6.4 Rates of Change in Quadratic Relations

Strand
Quadratic Relations of the Form $y = ax^2 + bx + c$

Student Text Pages
264–271

Suggested Timing
80 min

Tools
- graphing calculators
- grid paper

Related Resources
BLM 6.4.1 Practice: Rates of Change in Quadratic Relations
BLM 6.4.2 Achievement Check Rubric
BLM G1 Grid Paper

Specific Expectations

Identifying Characteristics of Quadratic Relations

In this section, students will

QR2.02 determine, through investigation using technology, that a quadratic relation of the form $y = ax^2 + bx + c$ $(a \neq 0)$ can be graphically represented as a parabola, and determine that the table of values yields a constant second difference

Link to Get Ready

Have students complete all of the Get Ready questions before proceeding with Section 6.4.

Warm-Up

1. Create a table of values for x-values from -3 to 3. Find the rate of change of the y-values.

a) $y = 2x - 1$

b) $y = -3x + 2$

Warm-Up Answers

1.

x	y	Rate of Change
-3	-7	
-2	-5	$-5 - (-7) = 2$
-1	-3	$-3 - (-5) = 2$
0	-1	$-1 - (-3) = 2$
1	1	$1 - (-1) = 2$
2	3	$3 - 1 = 2$
3	5	$5 - 3 = 2$

2.

x	y	Rate of Change
-3	11	
-2	8	$8 - 11 = -3$
-1	5	$5 - 8 = -3$
0	2	$2 - 5 = -3$
1	-1	$-1 - 2 = -3$
2	-4	$-4 - (-1) = 3$
3	-7	$-7 - (-4) = -3$

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class. (5–10 min)

Section Opener

- In this section, students extend the concept of rates of change to non-linear relations.
- Students learn that rates of change are also known as first differences and they are introduced to second differences.

Investigate

- Remind students that, in any linear relation, the rate of change of the y-values is constant. Students should recognize that the rate of change for a linear relation is the slope of the relation.

Common Errors

- Some students may not always subtract the *y*-values in the same order (i.e. 1st from 2nd, 2nd from 3rd, 3rd from 4th). They may do 1st from 2nd, 3rd from 2nd. This will cause their first differences to not be constant.

R_x Take time to reinforce how to find first differences, helping those that consistently are having trouble one on one.

Ongoing Assessment

- This may be a good opportunity to observe and record the individual students' learning skills: group work, work habits, organization, and initiative.

Accommodations

ESL—Have each student work with a partner and allow the use of a dictionary. Some students may require extra time.

Gifted and Enrichment—Have students research to collect data in context and write their own problem.

- Some students may speculate that second differences are the same because the *x*-value of a quadratic has an exponent of 2 and the *x*-value of a linear has an exponent of 1. This can actually be a memory aid for those that need that kind of thing to help recall concepts.
- Use **BLM 6.4.1 Practice: Rates of Change in Quadratic Relations** for extra practice or remediation.
- Have students copy the table in step 1 of part B of the Investigate. Discuss the idea of second differences with them. Have them complete the table as instructed in step 2 and answer the question posed in the first paragraph of step 3.
- The last two paragraphs of step 3 summarize the Investigate. Discuss the conclusions with the class.
- Use **BLM 6.4.1 Practice: Rates of Change in Quadratic Relations** for extra practice or remediation.

Investigate Answers (pages 264–265)

Part A

1.

Horizontal Distance (m)	Height (m)	First Differences
1	4.0	
2	3.8	$3.8 - 4.0 = -0.2$
3	3.6	$3.6 - 3.8 = -0.2$
4	3.4	$3.4 - 3.6 = -0.2$
5	3.2	$3.2 - 3.4 = -0.2$
6	3.0	$3.0 - 3.2 = -0.2$

The first differences are constant.

2. For each 1 m increase in horizontal distance, the height decreases by 0.2 m.

Part B

3–4. a), b)

Horizontal Distance (m)	Height (m)	First Differences	Second Differences
0	10.8		
1	7.5	$7.5 - 10.8 = -3.3$	
2	4.8	$4.8 - 7.5 = -2.7$	$-2.7 - (-3.3) = 0.6$
3	2.7	$2.7 - 4.8 = -2.1$	$-2.1 - (-2.7) = 0.6$
4	1.2	$1.2 - 2.7 = -1.5$	$-1.5 - (-2.1) = 0.6$
5	0.3	$0.3 - 1.2 = -0.9$	$-0.9 - (-1.5) = 0.6$
6	0	$0 - 0.3 = -0.3$	$-0.3 - (-0.9) = 0.6$
7	0.3	$0.3 - 0 = 0.3$	$0.3 - (-0.3) = 0.6$
8	1.2	$1.2 - 0.3 = 0.9$	$0.9 - 0.3 = 0.6$
9	2.7	$2.7 - 1.2 = 1.5$	$1.5 - 0.9 = 0.6$
10	4.8	$4.8 - 2.7 = 2.1$	$2.1 - 1.5 = 0.6$
11	7.5	$7.5 - 4.8 = 2.7$	$2.7 - 2.1 = 0.6$
12	10.8	$10.8 - 7.5 = 3.3$	$3.3 - 2.7 = 0.6$

The first differences decrease to a minimum, then increase again. Second differences are constant.

5. From a table of values, if first differences are constant, the graph is a line. If first differences are not constant but second differences are constant, the graph is a parabola.

Examples

- Instruct students to read and solve part a) of the Example without looking at the solution.
- The second part of the Example requires the use of graphing calculators; you may wish to do this part with the class on the overhead calculator.
- Draw students' attention to the MathConnect. Ensure students understand that the grid can be placed anywhere on the cross section of the stadium.

Key Concepts

- Mention that a quadratic is called a second-degree equation because it has, as its term with the largest exponent, an x-squared term.

Discuss the Concepts

- Have students jot down their ideas for the Discuss the Concepts before asking the class for answers. This will allow them to self-assess whether they understood the concepts from this section.

Discuss the Concepts Suggested Answers (page 268)

D1. Find the first differences, then find the second differences. If the second differences are constant, then the relationship is quadratic.

D2. I could place a grid over the photograph and record the coordinates of key points, such as the x- and y-intercepts and the vertex. I could enter these data in a graphing calculator and then use the QuadReg command to find the equation for the parabola that best fits the data.

Practise the Concepts (A)

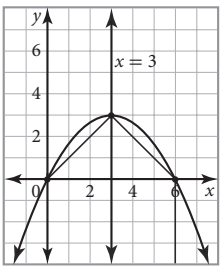
- Encourage students to refer back to the Investigate and the Example before asking for assistance.
- For question 1, students could find the values for the table by substituting x-values and calculating the corresponding y-values. You may wish to have students make each table for x-values from -3 to 3.

Apply the Concepts (B)

- Question 5 is a Literacy Connect. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy.
- Question 6 is an Achievement Check. It can be used as a form of diagnostic or formative assessment, or assigned as a small summative assessment piece. This provides an opportunity for formative or self-assessment, using **BLM 6.4.2 Achievement Check Rubric**. You may wish to use **BLM G1 Grid Paper** for this activity.

Achievement Check Answers (page 270)

6. a)–f)



Extend the Concepts (C)

- Assign the Extend the Concepts questions to students who are not being challenged by the questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.
- Students should be able to answer question 8a). The pattern for part b) can be found by finding first differences. Weaker students may need that hint. You may wish to use **BLM G1 Grid Paper** for this activity.