5.2 Solve Linear Systems by Substitution

Strand
Modelling Linear Relations

Student Text Pages
205–211

Suggested Timing
80–160 min

Tools
- graphing calculators
- grid paper

Related Resources
BLM 5.2.1 Practice: Solve Linear Systems by Substitution
BLM G1 Grid Paper

Specific Expectations

Solving and Interpreting Systems of Linear Equations

In this section, students will

ML3.02 solve systems of two linear equations involving two variables with integral coefficients, using the algebraic method of substitution or elimination

Link to Get Ready

In this section, students use an algebraic method of solving linear systems. It builds on the skills and knowledge from Section 5.1. The Get Ready segments Algebraic Expressions and Manipulate and Solve Equations provide the needed skills for this section. You may wish to have students review Get Ready questions 1–4 before proceeding with Section 5.2.

Warm-Up

1. Write an equation to represent each situation.
- **a)** The cost to put an advertisement in the community newspaper is $20.00 per day plus $0.30 per word.
- **b)** The cost to print T-shirts for your class is $5.75 per shirt plus a set-up fee of $50.00.

2. Solve each equation for each variable.

a) $5x + 10y = 15$ **b)** $3x - 6y = 9$ **c)** $2x + 5y = 11$

3. Substitute $y = x - 1$ into each equation. Simplify your answers to find the value of x.

a) $3x - y = 7$ **b)** $x - 2y = 5$ **c)** $2x - 5y = 8$

Warm-Up Answers

1. a) $C = 20 + 0.30w$ **b)** $C = 5.75s + 50$

2. a) $x = -2y + 3$; $y = -\frac{1}{2}x + \frac{3}{2}$

b) $x = 2y + 3$; $y = \frac{1}{2}x - \frac{3}{2}$

c) $x = -\frac{5}{2}y + \frac{11}{2}$; $y = -\frac{2}{5}x + \frac{11}{5}$

3. a) $x = 3$ **b)** $x = -3$ **c)** $x = -1$

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class. (5–10 min)

Section Opener

- Connect a discussion on determining the best selling price for products and services from the Section Opener with students' research on the Career Profile from the Chapter Opener.

Common Errors

- Some students may write $x = 3y$ instead of $y = 3x$ for question 1b).

R_x Have students use words first and then change to variables. For example: (Jane's age) plus (her mother's age) equals 60: $J + M = 60$, (her mother's age) is (three times Jane's age) $M = 3J$ therefore, when using x for Jane's age and y for her mother's age, the equations become $x + y = 60$, $y = 3x$.

- Some students may have difficulty setting up the two equations.

R_x Have students read the question once, and then read it again, to be sure they understand what is asked and what is given before they start writing.

- Some students may have difficulty deciding which variable to substitute in the equations.

R_x Have students look at the two equations. If one has one variable already isolated, then have them circle that variable in each equation. This is the value that they will substitute. If they must rearrange the equations, have them look to see which equation is easier to rearrange.

- Some students may have difficulty rearranging an equation to isolate a particular variable.

R_x Have students use colour and arrows to help in their initial work so they become more familiar with the conventions.

Ongoing Assessment

- Circulate as students work through the Investigate to see how well they understanding the concepts.
- You may wish to collect students' answers for D1 or D3 and use this as a formative assessment tool.

Accommodations

Memory—Allow students to work with a partner or in small groups to review the sequence of steps to follow.

Investigate

- Point out the definition of *substitution method*.
- Circulate while students complete the Investigate and provide help as needed.
- You may wish to have students work with a partner.
- Some students may need help with substitution. Encourage students to circle the expression they are substituting, and cross out the item they are replacing. Suggest that students use coloured pencils when working on substitution.
- For question 4, have students use graphing calculators or **BLM G1 Grid Paper** to graph the equations.
- Consolidate students' understanding by discussing their results.
- Use **BLM 5.2.1 Practice: Solve Linear Systems by Substitution** for extra practice or remediation.

Investigate Answers (page 205–206)

1. a) $x + y = 60$ **b)** $3x = y$

2. a) $x + 3x = 60$ **b)** $4x = 60$; $x = 15$. This value represents Jane's age.

3. $y = 45$. This value represents the age of Jane's mother.

4. The coordinates of the point of intersection (15, 45) represents the ages of Jane and her mother.

Examples

- Have students work through the Examples as a class before proceeding to Discuss the Concepts. Alternatively, have students complete the Examples independently or in small groups before reviewing them as a class.
- For Example 1, students must first rearrange the equations. Encourage students to use colour and to draw circles to mark what is being substituted into the equations. You may wish to photocopy this example onto an overhead to model the use of colour when showing students how to substitute.
- For Example 2, you may wish to point out that one of the equations is already set up for substitution. In addition, you may wish to stress the check at the end of the example.
- For Example 3, some students may need help setting up the equations. This can be the most challenging part of a word problem. Remind students to start by explicitly stating their variables and what they represent.

Key Concepts

- Discuss with students the difference between solving by graphing and solving by substitution.

Discuss the Concepts

- Give the class time to record their responses before conducting a discussion.
- You may wish to assign question D1 or D3 as a journal entry.

Discuss the Concepts Suggested Answers (page 209)

D1. At the point of intersection, the x- and y-values are the same for both lines. So, I can substitute the expression for y from one equation into the other equation.

D2. Answers will vary. Possible answer:

Rearrange one equation to isolate y.

$$y = -2x + 8$$

Substitute the expression $-2x + 8$ for y in the other equation and solve for x.

$$\begin{aligned} 4x + 3(-2x + 8) &= 12 \\ 4x - 6x + 24 &= 12 \\ -2x &= -12 \\ x &= 6 \end{aligned}$$

Substitute $x = 6$ into either equation and solve for y.

$$\begin{aligned} 2(6) + y &= 8 \\ 12 + y &= 8 \\ y &= -4 \end{aligned}$$

Finally, I would check the solution works in the other equation.

D3. In both cases, we identify the coordinates of the point of intersection of the lines. When we solve by graphing, we graph both lines on the same set of axes and read the coordinates of the point of intersection from the graph. When we solve by substitution, we use algebra to find the values of x and y that satisfy both equations.

Practise the Concepts (A)

- Encourage students to refer back to the Investigate and the Examples before asking for assistance.
- You may wish to have students work in groups on the first few questions to ensure that everyone is able to use the substitution method correctly. Alternatively, you may wish to have a few students post their work on the board as you have students work through the questions.

Apply the Concepts (B)

- You may wish to work through question 4 as a class, and then remind students that, for most of the rest of the questions, they are expected to choose variables and write equations. Encourage students to choose variables that correspond to the question. Point out that in question 6, students are asked to use C for the cost and n for the number of people attending.
- Question 7 is a Literacy Connect. You may wish to assign this question as a journal entry or to discuss the question as a class. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy.
- You may wish to have students also solve one of questions 8–14 by graphing to see how the two methods compare.
- Question 13 may bring you to a discussion of a variety of sports activities in which points are gained when you win, lose, or tie. You may wish to have students do an Internet search to find statistics that relate to hockey, baseball, etc.
- Question 14 links to the Chapter Problem. Remind students to keep the solution to this question handy as the methods they used may help them with the Chapter Problem Wrap-Up. Alternatively, you may wish to use the Chapter Problem as a summative assessment at the end of the chapter.

Extend the Concepts (C)

- Assign the Extend the Concepts questions to students who are not being challenged by questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.
- Questions 16 and 17 introduce students to the alternate solutions for linear systems—no solution and an infinite number of solutions. Have students read the MathConnect before doing question 16. Students should be able to explain that parallel lines never intersect.