7.2 Common Factoring

Specific Expectations
Manipulating Quadratic Expressions
In this section, students will
QR1.02 factor binomials (e.g., $4x^2 + 8x$) and trinomials (e.g., $3x^2 + 9x - 15$) involving one variable up to degree two, by determining a common factor using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil) and strategies (e.g., patterning)

Link to Get Ready
The Get Ready segments Polynomials and Algebraic Expressions provide the needed skills for this section. You may wish to have students complete questions 1 to 7 before proceeding with Section 7.2.

Warm-Up
1. Write each number as a product of its prime factors.
   a) 12   **b)** 45   **c)** 60

2. Find the GCF for the numbers in each set.
   a) 10, 12   **b)** 8, 20   **c)** 12, 18, 24

3. The area of a rectangle is 20 cm^2.
   a) Find all the possible dimensions of the rectangle.
   b) Which dimensions will give the rectangle the least perimeter? Verify your answer.

Warm-Up Answers

1. a) $2 \times 2 \times 3$ **b)** $5 \times 3 \times 3$ **c)** $2 \times 2 \times 3 \times 5$

2. a) 2   **b)** 4   **c)** 6

3. a) 1 cm by 20 cm, 2 cm by 10 cm, 4 cm by 5 cm
    b) 4 cm by 5 cm

Teaching Suggestions
Warm-Up
- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class. (5–10 min)

Investigate
- Model how to use a factor tree.
- Have students work through the questions in pairs or small groups, assisting each other when they get stuck.
- Perform one or two sample calculations with the CAS to ensure students understand how to use their calculators.
- For Investigate B, show students that they should divide each term by the GCF.
- Use **BLM 7.2.1 Practice: Common Factoring** for extra practice or remediation.
- You may wish to distribute copies of **BLM T2 The Computer Algebra System (CAS) on the TI–89 Calculator**.

- While students are working on the Investigate, circulate to see how well each works within a group. This may be an opportunity to begin observing and recording the individual student's learning skills: group work, work habits, organization, and initiative.
- This is a good section to use CAS to help students master the concepts without arithmetic errors getting in the way.

Accommodations

ESL—Encourage students to use a variety of resources to understand new words.

Gifted and Enrichment—Have students calculate the floor area of the classroom, then find the possible dimensions of a room with the same area.

Have students complete the Investigate B questions using paper and pencil, without a calculator.

Provide additional challenge by having students find the GCF for these trinomials: $4x^2 + 8x + 28$, $5x^2 - 10x + 45$, $-3x^2 - 6x - 21$, $-3x^2 - 9x - 30$.

Have students develop a new question, using question 18 of the Extend the Concepts as a model.

Memory—Have students develop a poster that shows factoring and expanding as opposite operations.

Perceptual—Encourage students to use patterning to understand how to find a common factor for a trinomial.

Visual—Provide algebra tiles for students to use in representing the factoring process.

Investigate Answers (page 290–291)

Investigate A

1. a)

b) 2 by $(2x + 4)$
4 by $(x + 2)$

c) The rectangle with dimensions 4 by $(x + 2)$ has one side that represents the greatest common factor of the area.

2. $4(x + 2)$

3. a)

b) x by $(3x + 9)$
$3x$ by $(x + 3)$

c) The rectangle with dimensions $3x$ by $(x + 3)$ has one side that represents the greatest common factor of the area.

4. $3x(x + 3)$

5. The area of a rectangle is the product of its length and width. Factoring an expression means to find two expressions whose product is the original expression. Factoring the area of a rectangle gives possible dimensions for the rectangle.

Investigate B

1. a) $3(x^2 + 2x + 3)$ **b)** $5(x^2 + 2x + 3)$
c) $7(x^2 + 2x + 3)$ **d)** $9(x^2 + 2x + 3)$

2. a) $x^2 + 2x + 3$ **b)** No
c) A trinomial **d)** The greatest common factor increases by 2 each time. The trinomial factors are equal.

Examples

- For Example 1, refer students to Warm-Up question 2.
- Remind students that a GCF may contain both a number and a variable.
- As students work through Example 2, point out that, once factored, the remaining binomial cannot be factored further.
- Remind students to check their answers by expanding.
- If students seem to struggle with Example 3, provide additional examples.
- Before introducing Example 4, review the formulas for determining the area and perimeter of rectangles.

Key Concepts

- You may wish to refer students back to **BLM 7.CO.1 Literacy Link: Concept Web** as they factor binomials and trinomials by finding the GCF.

Discuss the Concepts

- Have students work in pairs to answer the questions.
- Invite some students to write their answers on the board.

Practise the Concepts (A)

- Encourage students to refer back to the Examples before asking for assistance.
- For question 7, review how to evaluate algebraic expressions.
- Where technology is not available, use paper and pencil to decide on which method to use.

Apply the Concepts (B)

- Review how to evaluate algebraic expressions for questions 8, 11, and 14.
- Direct students' attention to the MathConnect, and have a brief discussion about FIFA and soccer. Have interested students conduct further research in the library or on the Internet and report their findings to the class.
- Question 10 is an Achievement Check. It can be used as a form of diagnostic or formative assessment, or assigned as a small summative assessment piece. This provides an opportunity for formative or self-assessment, using **BLM 7.2.2 Achievement Check Rubric**.
- Question 15 is a Literacy Connect. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy.
- Question 16 is a Chapter Problem. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up.
- You may wish to have students work with a partner to complete question 16.

> **Achievement Check Answers (page 296)**
>
> **10. a) Method 1** **Method 2**
>
> $A = 30(20) - 4(x^2)$ $A = (30 - 2x)(20 - 2x) + 2x(20 - 2x)) + 2x(30 - 2x)$
>
> $\quad = 600 - 4x^2$
>
> **b)** Use a CAS to simplify the expression from Method 2.
>
> $A = 600 - 60x - 40x + 4x^2 + 40x - 4x^2 + 60x - 4x^2$
>
> $\quad = 600 - 4x^2$
>
> **c)** $A = 600 - 4x^2$
>
> $\quad = 600 - 4(3)^2$
>
> $\quad = 600 - 36$
>
> $\quad = 564$
>
> If x is 3 cm, 564 cm^2 of tin are used.

Extend the Concepts (C)

- Assign the Extend the Concepts questions to students who are not being challenged by questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.