6.3 Key Features of Quadratic Relations

Strand
Quadratic Relations of the Form $y = ax^2 + bx + c$

Student Text Pages
254–263

Suggested Timing
80 min

Tools
- graphing calculators
- grid paper

Related Resources
BLM 6.3.1 Practice: Key Features of Quadratic Relations
BLM 6.3.2 Achievement Check Rubric
BLM G1 Grid Paper

Specific Expectations

Identifying Characteristics of Quadratic Relations

In this section, students will

QR2.03 identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the *y*-intercept, the zeros, and the maximum or minimum value), using a given graph or a graph generated with technology from its equation, and use the appropriate terminology to describe the features

Link to Get Ready

Have students complete questions 2, 4, and 5 from the Get Ready before proceeding with Section 6.3.

Warm-Up

1. Use a graphing calculator. Graph each equation, then identify the $x =$ and $y =$ intercepts.
a) $y = 2x + 4$
b) $y = -3x - 9$

2. What is a line of symmetry?

Warm-Up Answers

1. a) (0, 4) (−2, 0) **b)** (0, −9) (−3, 0)

2. A line of symmetry is a line dividing a shape into two equal parts, each of which is a mirror image of the other. The line acts like a mirror.

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class. (5–10 min)

Section Opener

- You may wish to encourage students to research microgravity simulation on the Internet. A lot of interesting information can be found at http://www.atlasaerospace.net/eng/events-r.htm?id=35.

Investigate

- For Investigate A, ensure students understand they are to compare the shapes of the parabolas in terms of wider/fatter or skinnier/thinner, or in terms of distance from the *y*-axis.
- For Investigate A questions 2 and 3, remind students that converting the fractions to decimals does not change the graph of the relation.
- For Investigate B, have students work in pairs.
- You may wish to use **BLM G1 Grid Paper** for this activity.
- Instruct students to only work on the first graph. Once they have finished the first graph, discuss the results as a class before they move on to the next two graphs. Students should note there are no *x*-intercepts on the first graph. Ask students to explain why.

Common Errors

- Some students may think a parabola that opens upward has a maximum and a parabola that opens downward has a minimum.

R_x Have students examine the graphs of different parabolas. From the graph, it should be clear that a parabola that opens upward has a minimum and a parabola that opens downward has a maximum.

- Some students may think the maximum or minimum is the ordered pair that describes the location of the vertex.

R_x Remind students that the maximum or minimum is the greatest or least possible *y*-value, so the maximum or minimum is the *y*-coordinate of the vertex.

Ongoing Assessment

- The Discuss the Concepts can be used as a diagnostic assessment or a self-assessment to assess students' readiness to move on.

Accommodations

ESL—Pair ESL students with non-ESL students for the Investigates.

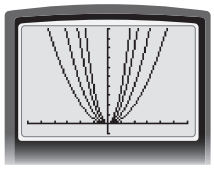
Motor—Provide students with copies of BLM 6.3.2 Investigate B: Features of a Parabola.

- Once students have completed the Investigate, have them copy the definitions into their notes. Have them sketch a graph of a parabola and label the features as shown on page 257 of the student book.
- Use **BLM 6.3.1 Practice: Key Features of Quadratic Relations** for extra practice or remediation.

Investigate Answers (pages 254–257)

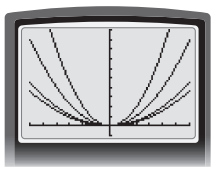
Investigate A

1. b)



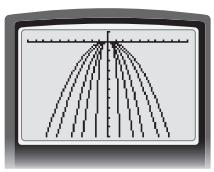
c) Each parabola is skinnier than the previous parabola.

2. b)



c) Each parabola is wider than the previous parabola.

3. b)



c) These parabolas open down, while the parabolas in questions 1 and 2 opened up.

4. Given a quadratic of the form $y = ax^2$, if a is positive, the parabola opens upward. If a is negative, the parabola opens downward. Ignoring the sign, if a is a fraction less than 1, the parabola is wider; if a is a number greater than 1, the parabola is skinnier.

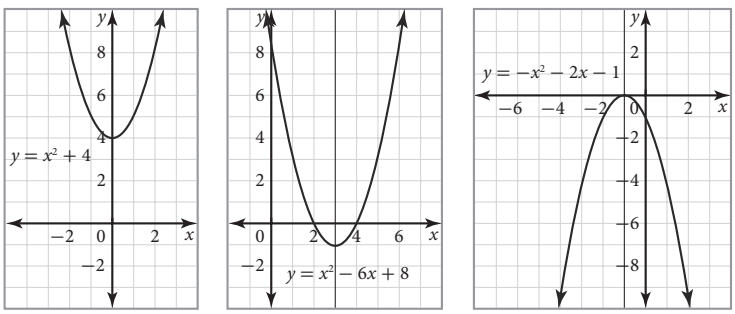
Investigate B

1. The graphs of $y = x^2 + 4$ and $y = x^2 - 6x + 8$ open upward. The graph of $y = -x^2 - 2x - 1$ opens downward. If the coefficient of x^2 is positive, the parabola opens upward. If the coefficient of x^2 is negative, the parabola opens downward.

2. For $y = x^2 + 4$, the vertex is (0, 4). For $y = x^2 - 6x + 8$, the vertex is (3, –1). For $y = -x^2 - 2x - 1$, the vertex is $(-1, 0)$.

3. For $y = x^2 + 4$, the minimum is 4. For $y = x^2 - 6x + 8$, the minimum is -1. For $y = -x^2 - 2x - 1$, the maximum is 0.

4.



5. For $y = x^2 + 4$, the parabola does not cross the *x*-axis. This parabola has no *x*-intercepts. For $y = x^2 - 6x + 8$, the *x*-intercepts are 2 and 4. For $y = -x^2 - 2x - 1$, the *x*-intercept is 0.

6. The vertex of a parabola is the ordered pair that describes the point where the parabola changes from increasing to decreasing or from decreasing to increasing. The maximum or minimum value is the *y*-coordinate of the vertex. The axis of symmetry is a vertical line through the vertex. The *x*-intercepts are the *x*-coordinates of the points where the parabola intersects the *x*-axis.

Examples

- The Examples will help students solidify the concepts they discovered in the Investigates.
- Have students cover the solutions in the text and solve the worked examples. Have students compare their solutions to the solutions in the text.
- For Example 1, draw students' attention to the MathConnect. Ensure they understand how to find the equation of a vertical line.

Key Concepts

- The Key Concepts sum up the Investigates.
- Discuss these questions with the class:
  - How can you tell, from the equation, if a parabola opens upward or downward?
  - What is the vertex of a parabola?
  - What do the vertex and the maximum/minimum value have in common?
  - What do the axis of symmetry and the vertex have in common?

Discuss the Concepts

- Pose each question to the class first, give them a chance to think about their responses, then discuss the answers as a class.

Discuss the Concepts Suggested Answers (page 260)

D1. He is partially right, the y-coordinate of the vertex is the maximum/minimum value.

D2. If the coefficient of the squared term is negative, the parabola has a maximum. If the coefficient of the squared term is positive, the parabola has a minimum

D3. The student is correct. A parabola can have two x-intercepts; it could also have one or none. If the vertex is below the x-axis and the coefficient of the squared term is positive, the parabola will have two x-intercepts. If the vertex is above the x-axis and the coefficient of the squared term is negative, the parabola will have two x-intercepts. If the vertex is below the x-axis and the coefficient of the squared term is negative, the parabola will have no x-intercepts. If the vertex is above the x-axis and the coefficient of the squared term is positive, the parabola will have no x-intercepts. If the vertex is on the x-axis, the parabola will have one x-intercept.

Practise the Concepts (A)

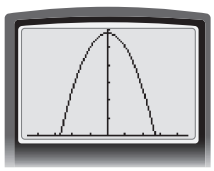
- Encourage students to refer back to the Examples before asking for assistance.
- Use question 1 to help students solidify the concepts from the lesson.

Apply the Concepts (B)

- Questions 2 and 3 require the use of graphing calculators. Students should be given time to complete these questions during class time.
- Question 5 is a Literacy Connect. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy.
- Questions 4 and 6 ask students to interpret information; some level 1 and 2 students may find this challenging.
- Question 6 links to the Chapter Problem. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up.
- Question 7 is an Achievement Check. It can be used as a form of diagnostic, or formative assessment, or assigned as a small summative assessment piece. This provides an opportunity for formative or self-assessment using **BLM 6.3.2 Achievement Check Rubric**.

Achievement Check Answers (page 263)

7. a)



b) The vertex is (0, 290).

c) The y-coordinate of the vertex is the height of the bridge, so the bridge is about 290 ft high.

d) The span of the bridge is the horizontal distance between the x-intercepts. The x-intercepts are -135 and 135, so the span of the bridge is $135 - (-135)$, or 270 ft.

Extend the Concepts (C)

- Assign the Extend the Concepts question to students who are not being challenged by questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.
- You may wish to use **BLM G1 Grid Paper** for part c).