as its denominator.</p> | <p>2. Algebra</p> <p>Rearrange each equation to isolate y.</p> <p>a) $2x + y = 7$</p> <p>b) $6x + 3y + 12 = 0$</p> |
| <p>3. Linear Relations</p> <p>a) Draw an example of a graph that represents a linear relation.</p>  <p>b) What would the point $(0, 1)$ represent on the graph of a linear equation?</p> | <p>4. Algebraic Expressions</p> <p>Evaluate each expression for the given value of x.</p> <p>a) $y = 3x - 7$ $x = 4$ b) $2x - y = 4$ $x = -1$ c) $y = -\frac{1}{3}x + 5$ $x = 6$</p> |
| <p>5. Estimation</p> <p>If an object is moving at a constant rate of 2.2 m/s, approximately how far will it travel in 1 min?</p> | <p>6. Math Literacy</p> <p>Rearrange the linear equation $-2x + y = 5$ into slope y-intercept form.</p> |

Practise: Solve Linear Systems by Graphing



1. Provide an example of each of the following.
- a) a linear equation b) a linear system c) the point of intersection of two lines

2. a) Rearrange the following equations into slope y-intercept form.

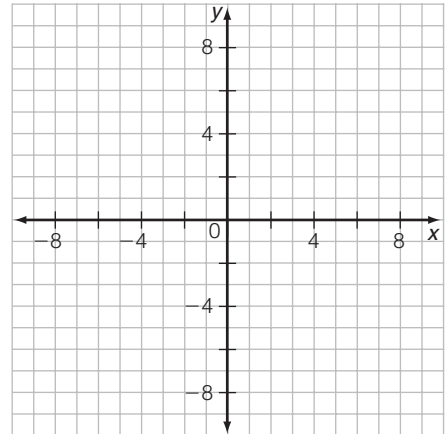
$$-2x = -y + 1 \text{ ① } \underline{\hspace{4cm}}$$

The slope is _____ and the y-intercept is _____.

$$x = -y + 4 \text{ ② } \underline{\hspace{4cm}}$$

The slope is _____ and the y-intercept is _____.

- b) Use the slope and y-intercept to graph each line on the same coordinate grid. Label each line with its equation.



- c) What is the point of intersection of the two lines?

(____, ____)

- d) Check these coordinates in each of the original equations.

Equation ①

LS =

RS =

Equation ②

LS =

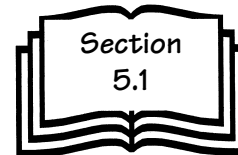
RS =

- e) What is the solution to this linear system? (____, ____).

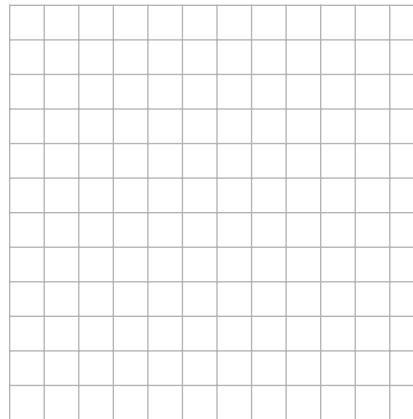
3. Consider the linear system $y = 20x + 150$ and $y = 30x + 100$.

- a) Use a graphing calculator to graph both equations in the same window. Choose an appropriate viewing window.

- b) What is the point of intersection of the two linear equations? (____, ____)



4. Solve this linear system by graphing. You can use either the grid provided or a graphing calculator



a) $y = 3x + 4$ ① and $y = 2x + 6$ ②

b) The point of intersection is

(____, ____)

c) Check these coordinates in each of the equations.

Equation ①

LS =

RS =

Equation ②

LS =

RS =

5. Riverside Golf and Country Club runs a women’s league in the summer. The cost to join is \$300 plus \$25 per round of golf played. South Shore Golf Course also offers a women’s league. Their membership fee is \$400, but they charge only \$20 per round played.

a) Write an equation to model the cost to join the women’s league at Riverside.

Let y represent the total cost and x represent the number of rounds played.

$y = ___ x + ___$

b) Write an equation to model the cost to join the women’s league at South Shore.

Let y represent the total cost and x represent the number of rounds played.

$y = ___ x + ___$

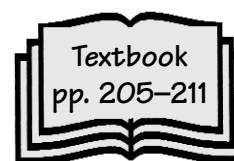
c) Use a graphing calculator to find the point of intersection of the two equations.

What does this point represent in terms of the cost of each membership?

The point of intersection is (____, ____).

This means that _____

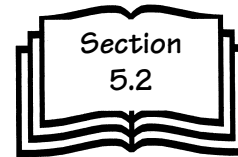
5.2

Solve Linear Systems
by Substitution

Warm-Up

| <p>1. Number Operations</p> <p>Evaluate.</p> <p>a) $3(4) + 12 - 4(2 + 4)$</p> <p>b) $(1 + 2 + 3) - (4 + 5 + 6)$</p> | <p>2. Algebra</p> <p>Identify which variable would be easier to solve for. Explain why.</p> <p>a) $3x + y = 11$</p> <p>b) $x - 2y + 14 = 0$</p> | | | | | | | | | | | | |
|---|---|-----|-----|----|--|----|--|---|--|---|--|---|--|
| <p>3. Linear Relations</p> <p>Consider the line defined by the equation $2x + y = 4$.</p> <p>a) Rewrite the equation in the slope y-intercept form.</p> <p>b) What is the slope?</p> <p>c) What is the y-intercept?</p> | <p>4. Math Literacy</p> <p>Describe two ways to graph the line represented by the equation $y = \frac{1}{4}x - 3$.</p> | | | | | | | | | | | | |
| <p>5. Estimation</p> <p>If Celia earns \$8.95/h at her job, about how much does she make in 8.5 h?</p> | <p>6. Tables of Values</p> <p>Complete the table of values for the relation $y = -x + 5$.</p> <table border="1" data-bbox="922 1587 1065 1843"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>-2</td> <td></td> </tr> <tr> <td>-1</td> <td></td> </tr> <tr> <td>0</td> <td></td> </tr> <tr> <td>1</td> <td></td> </tr> <tr> <td>2</td> <td></td> </tr> </tbody> </table> | x | y | -2 | | -1 | | 0 | | 1 | | 2 | |
| x | y | | | | | | | | | | | | |
| -2 | | | | | | | | | | | | | |
| -1 | | | | | | | | | | | | | |
| 0 | | | | | | | | | | | | | |
| 1 | | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | | |

Practise: Solve Linear Systems by Substitution



1. Why is the substitution method for solving a linear system given that name?

2. Solve the following linear system by substitution.

$$y = 2x + 1 \quad \textcircled{1} \qquad y = 3x - 13 \quad \textcircled{2}$$

- a) Substitute the expression for y from $\textcircled{1}$ into the other equation.

$$\textcircled{2} \quad \underline{\hspace{2cm}} = 3x - 13$$

- b) Solve for x by arranging the variable terms on one side of the equation and the constant terms on the other side.

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{1cm}} x = \underline{\hspace{1cm}}$$

$$x = \underline{\hspace{1cm}}$$

- c) Substitute the value for x into one of the original equations and solve for y .

Using $y = 2x + 1$,

$$y = 2(\quad) + 1$$

$$y = \underline{\hspace{1cm}} + 1$$

$$y = \underline{\hspace{1cm}}$$

- d) Check your answer by substituting the x - and y -values that you determined in parts c) and d) into the other equation.

For $y = 3x - 13$,

LS =

RS =

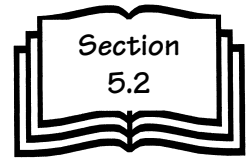
Is the left side equal to the right side? _____

The solution to this linear system is (,).

3. Solve each linear system by substitution.

a) $y = x + 3$ and $2x + 4y = 4$

b) $y = x - 5$ and $3x - 5y = 9$



4. Dianne is looking for a banquet hall for her school's winter semi-formal dance. The first hall she calls charges \$1000 plus \$14 per person, while a second hall charges \$800 plus \$16 per person. She needs to compare the costs of the two halls to decide which offers a better deal for the school dance.

- a) Write linear equations to model the total cost of each hall. Let y represent the total cost for each hall and x represent the number of people attending the dance.

First hall: $y = \underline{\quad} x + \underline{\quad}$

Second hall: $y = \underline{\quad} x + \underline{\quad}$

- b) Solve the system of linear equations using substitution.

- i) Substitute the expression for y from one equation into the other equation. Solve for x .

- ii) Substitute the value of x determined in part i) into one of the equations and solve for y .

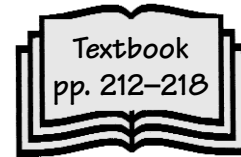
- iii) Check your answer by substituting x and y into the other equation.

- iv) Therefore, the solution is $(\underline{\quad}, \underline{\quad})$.

- c) What does the solution in part b) represent?

- d) If Dianne's school anticipates that 150 people will attend the dance, which hall should they choose?

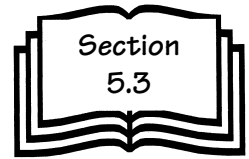
5.3

Solve Linear Systems
by Elimination

Warm-Up

| | |
|--|---|
| <p>1. Number Operations</p> <p>Evaluate.</p> <p>a) $4(3) + 2(6) - 12$</p> <p>b) $(2 \times 3 \times 1) + (2 \times 5) - (3 \times -2)$</p> | <p>2. Algebra</p> <p>Simplify.</p> <p>a) $7x + 12x - 2 + 24$</p> <p>b) $-2y + 6x + 2y + 34$</p> |
| <p>3. Linear Systems</p> <p>Is the point $(0, 2)$ the solution to the linear system $y = 2x - 4$ and $y = -x + 2$? Explain.</p> | <p>4. Math Literacy</p> <p>What is considered to be the break-even point for a business product?</p> |
| <p>5. Estimation</p> <p>Estimate the amount of GST that would be added to each price. GST is 6%.</p> <p>a) \$150.00</p> <p>b) \$19.99</p> | <p>6. Manipulate Linear Equations</p> <p>Rearrange the equation to isolate x.</p> $5x - 3y = 10$ |

Practise: Solve Linear Systems by Elimination



1. Why is the elimination method for solving linear systems given that name?

2. Solve the following linear system by elimination.

$$2x + y = 8 \quad \textcircled{1}$$

$$-2x + 3y = 0 \quad \textcircled{2}$$

- a) Add or subtract the equations to eliminate one variable.

- b) Solve the equation in part a) for the remaining variable.

- c) Substitute the value determined for the variable in part b) into equation $\textcircled{1}$ and solve for the other variable.

- d) Check your answer by placing the values for x and y into equation $\textcircled{2}$.

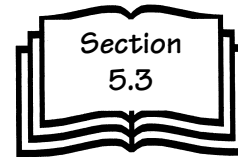
The solution to this linear system is (____, ____).

3. Solve the following linear system by elimination.

$$8x - 11y = 5 \quad \textcircled{1} \quad 4x + 17y = 25 \quad \textcircled{2}$$

- a) Multiply equation $\textcircled{2}$ by _____
to eliminate the variable _____.
when the equations are _____.
The new equation $\textcircled{3}$ is _____.

Hint: Adding or subtracting these equations will not eliminate a variable for this linear system.



- b) Add or subtract equations $\textcircled{1}$ and $\textcircled{3}$ to eliminate the variable.

- c) Solve the equation in part a) for the remaining variable.

- d) Substitute the value determined in part c) into equation $\textcircled{1}$ and solve for the other variable.

- e) Check your answer by placing the values for x and y into equation $\textcircled{2}$.

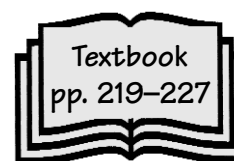
The solution to this linear system is (____, ____).

4. Describe what you would do with the following linear systems to eliminate one variable.

a) $2x + y = 2 \quad \textcircled{1}$ $3x - 2y = 10 \quad \textcircled{2}$

b) $3x - y = 14 \quad \textcircled{1}$ $2x - y = 10 \quad \textcircled{2}$

5.4

Solve Problems Involving
Linear Systems

Warm-Up

1. Number Operations

Evaluate.

a) $2(6 + 2) + 4(3) - 7$

b) $\frac{(6 + 2)}{4 - 2(-3)}$

2. Algebra

Add the equations in each pair.

a)
$$\begin{array}{r} -2x + y = 12 \\ + 2x + 3y = 2 \\ \hline \end{array}$$

b)
$$\begin{array}{r} 4x + 3y = 22 \\ + x - 3y = 7 \\ \hline \end{array}$$

3. Linear Relations

Do the data in the table below represent a linear relation? Explain your answer.

| Time (s) | Distance (m) |
|----------|--------------|
| 10 | 1.0 |
| 15 | 1.5 |
| 20 | 2.5 |
| 30 | 4.0 |

4. Math Literacy

Throughout this chapter you have been asked to solve given linear systems. Describe what that means, regardless of the method used to determine the solution.

5. Estimation

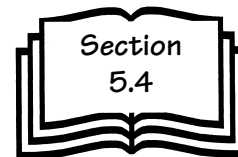
In 2006, a company experienced a 5.1% drop in profits compared to their \$125 000 profit in 2005. Estimate this company's profits in 2006.

6. Linear Systems

Solve this linear system.

$$\begin{array}{l} y = 3x + 4 \\ y = -2x + 9 \end{array}$$

Practise: Solve Problems Involving Linear Systems



1. Rearrange the linear system $2x + y = -3$ ① and $3x - y = 8$ ② to isolate y for each equation.
Solve by graphing with a graphing calculator.

The solution is (____, ____).

2. Solve the linear system in question 1 using the elimination method.
 - a) Add or subtract the equations to eliminate a variable.

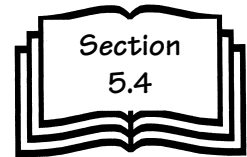
b) Solve the equation in part a) for the remaining variable.

c) Substitute the value determined in part b) into equation ① and solve for the other variable.

d) Check your answer by placing the values for x and y into the original equation ②.

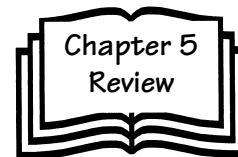
The solution to this linear equation is (____, ____).

For questions 3 to 5, use your preferred method for solving linear systems.



- 3.** Jordan's hockey team earns 2 points for a win and 1 point for a tie. They receive no points for a loss. Last season the team played 28 games and lost 9 of them. They had a total of 34 points. Let x represent the number of wins and y represent the number of ties.
- Write an equation to represent the total number of games won and tied.
 - Write an equation to represent the total number of points earned by Jordan's team.
 - Solve this linear system to find the number of games won and the number of games tied.
 - Which method for solving linear systems did you use for part **c)**? Why?
- 4.** Tony has started a lawn-mowing business for the summer. He charges customers a flat fee of \$50 to sign up and \$5 per week. He knows that his biggest competition is his neighbour Mike, who charges \$10 per week and no flat fee.
- Write the linear equations that represent Tony's and Mike's fees.
 - Solve the linear system in part **a)**.
 - Which method for solving linear systems did you use for part **b)**? Why?
 - What does your answer in part **b)** mean?
- 5.** Wendy rented a car for 6 days and drove it 480 km. The rental cost Wendy \$361.50. Charles rented the same car for 2 days and drove it 300 km. His rental cost \$173.00. Write a linear system for the car rental costs and solve by graphing on a graphing calculator.

Chapter 5 Review



1. Is $(5, 3)$ the solution to each of the given linear systems?

Explain your answer.

a) $3x - 4y = 3$ and $x + y = 8$

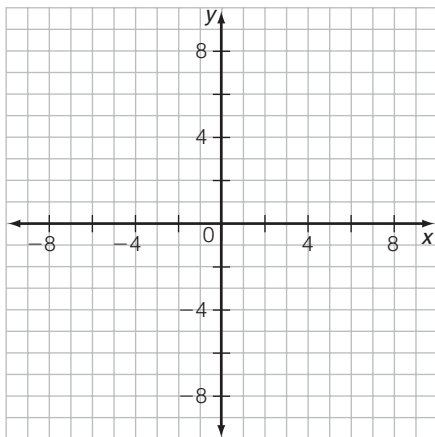
b) $x - y - 2 = 0$ and $y = -x - 3$

5.1 Solve Linear Systems by Graphing, textbook pages 198–204

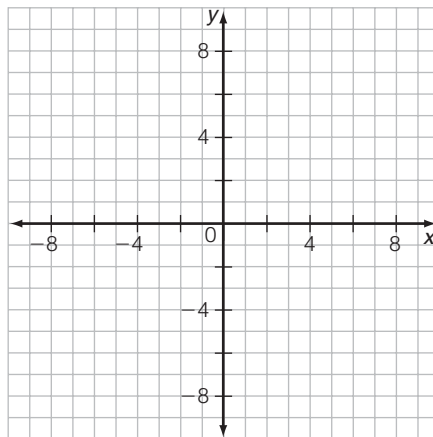
2. Solve each system of linear equations by graphing.

a) $y = 2x + 3$ and $y = x + 4$

b) $y = 3x - 4$ and $y = x - 2$



The solution is (____, ____).



The solution is (____, ____).

3. Use a graphing calculator to solve each linear system.

a) $y = 2x - 27$ and $y = -x + 12$

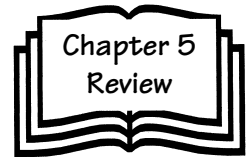
The solution is (____, ____).

b) $y = 4x + 20$ and $y = 2x + 30$

The solution is (____, ____).

- c) Why is it easier to solve the linear systems in this question with a graphing calculator than by drawing a graph by hand?

4. Rearrange each linear equation to isolate y .
- a) $x - y + 4 = 0$ b) $2x - 3y - 5 = 0$ c) $12x + 4y = 20$



5.2 Solve Linear Systems by Substitution, textbook pages 205–211

5. A high school student council is selling sweatshirts to raise money for their school. The supplier will charge a \$250 flat fee plus \$10 per sweatshirt made. The student council wants to sell the sweatshirts for \$20 each, but needs to know how many sweatshirts they must sell in order to break even. Show how they could determine this.
- a) For the student council to break even, what does the total cost of having the sweatshirts made need to be equal to?
- b) Write an equation that models the costs of having the sweatshirts made.
- c) Write an equation that models the revenue that the student council will make by selling the sweatshirts for \$20 each.
- d) Solve the linear system using your equations from parts b) and c).
- e) What is your solution from part d)? How many sweatshirts does the student council need to sell to break even?

5.3 Solve Linear Systems by Elimination, textbook pages 212–218

6. Describe what must be done first to solve the system of linear equations $x - 2y = 1$ ① and $x + y = 4$ ②.
- a) If x is to be eliminated: _____
- b) If y is to be eliminated: _____

5.4 Solve Problems Involving Linear Systems, textbook pages 219–227

7. Solve the linear system algebraically, using the elimination or substitution method. Briefly describe why you chose the method that you did.
- $y = 2x + 1$ ① $3x - 5y = 9$ ②

Method used:

Reason: