1.3 Similar Triangles

Strand
Measurement and Trigonometry

Student Text Pages
19–29

Suggested Timing
80–160 min

Tools
- cardboard
- coloured pencils or markers
- computers
- grid paper
- plain white paper
- protractors
- rulers
- scissors
- *The Geometer's Sketchpad®*

Related Resources
BLM 1.3.1 Practice: Similar Triangles
BLM 1.3.2 Achievement Check Rubric
BLM G1 Grid Paper
BLM G3 Protractor
BLM T1 *The Geometer's Sketchpad®* 4

Specific Expectations

Solving Problems Involving Similar Triangles

In this section, students will

MT1.01 verify, through investigation (e.g., using dynamic geometry software, concrete materials), properties of similar triangles (e.g., given similar triangles, verify the equality of corresponding angles and the proportionality of corresponding sides)

MT1.02 determine the lengths of sides of similar triangles, using proportional reasoning

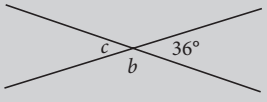
Link to Get Ready

The skills of finding ratios of proportional sides, necessary for similar triangles, are reviewed in questions 3 and 4 of the Get Ready. Students also will need to recall angle properties to find missing angles; these are reviewed in question 5 of the Get Ready.

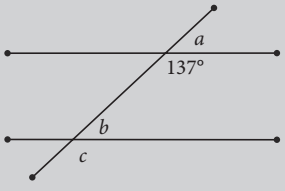
Warm-Up

1. The measures of two angles in a triangle are 43° and 62°. What is the measure of the third angle?

2.



3. Find the measure of all missing angles.



4. Solve $x:2 = 3:6$.

Warm-Up Answers

1. 75°
2. $b = 144°$; $c = 36°$
3. $a = 43°$, $b = 43°$, $c = 137°$
4. $x = 1$

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class. (5–10 min)

Common Errors

- Some students may not understand the meaning of corresponding angles and sides.

R_x Use large cutouts of a pair of similar triangles or transparencies of similar triangles. Demonstrate to students that corresponding angles coincide when one triangle is placed over the other. Shift the smaller triangle to show each pair of corresponding angles. Once the corresponding angles have been identified, students should recognize that corresponding sides are between pairs of corresponding angles.

- Some students may use the protractor incorrectly to measure the angles.

R_x Instruct students to place the baseline of the protractor on one arm of the angle. The centre of the baseline aligns with the vertex of the angle. Have students read the scale that starts with 0°.

- Some students may not be consistent in keeping the same triangle sides as the numerator/denominator in the side ratios.

R_x Have students line up the two triangle names one under the other. For example,
ABC
DEF

Ongoing Assessment

- The Discuss the Concepts can be used as a diagnostic assessment or self-assessment to assess students' readiness to move on.
- This section has many opportunities for students to do project work or assignments. Doing some in-class assignments will prepare students for lessons that involve assignments outside the classroom.

Section Opener

- Conduct a discussion on architects from the Section Opener with students' research on the draftsperson Career Profile from the Chapter Opener. Discuss similarities and differences between the two careers.

Investigate

Method 1: Use Pencil and Paper

- Instruct students to use rulers and grid paper or **BLM G1 Grid Paper** to construct the three triangles. Then, show students how to use their protractors to measure the angles. You may wish to use **BLM G2 Protractor** for this activity.
- Circulate while students complete the Investigate, and provide help as needed.
- You may wish to have students work in pairs.
- Encourage students who complete the Investigate early to assist others. Have students compare their conclusions with one another, or do the Investigate again with different triangles.
- Consolidate the Investigate by writing the properties of similar triangles on the board or on an overhead.

Method 2: Use *The Geometer's Sketchpad®*

- Distribute **BLM T1 *The Geometer's Sketchpad®* 4**, and review the instructions with students. If a computer projector is available, review *The Geometer's Sketchpad®* screens and menus.
- The dynamic software allows students to drag points and immediately see the changes in the diagram and the related measures.
- Once students have completed the Investigate, encourage them to manipulate their triangle and observe what happens to the related angles and ratios.
- Use **BLM 1.3.1 Practice: Similar Triangles** for extra practice or remediation.

Investigate Answers (pages 19 to 21)

Method 1: Use Pencil and Paper

2. Students' triangles may differ. The table below shows measures for the triangles in the student book. Ensure that the angles in each row of students' tables are equal, and that the side lengths in column 1 are proportional to the side lengths in columns 2 and 3.

$\angle A = 39°$	$\angle D = 39°$	$\angle G = 39°$
$\angle B = 51°$	$\angle E = 51°$	$\angle H = 51°$
$\angle C = 90°$	$\angle F = 90°$	$\angle I = 90°$
AB = 2.1 cm	DE = 4.2 cm	GH = 3.2 cm
BC = 1.3 cm	EF = 2.6 cm	HI = 2.0 cm
AC = 1.7 cm	DF = 3.4 cm	GI = 2.6 cm

3. The angles in each row of the table should be equal. The lengths of the sides are different.

4. Answers will vary, depending on students' triangles. However, ratios will all be equal as will the measures of the corresponding angles.

Method 2: Use *The Geometer's Sketchpad®*

7. The corresponding angle measures are the same. The lengths of the corresponding sides are proportional.

8. The ratios are all equal. Yes, all pairs of similar triangles will have equal ratios.

Accommodations

ESL—Pair ESL students with non-ESL students for the Investigate.

Gifted and Enrichment—Have interested students read the MathConnect in Extend the Concepts, and then conduct research in the library or on the Internet to find out more about the use of similar triangles in radiation treatment for cancer patients. Have students report their findings to the class.

Language—Assist or have other students assist students having difficulty drawing diagrams from verbal or written instructions.

Perceptual—Provide a colour diagram to assist students with the understanding of corresponding angles and sides.

Provide a handout of the conclusions.

9. All the corresponding angle measures are the same. The ratios of the corresponding sides are equal.

Students' conclusions may vary. A sample response: The corresponding angles in similar triangles have equal measures. The ratios of the corresponding sides will always be equal to each other.

Examples

- Point out to students the MathConnect showing the symbol ~ for "similar to" and the symbol ≐ for "approximately equal to."
- Work through the Examples as a class before proceeding to Discuss the Concepts. Alternatively, have students complete the Examples independently or in small groups before reviewing them as a class.
- After students have completed Example 1, ask them what information they would need to be able to find the measure of side EF. (Answer: the measure of side BC)

Key Concepts

- Ensure that students are able to write the proportions for two similar triangles.
- In addition to the corresponding angles, ensure students are able to identify corresponding sides. For example, if $\triangle ABC \sim \triangle DEF$, then $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ or $\frac{c}{f} = \frac{b}{e} = \frac{a}{d}$. This is the proportionality statement for the corresponding side ratios.

Discuss the Concepts

- Give students time to formulate their own answers before conducting a discussion.
- For questions D2 and D3, suggest to students that they use a concrete example if they are having trouble formulating an answer.

Discuss the Concepts Suggested Answers (page 25)

D1. a) The letters of the corresponding angles are listed in order; $\angle P = \angle X$, $\angle Q = \angle Y$, $\angle R = \angle Z$

b) The letters of the corresponding sides are listed in order; PQ and XY, QR and YZ, PR and XZ

D2. Write the three ratios and reduce. If the values are the same, the triangles are similar.

For example:

Given $\triangle MNP$ and $\triangle HJK$, where $MP = 6$, $NP = 7$, $MN = 4$, $HJ = 10$, $JK = 17.5$, $HK = 15$, the three ratios are:

$\frac{h}{m} = \frac{17.5}{7}$ $\quad$ $\frac{k}{p} = \frac{10}{4}$ $\quad$ $\frac{j}{n} = \frac{15}{6}$

$= 2.5$ $\quad$ $= 2.5$ $\quad$ $= 2.5$

Since the ratios of corresponding sides are equal, $\triangle MNP \sim \triangle HJK$.

D3. If the corresponding angles are equal, the triangles are similar.

Practise the Concepts (A)

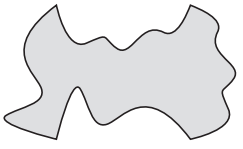
- Encourage students to refer back to the Investigate and the Examples before asking for assistance.
- Question 7 is a Literacy Connect. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy. You may wish to conduct a class discussion for this question.

Apply the Concepts (B)

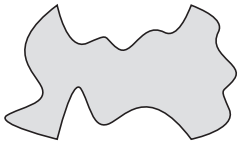
- There are no diagrams provided for questions 8, 9, 10, and 11. This may be the first time students see questions of this type. Instruct students to sketch a diagram before proceeding with the questions. You may wish to provide **BLM G1 Grid Paper** for these questions. Use **BLM 1.3.1 Practice: Similar Triangles** for additional practice as needed.
- Question 12 requires the use of *The Geometer's Sketchpad®*. When assigning this question, leave an appropriate amount of time for its completion based on the availability of the technology. You may wish students to use **BLM T1 *The Geometer's Sketchpad®* 4** for this activity.
- Question 13 is an Achievement Check question. Students may wish to use graphing software, such as *The Geometer's Sketchpad®*, for this activity. The Achievement Check can be used as a form of diagnostic or formative assessment or assigned as a small summative assessment piece. You may wish to use **BLM 1.3.2 Achievement Check Rubric** to assist you in assessing your students. As an extension of this activity, have students research on the Internet or in the library for more of M. C. Escher's work.

Achievement Check Answers (page 28)

13.



Design A



Design B

The similar figures are featured such that Design B must rotated 90° counterclockwise to fit Design A–like a puzzle. Tessellations examplify the four types of symmetry: rotation, translation, reflection and glide reflection.

Extend the Concepts (C)

- Assign the Extend the Concepts questions to students who are not being challenged by questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.
- You may wish to use this question as an extension to Investigate Method 1.
- Students who may find question 14 challenging may be able to complete it using technology, such as *The Geometer's Sketchpad®*.
- Questions 15 and 16 should only be assigned to students who have completed question 14.
- Refer students to the MathConnect in Extend the Concepts, and read it aloud to the class. Conduct a brief class discussion on how doctors may use the properties of similar triangles in their treatment of cancer patients. Have interested students research the topic further in the library or on the Internet, and report their findings to the class.