2.3 The Sine and Cosine Ratios

Strand
Measurement and Trigonometry

Student Text Pages
63–73

Suggested Timing
80 min

Tools
- computers
- *The Geometer's Sketchpad®*
- grid paper
- protractors
- rulers

Related Resources
BLM 2.3.1 Practice: The Sine and Cosine Ratios
BLM 2.3.2 Achievement Check Rubric
BLM G1 Grid Paper
BLM G2 Protractor
BLM T1 *The Geometer's Sketchpad®* 4

Specific Expectations

Solving Problems Involving the Trigonometry of Right Triangles

In this section, students will

MT2.02 determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios and the Pythagorean theorem

Link to Get Ready

The Get Ready segments Solving Proportions, Rounding, and Squares and Square Roots provide the needed skills for this section. Have students complete these questions before proceeding with Section 2.3.

Warm-Up

1. Sketch two similar triangles and explain why they are similar.

2. Express each ratio as a decimal. Round each answer to four decimal places.

a) $x = \frac{3}{7}$ **b)** $x = \frac{16}{33}$

c) $x = \frac{4}{11}$ **d)** $x = \frac{2}{13}$

3. Find the missing value. Round each answer to four decimal places.

a) $3 = 5 \div x$ **b)** $9 = x \div 11$

c) $x = 4 \div 5$ **d)** $11 = 51 \div \text{x}$

Warm-Up Answers

1. Two triangles are similar when corresponding angles are equal or when the ratios of corresponding sides are equal.

2. a) 0.4286 **b)** 0.4848 **c)** 0.3636 **d)** 0.1538

3. a) 1.6667 **b)** 99 **c)** 0.8000 **d)** 4.6364

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class. (5–10 min)

Section Opener

- Show pictures or a short video of kite skiing or surfing. Although many people have flown a kite, students may be surprised by this exciting application that uses parachute-sized kites to pull a person. Skis, snowboards, surfboards, boats, and buggies make this system effective on almost any surface. This sport is gaining popularity in Canada, especially in the province of Quebec.
- Ask students to read the definition of trigonometry, and have them add the term to their word wall.

Common Errors

- Some students may inadvertently switch calculators to radian mode.

R_x Without providing a detailed explanation of radian measure, ensure that students' calculators are set on degree mode.

- Some students may have trouble remembering how to key problems into their calculator, especially when solving for the inverse of sine or cosine.

R_x Encourage students to keep notes on how to key problems into their calculator.

- Some students may mix up operations when manipulating a formula to solve for a different variable.

R_x Introduce students to a function triangle as a memory aid. Divide a pyramid into top and bottom. In the top space, write "opposite." Divide the lower half of the pyramid in two, writing "sine" in the left space and "hypotenuse" in the right. In this structure, the top-bottom split represents division and the left-right split represents multiplication. To manipulate this sine equation, students can first put their fingers over the variable they wish to solve for, then write out the equation as they see it.

Investigate

- Have each student work with a partner to check each other's work and ensure that the triangles are similar. Watch carefully to ensure that students are comparing the sides in the correct order.
- You may wish to have students work with partners on the board to provide a visual for the rest of the class. Alternatively, you may wish to display an overhead of the table and one example as the class works through the activity so students can check their work.
- You may wish to have students use **BLM G1 Grid Paper** and/or **BLM T1 *The Geometer's Sketchpad*® 4** in this activity.
- Ensure that students read the MathConnect and understand the abbreviations.
- Note that for these questions, angles are referred to using different terminology. Where students have seen ∠C before, they will now see ∠ACB. Explain to students that, with the three-letter coordinate, the middle letter represents the angle being discussed.
- Have students read the definitions for sine ratio and cosine ratio. Have them add the key terms to their word wall.
- Use **BLM 2.3.1 Practice: The Sine and Cosine Ratios** for extra practice or remediation.

Investigate Answers (page 63–67)

Method 1

4. I notice that sin x is not the same as sin y.

5. I notice that cos x is not the same as cos y.

6. I notice that sin x is the same as cos y, because length of side opposite x is equal to side adjacent to y. This is also true of sin y and cos x.

Method 2

12. Yes, both the angles being measured and the lengths of the sides change.

20. They're the same.

22. The ratio is the same for sin ∠ACB and cos ∠CAB, but the value is different. This is also true for sin ∠CAB and cos ∠ACB.

Examples

- For Example 1, draw students' attention to the MathConnect and have them test their calculators to make sure they are using them correctly.
- For Example 2, you may wish to discuss the idea of a horizontal distance versus the string length, which is a diagonal distance (hypotenuse). This prepares students for situations where it is not explicitly stated to solve for the horizontal distance. Again, stress the fact that the diagram is given here, but often we are finding a value based on only a word problem and so need to know how to draw the diagram as well.
- In Example 4, students solve for the angle, given the lengths of two sides. Let students know that the sine ratio can be used in both situations. You could make this question relevant by discussing a storm that caused such damage. Crews must fix leaning poles but may give priority to those angled over a certain degree.

Key Concepts

- Review the concepts with students to ensure they understand the difference between finding the length of a side and finding the value of the angle.

Ongoing Assessment

- This may also be a good opportunity to observe and record the individual student's learning skills: group work, work habits, organization, and initiative.
- You may wish to assign the Chapter Problem question as a formative assessment now, or assign all of the Chapter Problem questions as a summative assessment after students have finished the chapter.

Accommodations

Memory—Help students remember the trigonometric ratios with the mnemonic SOH CAH. The first letter in each syllable represents the trigonometric function (i.e., sine, cosine), and the next two letters stand for one of the triangle's sides. The mathematical operation in each case is division. Thus, SOH means Sine equals Opposite over Hypotenuse and COH means Cosine equals Adjacent over Hypotenuse.

Discuss the Concepts

- Have each student work with a partner to answer the questions and read their answers to another pair. Have these groups of four work together to develop better answers. Then, have each of these groups of four work with another group of four to improve the answers, and so on until the whole class is involved. As a class, come up with one "ideal" answer that everyone can agree on and understand.
- Another possibility is to use these questions to create a journal entry. Collect the journals the following day to see who really understands the concepts presented.

Discuss the Concepts Suggested Answers (page 71)

D1. The value of the sine ratio depends on the angle because the angle does not depend on the length of the sides. See the Investigate: Method 1 on page 54 for a visual.

D2. No, the triangle can be any size and still have the same angles.

D3. If you know one angle in a right triangle, you can find the length of either the side opposite the angle or the hypotenuse by using the formula $\sin \angle = \text{opposite} \div \text{hypotenuse}$.

D4. If you know the length of the hypotenuse and one side of a right triangle, you can solve for the angle adjacent to the side by using the formula $\cos \angle = \text{adjacent} \div \text{hypotenuse}$.

Practise the Concepts (A)

- Encourage students to refer back to the Examples before asking for assistance.
- Circulate as students answer questions 1 and 2 to ensure all students are able to use their calculators effectively.
- Direct students' attention to the MathConnect. Have a brief discussion about geoscience and how scientists might use trignometry in these applications. Have interested students conduct further research in the library or on the Internet and report their findings to the class.

Apply the Concepts (B)

- Question 9 relates to something students will find of interest. This question could lead to a discussion of other skateboard ramps in the area or at the school.
- Question 12 is a Chapter Problem. It allows the students to see a continuation of the information presented at the outset of the chapter. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up.
- Question 13 is a Literacy Connect. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy.
  Be sure that students are able to articulate their understanding of the concepts. You may wish to have the students hand in their answers to this question and use each one as a formative assessment.

- Question 15 is an Achievement Check. It can be used as a form of diagnostic or formative assessment, or assigned as a small summative assessment piece. This provides an opportunity for formative or self-assessment, using **BLM 2.3.2 Achievement Check Rubric**. You may wish to use **BLM G1 Grid Paper** and **BLM G2 Protractor** to help students solve these questions.

Achievement Check Answers (page 73)

15. a) maximum reach = 5.91 m
minimum reach = 5.64 m

b) Yes, because it is at a 73.4° angle.

c) Yes, because the ladder in part b) is twice the length of the one in part a), we can expect that a safe range will be twice the safe range in the first question.

Extend the Concepts (C)

- Assign the Extend the Concepts questions to students who are not being challenged by questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.