3.1 Slope as a Rate of Change

Strand
Modelling Linear Relations

Student Text Pages
100–110

Suggested Timing
75–150 min

Tools
- calculators
- graphing calculators
- grid paper
- rulers

Related Resources
BLM 3.1.1 Practice: Slope as a Rate of Change
BLM 3.1.2 Achievement Check Rubric
BLM G1 Grid Paper
BLM G4 Four-Quadrant Grid

Specific Expectations

Graphing and Writing Equations of Lines

In this section, students will

ML2.01 connect the rate of change of a linear relation to the slope of a line, and define the slope as the ratio $m = \frac{\text{rise}}{\text{run}}$

ML2.02 identify, through investigation, $y = mx + b$ as a common form for the equation of a straight line, and identify the special cases $x = a$, $y = b$

Link to Get Ready

Students will learn about constant rates of change in this section and need to be comfortable with common factors and operations with fractions and decimals. Have students complete questions 1, 2, and 3, before proceeding with Section 3.1.

Warm-Up

1. Continue the pattern. 3, 5, 7, __, __, __

2. Express 71" in feet and inches.

3. If snow is falling at 3 cm/h, how much snow will fall in one day?

Warm-Up Answers

1. 9, 11, 13 **2.** 5'11" **3.** 72 cm

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class. (5–10 min)

Section Opener

- Have students look at the photo in the Section Opener and read the questions. Introduce the section by asking students if they have cell phones, whether and how often they use text messaging, and how much their text messaging costs. This provides an introduction to the Investigate.

Investigate

- Have students copy the table in question 1a) in their notebooks before assigning the Investigate questions.
- Investigate questions 2 and 3 can be completed by students orally or in their notebooks.
- Distribute grid paper and coach students through questions 4 and 6 if necessary. You may wish to use **BLM G1 Grid Paper** for this activity. Or, you may prefer to use **BLM G4 Four-Quadrant Grid**.

Common Errors

- Some students may confuse the decimal places for dollars and cents when making their calculations. For example, some students may claim that 20 text messages cost $300 (i.e., 20 × 15).

R_x Explain to students that if they input "15" into their calculators for 15¢, their answers will be in cents; if they input "0.15" for 15¢, their answers will be in dollars.

- Some students may miss a step while using their graphing calculators, particularly in question 1, to complete the Investigate questions.

R_x Pair students who are confident using their graphing calculators with weaker students.

- Some students may have computational errors in their solutions. In Example 3, for example, students may have difficulty subtracting negative integers (e.g., $1 - -2$, and not getting 3).

R_x Have students count squares instead of subtracting integers.

- Some students may have difficulty determining positive and negative directions of movement on a graph (e.g., up the slope is positive).

R_x Have these students create the graphs on paper, rather than using a graphing calculator.

Accommodations

Language—Students may benefit from a one-on-one discussion of the concepts.

Motor—Have students who have difficulty graphing by hand work with a partner and/or use a graphing calculator.

Perceptual—Have students work in pairs when using a graphing calculator for the Investigate activity.

Assessment

- Observe students' facility with graphing by hand and/or using graphing technology.
- Assess students' understanding of the concepts presented thus far, including their ability to connect the rate of change and the slope, by taking up their answers to the assigned questions.
- Assess students' ability to use technology, understand the concepts, and work together in their completion of the Investigate.
- Assess students' ability to work through Example 2 independently.
- Distribute graphing calculators, and coach students through question 5 if necessary.
- Use an overhead to take up the questions as a class and discuss students' results.
- Not all parts of every question need be assigned to each student. For some students, the Student Workbook will facilitate their learning better than the student text.
- Use **BLM 3.1.1 Practice: Slope as a Rate of Change** for extra practice or remediation.

Investigate Answers (pages 100–101)

1. a)

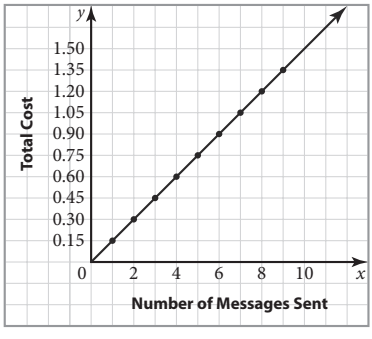
Messages	Cost
0	0
1	0.15
2	0.30
3	0.45
4	0.60
5	0.75
6	0.90
7	1.05
8	1.20
9	1.35
10	1.50

b) The rate of change is constant and always increases by 0.15. Each text message costs 15¢, therefore the total cost keeps going up by 15¢.

2. a) $3 **b)** $4.50 **c)** $6
d) $7.50 **e)** $15 **f)** 0.15x$

3. a) 15¢ **b)** 15¢ **c)** 15¢

4. a)



b) If connected, the points would form a (straight) line. Each point is over 1 and up 15¢ from the previous point.

5. b) Graphs should appear relatively the same.
c) Line goes through all points.

6. b) Rise and run answers will vary, however, slope will always be 0.15.
c) 0.15; The lines have the same slope. There is only one slope for a (straight) line.
d) The slope of the line formed by joining the points from the table is 0.15. In the equation $y = 0.15x$, 0.15 is the coefficient of x; it is the cost per message or the rate of change in total cost.

Examples

- Write the three Examples on the board or on an overhead, and work through them as a class. Alternatively, have students complete the Examples independently or in small groups before reviewing them as a class.
- Example 1 can be treated as a hands-on investigation. Have students use tiles to illustrate the relationship between the rate of change of tiles to the slope of the line of the graph.
- Example 1 is relatively simple. Make note of any student who struggles with this example to determine who will need extra attention.
- Have students with a good grasp of the concepts in Example 1 extend the pattern to determine, without tiles, the number of squares in the 20^{th} term (diagram) of the pattern.
- Example 2 is a relatively straightforward approach to determining the rate of change in a linear relation. Have students copy and complete the table, and analyze the difference in the *y*-values.
- The purpose of Example 3 is to have students connect an equation to its graph and, therefore, its slope. Emphasize the method for determining rise and run, and for calculating the slope.
- Example 3 provides a good opportunity to assess students' facility with integers.
- Example 4 is an application of how rate of change is used in an everyday situation. Ask students to think of other situations that involve the rate of change.
- Example 4 also provides a good opportunity to assess students' understanding of the application of linear relations to real situations.

Key Concepts

- Read and discuss the Key Concepts as a class.
- Ask students to rewrite the key concepts in their own words.

Discuss the Concepts

- Conduct a class discussion or break the class into smaller groups to discuss the concepts.
- Ask students to jot down their thoughts before beginning the discussion. Students may be able to think of an example of a context that would generate a negative slope.

Discuss the Concepts Suggested Answers (page 106)

D1. Positive slopes move up and to the right across the graph, negative slopes move down and to the right.

D2. As long as the *x*-values go up by an increment of one, the rate of change is the incremental difference in the *y*-values. Otherwise, the rate of change is the incremental change in *y* divided by the incremental change in *x*.

D3. When finding the slope of a line from the graph, it doesn't matter which points are selected, because all the points on the line will have the same slope.

Practise the Concepts (A)

- Encourage students to refer back to the Examples before asking for assistance.
- Take up questions as a class frequently for ongoing instruction and quick feedback to students.
- Some students will be able to work at a faster pace and may not need to do all of the questions in this section.
- Complete part a) of some questions on the board as an example of what is expected.

Apply the Concepts (B)

- Use the board or an overhead to work through questions that prove to be problematic for many students.
- Extend question 9 to include a discussion about the storage capacity of portable mp3 players and/or how the capacity is measured (e.g., in MB or GB). Students can calculate the capacity of their players, or one they would like to have, in terms of time.
- For question 10, some students may benefit from constructing a ramp similar to the one in the text (e.g., using a metre stick and some books). Discuss the fact that whether the slope is positive or negative depends on which way you look at the ramp.
- Question 11 links to the Chapter Problem. Point out to students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up.
- Question 11c) is a Literacy Connect. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy/.
- Question 15 is an Achievement Check. It can be used as a form of diagnostic or formative assessment, or assigned as a small summative assessment piece. This provides an opportunity for formative or self-assessment, using **BLM 3.1.2 Achievement Check Rubric**.
- At the end of the section, have students choose three questions to answer using a graphing calculator and to compare their answers to their original answers. Ensure that students select appropriate questions that can be solved using the technology.

Achievement Check Answers (page 110)

15. a) left side: 2, right side: -2.

b) no

c) yes

d) $10 \times 0.5 \text{ m} = 5 \text{ m}$

e) $(5 - (-5)) \times 0.5 \text{ m} = 5 \text{ m}$

f) left side: 2, right side: -2.
They are the same because the steepness of the slope does not change.

Extend the Concepts (C)

- Assign the Extend the Concepts questions to students who are not being challenged by questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.