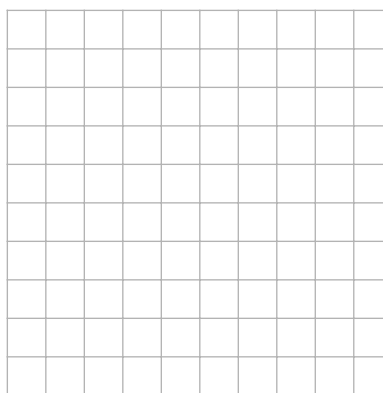# CHAPTER 8 · Represent Quadratic Relations

Get Set

Answer these questions to check your understanding of the Get Ready concepts on pages 318-319 of the *Foundations of Mathematics 10* textbook.
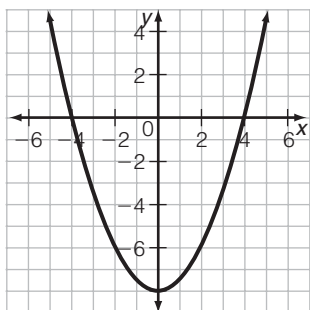
Relations

1. Graph the relation, then identify the relation as linear, quadratic, or neither.

   _____

x	y
−4	0
−3	−12
−2	−20
−1	−24
0	−24
1	−20
2	−12
3	0

Key Features of Quadratic Relations

2. Identify the information indicated.

   Coordinates of vertex: (_____, _____)

   Equation of axis of symmetry: _____

   x-intercepts: _____

   y-intercept: _____

Algebraic Operations

3. Substitute each given value for x, then solve for y.

   a) $y = (x - 3)^2 + 1$ for $x = 2$: _____

   b) $y = 2x^2 + 5x + 5$ for $x = -3$: _____
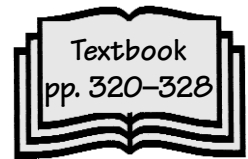
4. Expand and simplify.
   a) $(x - 4)^2$ b) $(x + 4)(x - 1)$

5. Factor.
   a) $4x^2 - 12$ b) $x^2 - 7x + 10$

8.1 Interpret Quadratic Relations

Warm Up

1. Linear Systems

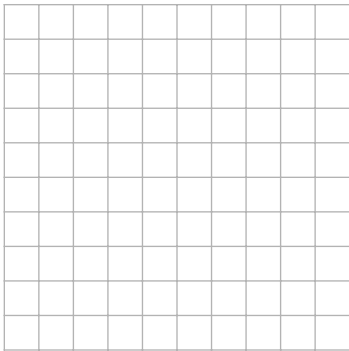Solve the linear system $y = 2x + 3$ and $y = -x + 12$.

2. Identify Quadratic Relations

Use first and second differences to determine if the relation is quadratic.

x	y	First Differences	Second Differences
-3	20		
-2	13		
-1	8		
0	5		
1	4		
2	5		
3	8		

3. Graph Quadratic Relations

Graph the relation $y = \dfrac{1}{2}x^2 - 4$.

4. Algebraic Operations

Re-arrange each equation into slope and y-intercept form.

a) $x + y - 3 = 0$ **b)** $4x - 2y + 5 = 0$

5. Math Literacy

Explain how to determine if two lines are parallel or perpendicular. Include an example of each situation.

_____

_____

6. Common Factoring
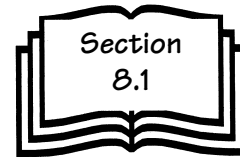
Factor completely.
$3a^3 - 15a$

7. Factor Trinomials
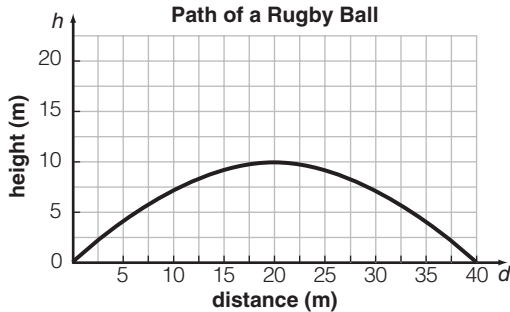
Factor $x^2 - x - 2$.

8. Substitute and Solve

Given the equation $y = 3x^2 - 4$, find the value of y when $x = -2$.

Practise: Interpret Quadratic Relations

1. The graph shows the path that a rugby ball followed after it was kicked. The *y*-axis represents the height of the ball in metres while the *x*-axis represents the horizontal position of the ball in metres.

Path of a Rugby Ball

a) What was the maximum height reached by the ball? _____

b) At what horizontal distance did the ball reach its maximum height? _____

c) How far did the ball travel before it hit the ground? _____

d) If the kicker was 32 m out from the cross bar of a goal post, would the ball pass over the cross bar if it is 3 m off of the ground? _____

e) What is the farthest distance the ball can be kicked and still pass over the 3 m cross bar?

_____

2. The quadratic relation $h = -5t^2 + 210$ describes the path of a rock that falls from the top of a cliff, with h representing the height in metres and t representing the time in seconds.

a) Complete the table. Then graph the relation.

$h = -5t^2 + 210$

t (s)	h (m)
0	
1	
2	
3	
4	
5	
6	
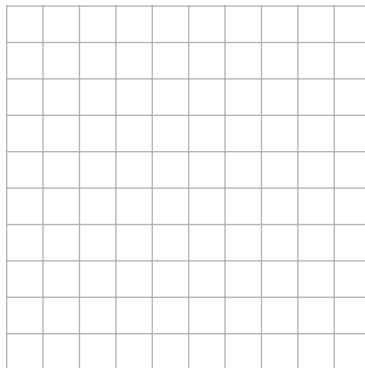
b) What is the height of the cliff? _____

c) How long will it take the rock to reach the bottom of the cliff?

Round your answer to the nearest tenth of a second. _____

d) How far from the bottom of the cliff is the rock when half of the time has passed?

_____

Date: _____

3. The table shows the curve of a clothes line that hangs between two poles 35 m apart.
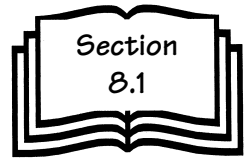
Horizontal Distance from First Pole (m)	0	5	10	15	20	25	30	35
Height of Line Above Ground (m)	2.14	1.77	1.52	1.49	1.46	1.51	1.73	2.12

a) Use a graphing calculator to plot the data.

b) Find the equation that models the curve of the clothes line. _____

c) Answer to the nearest tenth of a metre. How far from the ground is the lowest point?

_____

At what horizontal distance does it occur? _____

d) How far from each end could a person 1.6 m tall stand so their head just touches the line?

_____

4. Describe two methods that can be used to determine if a relation is quadratic.

_____

_____

_____

5. A harbour ferry service has 240 000 riders per month who pay a fare of $2. The fare is to increase in the new year. Previous fare increases have shown that for every $0.10 increase in the fare, the number of riders will drop by 10 000.
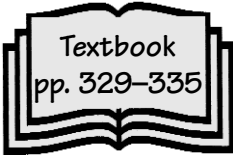
a) Complete the table.

Fare ($)	Riders	Total Revenue ($)
2.00	240 000	480 000
2.10		
2.20		
2.30		
2.40		
2.50		
2.60		

b) Plot revenue versus fare using a graphing calculator.

c) What fare price would generate the most revenue? _____

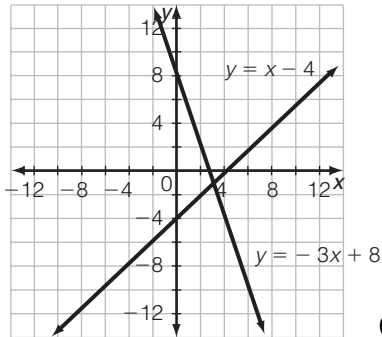**d)** What total revenue would this generate? _____

8.2 Represent Quadratic Relations in Different Ways

Warm Up

1. Linear Systems

State the point of intersection.

$y = x - 4$

$y = -3x + 8$

(____, ____)

2. Quadratic Relations

Complete a table of values from $x = -3$ to $x = 3$ for the relation $y = 3x^2 - 3x - 6$.

x							
y							

3. Solve a Linear System Algebraically

Solve the linear system
$y = x + 2$ and
$y = -x - 8$.

4. Math Literacy

Explain how you could show that the relations $y = (x - 2)(x + 1)$ and $y = x^2 - x - 2$ are different forms of the same equation, using a graphing calculator.

_____

_____

_____

_____

_____

5. Factor Trinomials

Factor completely.
$x^2 - 9x + 8$

6. Identify the x-Intercepts

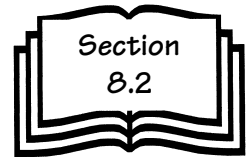What are the x-intercepts of the relation $y = (x - 3)(x + 6)$?

7. Compare Quadratic Equations

Do the equations $y = x^2 + x - 6$ and $y = (x - 3)(x + 2)$ represent the same relation? How do you know?

8. Analyze a Quadratic Relation

Given the equation $y = x^2 + 7x + 6$, identify the x- and y-intercepts.

Practise: Represent Quadratic Relations in Different Ways

1. Find the *x*-intercepts of each quadratic relation without graphing.
   a) $y = x^2 - 3x - 28$

   b) $y = c^2 + 2c - 15$

   c) $y = x^2 + x - 12$

2. Identify the *y*-intercept of each quadratic relation.
   a) $y = -x^2 + 2$

   b) $y = 3x^2 - 6x - 4$

   c) $y = -3x^2 + 4x - 6$

3. Explain how you can determine whether a quadratic relation will have a minimum or a maximum value, without graphing the relation.
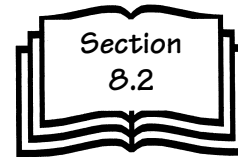
   _____

   _____

   _____

4. The area of a rectangle can be represented by the relation $A = 8x - x^2$.

   a) Factor the expression for area. _____

   b) What value of *x* will generate the rectangle with the greatest area? _____

x (cm)									
Area (cm²)									

   c) What will be the greatest area? _____

5. A rectangle has dimensions $x + 11$ and $2x + 5$, both measured in centimetres.

a) Draw the rectangle and label the sides.

b) Write the area inside the rectangle.

_____

c) Find the value of x that will produce an area of 117 cm².

> Section
> 8.2

> **Hint**: this means that the year 2000 is $t = 0$ and 2003 would be $t = 3$.

6. A model for the population in a small city is given by the relation $P = 14t^2 + 820t + 42\ 000$, where t is the time in years measured from the year 2000.

a) Calculate what the population will be in 2009.

b) Calculate what the population was in 1997.

c) Graph the relation $P = 14t^2 + 820t + 42\ 000$ using a graphing calculator.

d) Find when the population was the least. _____
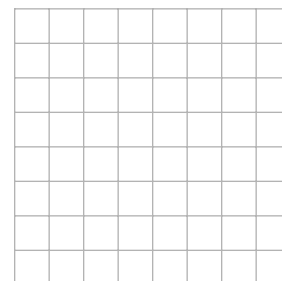
e) What was the least population? _____

7. The path of a soccer ball can be defined by the relation $h = -0.025d^2 + d$, where h represents the height in metres and d represents the horizontal distance in metres the ball travels before it hits the ground.

a) Find the d-intercepts. _____

b) Sketch a graph of the relation.

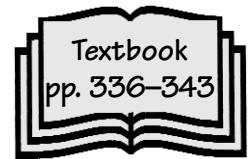| distance (m)									
height (m)									

c) What is the maximum height of the soccer ball? _____

d) How far will the soccer ball have travelled horizontally at its maximum height?

_____

8.3 The Quadratic Relation $y = ax^2 + c$

Warm Up

1. Linear Systems	2. Quadratic Relations

1. Linear Systems

Solve the linear system.

$y = 4x - 5$

$y = x + 1$

2. Quadratic Relations

Complete the table of values for the relation.

$y = x^2 + 3x + 2$

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
y											

3. Quadratic Relations

Complete the table of values for the relation.

$y = (x + 1)(x + 2)$

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
y											

4. Graph Quadratic Relations

Graph the relations $y = x^2 + 3x + 2$ and $y = (x + 1)(x + 2)$ on the same grid.

5. Math Literacy

Refer to the graphs from question 4. Explain why the graphs look the way they do.

6. Compare Quadratic Relations

Which of these equations represent parabolas that open downward? How do you know?

a) $y = 3x^2 - 6$

b) $y = -x^2 + 3$

c) $y = -\frac{1}{2}x^2$

7. Manipulate Quadratic Equations

The graph of the quadratic relation $y = -3x^2 + c$ passes through the point $(1, 2)$. Find the value of c.

8. Manipulate Quadratic Equations

The graph of the quadratic relation $y = ax^2 - 26$ passes through the point $(3, 1)$. Find the value of a.

Practise: The Quadratic Relation $y = ax^2 + c$
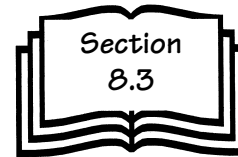
1. Without graphing, order the quadratic relations from narrowest
to widest.

a) i) $y = \dfrac{1}{2}x^2 + 3$ ii) $y = x^2 + 3$ iii) $m = 2p^2 + 3$

Order: _____

b) i) $y = -\dfrac{1}{4}x^2 - 4$ ii) $r = -\dfrac{1}{2}s^2 - 4$ iii) $y = -\dfrac{1}{3}x^2 - 4$

Order: _____

2. For each quadratic relation, find the y-intercept and determine if it is a minimum or a
maximum value.

Quadratic Relation	y-intercept	Maximum or Minimum
a) $y = x^2 + 5$		
b) $y = -\dfrac{1}{3}x^2 - 7$		
c) $y = -3x^2 + 27$		
d) $y = \dfrac{1}{4}x^2 - 1$		

3. Use a graphing calculator to graph each relation in question 2, then find the x-intercepts
of each relation.

a) _____   b) _____   c) _____   d) _____

4. A penny is dropped into a tank of water at the water's surface.
It falls to the bottom according to the relation $d = -3.5t^2 + 35$,
where d is the depth of the water measured in metres and t is the
time after the penny was dropped, measured in seconds.

time (s)	depth (m)
0	
1	

a) Complete the table of values for the relation $d = -3.5t^2 + 35$.
Round your answer to one decimal place.

b) How deep is the tank of water? _____

c) How long will it take for the penny to reach the bottom of the tank?

_____
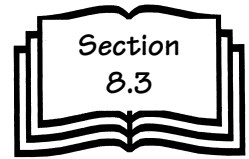
5. Graph each pair of quadratic relations on the same grid and describe the graphs.
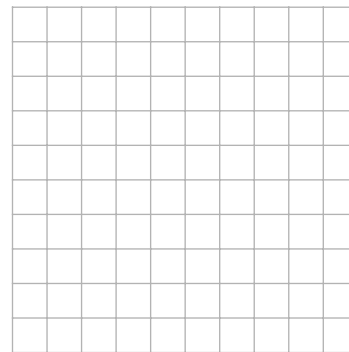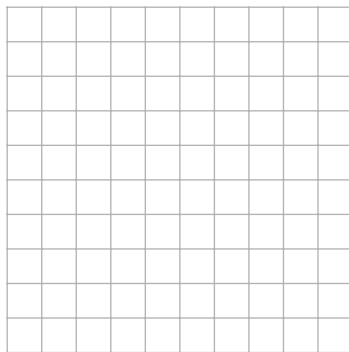
a) $y = 2(x + 3)(x - 3)$ and
$y = 2x^2 - 18$

b) $y = -4x^2 + 4$ and
$y = -4(x - 1)(x + 1)$

x	y
−4	

x	y

x	y
−3	

x	y

_____

_____

_____

_____
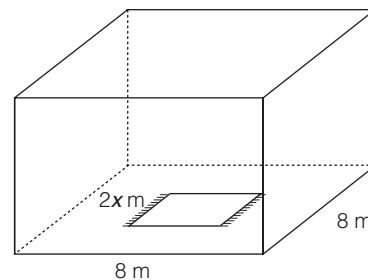
6. A square room of side length 8 m contains a square carpet of side length $2x$ m centred in the room.

a) Draw a diagram modelling the floor of the room.
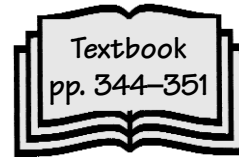
b) What is the area of the floor of the room?

_____

c) What is the area of the square carpet?

_____

2x m

8 m

8 m

d) Write an expression to represent the area of the bare floor that surrounds the carpet.

_____

8.4 Solve Problems Involving Quadratic Relations

Warm Up

1. Factor Trinomials	2. Quadratic Relations
Factor $x^2 + 5x - 14$.	Graph the relations below on the same set of axes using a graphing calculator. $y = -2x^2 + 3x + 9$ $y = (x - 3)(2x + 3)$
3. Factor a Difference of Squares	**4. Interpret Quadratic Relations**
Factor $4h^2 - 81$.	Use technology to identify the maximum or minimum of the quadratic relation defined by the equation $a = -45b^2 + 750c + 11\ 250$.
5. Math Literacy	**6. Interpret Quadratic Relations**
The equation that models the revenue for a product is $R = -38x^2 + 500x + 2500$, where R represents the revenue in dollars and x represents the price of the product. Explain how you can use the equation to determine the price that gives the maximum revenue.	Without graphing, identify the y-intercept of the parabola defined by the equation $y = (x + 2)(x - 9)$.
7. Interpret Quadratic Relations	**8. Trajectory of a Rocket**
Without graphing, identify the x-intercepts of the parabola defined by the equation $y = x^2 - 16$.	A model rocket follows a path described by the relation $h = -5t^2 + 200t$, with h representing the height in metres and t the time in seconds after the rocket was fired. **a)** How long is the rocket in the air? _____ **b)** When does it reach its maximum height? _____

Practise: Solve Problems Involving Quadratic Relations

Section
8.4
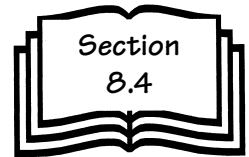
1. The path of a basketball can be modelled by the relation $h = -0.06d^2 + 0.6d + 3$, where h is the height of the ball in metres and d is the horizontal distance the ball travels in metres.

   a) Find the maximum height reached by the ball using technology.

   _____

   b) What is the horizontal distance the ball has travelled when it reaches this maximum height?

   _____

distance (m)	height (m)

2. On the set for an upcoming movie, a stunt woman jumped from a window of a burning building. The path followed by the stunt woman can be modelled by the relation $h = -4.9t^2 + 71.7$, where h is her height, in metres, above the safety net and t is the time, in seconds, since she jumped.

   a) How far below the window is the safety net?

   _____

   b) Calculate how far the stunt woman falls in the first 1.6 s after jumping. Round your answer to three decimal places.

   c) Calculate how long it will take her to fall to the safety net. Round your answer to two decimal places.
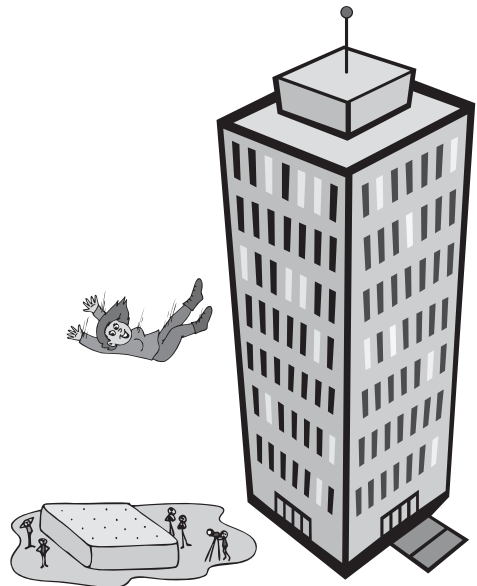
3. Jeremy kicked a football that follows a path that can be modelled by the relation $h = -4.9t^2 + 26t + 0.25$, where h represents the height, in metres, and t represents the time, in seconds, after Jeremy kicked the ball. Round your answers to two decimal places.

   a) Find the zeros of the relation using a graphing calculator. Interpret their meaning.

   _____

   b) How long after the ball was kicked did it reach its maximum height?

   _____

   c) What is the maximum height? _____
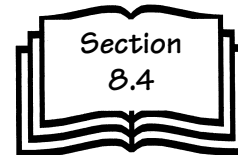
4. A baseball hit by a batter follows a path that can be described by the quadratic relation $h = -5t^2 + 10t + 1$, where h represents the height, in metres, and t represents the time, in seconds, after the ball was hit.

Section 8.4

a) Complete the table of values for the relation.

$h = -5t^2 + 10t + 1$

| time (s)	0	0.5	1	1.5	2	2.5	3
height (m)							

b) How high off the ground was the ball when it was hit? _____

c) What was the maximum height of the ball? _____

How long did it take for the ball to reach its maximum height? _____

d) Calculate the total time the ball was in the air to the nearest tenth of a second.

5. The organizers of a spring fair have developed a profit relation (P) that depends on the ticket price (t) charged per person. The profit is modelled using the relation $P = -37t^2 + 1258t - 7700$.
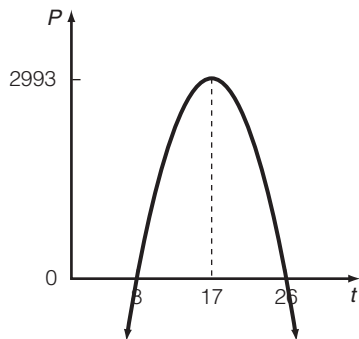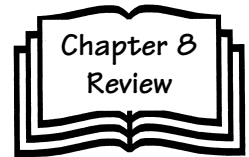

a) What does the third term represent in this relation? _____

b) Find the ticket price that would produce a maximum profit. _____

c) What will be the maximum profit? _____

Chapter 8 Review

8.1 Interpret Quadratic Relations

1. The data in the table represent the path of a
disc after it was thrown into the air.

   a) Use a graphing calculator. Graph the data
   and find the curve of best fit.

   b) How far did the disc travel horizontally
   before it hit the ground? Round your answer

   to one decimal place. _____

   c) What was the disc's maximum height?
   Round your answer to one decimal place.

   _____

| Horizontal Distance (ft)	Height (ft)
0	0.8
10	5.0
20	13.4
30	20.0
40	22.7
50	23.5
60	22.4
70	19.8

   d) How far had it travelled horizontally when it reached this height?

   Round your answer to one decimal place. _____

2. A hamburger stand sells a total of
300 hamburgers per day at $3.50 each.
Market research has shown that for
every $0.25 increase in price, 15 fewer
hamburgers will be sold.

   a) Complete the table.

   b) Plot revenue versus price using
   a graphing calculator.

   c) What price would generate the

   highest total revenue?

   _____

   d) What total revenue would this generate? _____

Price ($)	Number Sold	Revenue ($)
3.50	300	1050
3.75	285	
4.00		
4.25		
4.50		
4.75		
5.00		
5.25		
5.50		

8.2 Represent Quadratic Relations in Different Ways
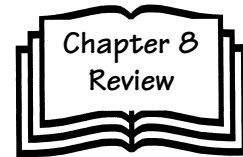
3. Identify the x-intercepts.

   a) $y = x^2 + 5x + 6$ **b)** $y = x^2 - 8x$

   c) $y = x^2 - 4x - 12$ **d)** $y = 12x^2 - 20x$

4. A car rental agency rents 400 cars a week at $80 per car. Industry research has shown that for every $2 increase in rental price, an agency will rent 8 fewer cars.

   a) Write an expression for the number of cars rented if the price is raised x times.

   _____

   b) Write an expression for the amount per car if the price is raised x times.

   _____

   c) Multiply the answers for parts **a)** and **b)** to get an expression that represents the revenue.

   d) Find the maximum revenue. _____

   e) For this revenue, how many cars are rented? _____

   What is the rental price per car? _____

5. How can you determine if a quadratic relation of the form $y = ax^2 + bx + c$ is the same as or different from a relation of the form $y = (x - s)(x - t)$?

   _____

   _____

   _____

8.3 The Quadratic Relation $y = ax^2 + c$

6. a) Complete the second and third columns in the table without a calculator.

Quadratic Relation	y-Intercept	Maximum or Minimum	x-Intercepts
a) $y = 2x^2 - 32$			
b) $y = \dfrac{1}{3}x^2 - 3$			
c) $y = x^2 + 9$			
d) $y = -\dfrac{3}{4}x^2 - 5$			

b) Graph each relation in part a using a graphing calculator and determine its *x*-intercepts. Fill in the last column of the table with your answers.

7. A roadway on a bridge is supported by two towers with a cable that joins them.
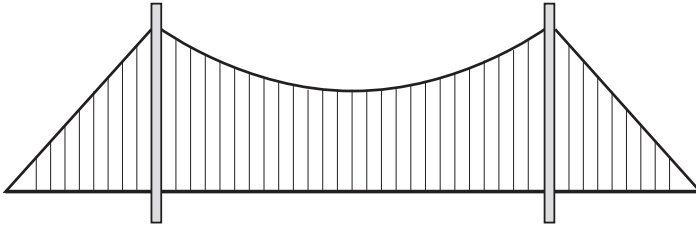
The cables between the towers hang in a parabolic shape that can be represented with the equation $y = 0.00036x^2 + 4$.

a) Graph the relation $y = 0.00036x^2 + 4$ on a graphing calculator.

b) Identify the minimum or maximum value and the coordinates of the vertex.

_____

c) Identify the *y*- and *x*-intercepts.

_____

8.4 Solve Problems Involving Quadratic Relations

8. The cost (in thousands of dollars) to produce items in a computer component manufacturing plant is given by the relation $C = 2x^2 - 29x + 100$, where *x* represents the number of hundreds of items produced. The revenue these items produce (in thousands of dollars) is given by the relation $R = x^2 - 10x + 250$, where *x* represents the number of hundreds of items sold.

a) Profit is defined as the difference between the revenue and the cost. Using $P = R - C$, as well as the two relations above, develop a profit relation for the company.
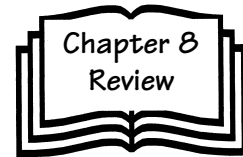
b) Graph your relation from part a on a graphing calculator.

c) Determine the zeros of the profit relation. _____

d) How many items should be produced to maximize profit? _____

e) What will be the maximum profit? _____

9. The local community theatre group sold 1200 tickets for the holiday concert at $20 per ticket. The committee plans to increase prices this year by $2 per ticket but they also believe that for each $2 increase, 60 fewer tickets will be sold.

a) Define your variables, then write the revenue relation that describes the ticket sales.

_____

_____

_____

_____

b) Complete the table of values.

Ticket price ($)	20	22							
Tickets sold (n)	1200								
Revenue ($)	24 000								

c) What selling price per ticket should maximize total revenue? _____

d) How many tickets must be sold to reach the maximum total revenue? _____

e) What is the maximum total revenue? _____