8.2 Represent Quadratic Relations in Different Ways

Strand
Quadratic Relations of the Form $y = ax^2 + bx + c$

Student Text Pages
329–335

Suggested Timing
80 min

Tools
- graphing calculators

Related Resources
BLM 8.2.1 Practice: Represent Quadratic Relations in Different Ways

Specific Expectations

Identifying Characteristics of Quadratic Relations

In this section, students will

QR2.04 compare, through investigation using technology, the graphical representations of a quadratic relation in the form $y = x^2 + bx + c$ and the same relation in the factored form $y = (x - r)(x - s)$ (i.e., the graphs are the same), and describe the connections between each algebraic representation and the graph (e.g., the y-intercept is c in the form $y = x^2 + bx + c$; the x-intercepts are r and s in the form $y = (x - r)(x - s)$)

Link to Get Ready

Students should have completed all Get Ready questions before proceeding with Section 8.2.

Warm-Up

1. Evaluate each quadratic expression for $x = 0$.
  a) $x^2 - 6x + 7$
  b) $3x^2 - 6x + 9$
  c) $(x - 5)(x + 6)$
  d) $-2(x + 3)(x - 7)$

2. Factor each polynomial.
  a) $x^2 + 9x + 20$
  b) $x^2 - 6x - 10$
  c) $x^2 - 49$

3. Expand and simplify.
  a) $(x + 2)(x + 4)$
  b) $(x - 3)(x + 4)$
  c) $-3(x + 1)(x - 2)$
  d) $(x + 5)^2$

Warm-Up Answers

1. a) 7
  b) 9
  c) -30
  d) 42

2. a) $(x + 4)(x + 5)$
  b) $(x - 8)(x + 2)$
  c) $(x + 7)(x - 7)$

3. a) $x^2 + 6x + 8$
  b) $x^2 + x - 12$
  c) $-3x^2 + 3x + 6$
  d) $x^2 + 10x + 25$

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class. (10–15 min)

Common Errors

- Some students may have difficulty expressing a quadratic relation of the form $y = ax^2 + bx + c$ in the form $y = (x - r)(x - s)$.

R_x Give students extra practice factoring trinomials of the form $x^2 + bx + c$. Ensure they understand that the value of b is the sum $-r + (-s)$ and the value of c is the product $(-r)(-s)$.

- Some students may think the zeros of a quadratic relation of the form $y = (x - r)(x - s)$ are $(-r)$ and $(-s)$. For example, they may think the zeros of the relation $y = (x + 4)(x - 1)$ are 4 and -1.

R_x Remind students that the zeros are the x-intercepts; the values of x when $y = 0$. Have them practice setting each binomial factor equal to 0 and solving for x.

Ongoing Assessment

- While students are working on the Investigate, you can circulate to see how well each works within a group. This may be an opportunity to begin observing and recording individual students' learning skills: group work, work habits, organization, and initiative. By this time, students should be very familiar with graphing calculators. TI-SmartView™ can be used to take up the investigation as a class.

Accommodations

Motor—Give students extra time to complete the questions in this section and encourage them to work with a partner when graphing quadratic relations.

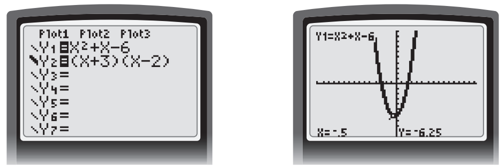
Visual—Encourage students to graph each relation and solve the problem graphically, then solve it algebraically using the graphical solution as a guide.

Investigate

- Have students work with a partner.
- Emphasize that the x-intercepts are the x-values of the points where the parabola intersects the x-axis—they are the values of x when $y = 0$.
- You may wish to review the methods for factoring trinomials and differences of squares. (When factoring a trinomial of the form $x^2 + bx + c$, find two integers whose product is c and whose sum is b.)
- Use **BLM 8.2.1 Practice: Represent Quadratic Relations in Different Ways** for extra practice or remediation.

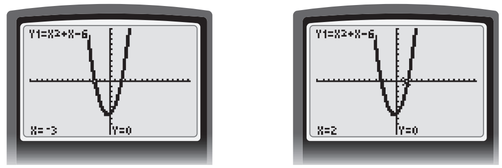
Investigate Answers (pages 329–331)

1. a)–f)



The graphs are the same. Both parabolas have a minimum value of -6.25.

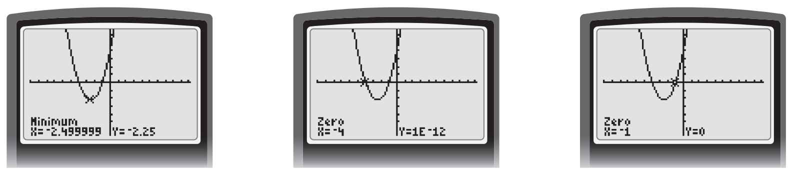
g) The x-intercepts are –3 and 2.



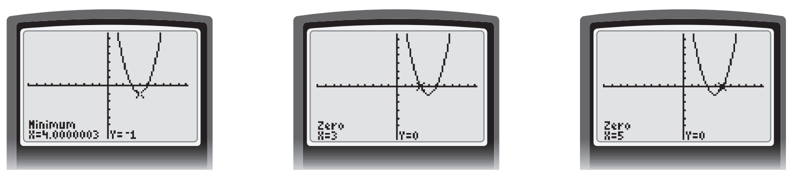
h) The zeros are –3 and 2. The zeros are the same for both relations.

i) When the equation of the parabola is written in the form $y = ax^2 + bx + c$, the value of c is the y-intercept. When the equation is written in the form $y = (x - r)(x - s)$, the values of r and s are the zeros.

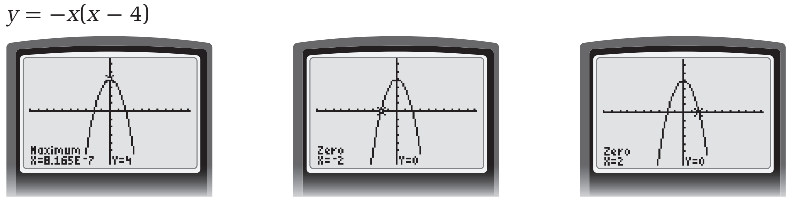
2. a) $y = (x + 4)(x + 1)$



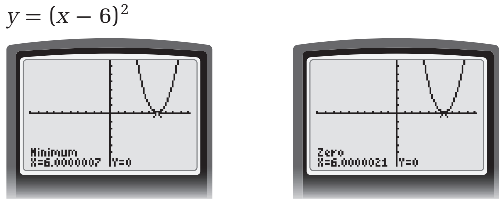
b) $y = (x - 5)(x - 3)$



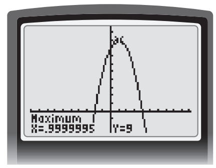
c) $y = -x(x - 4)$

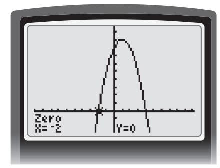


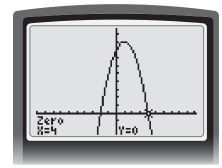
d) $y = (x - 6)^2$



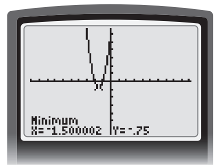
e) $y = -(x + 4)(x - 2)$

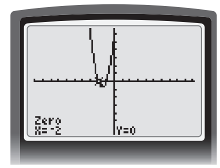


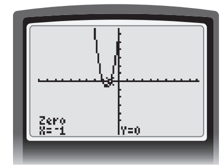




f) $y = 3(x + 2)(x + 1)$







In each case, the graph of the quadratic relation in factored form coincides with the graph of the relation in the form $y = ax^2 + bx + c$. Each time, the graphs have the same maximum or minimum and the same zeros.

3. b) $y = x^2 + x - 2$

d) The parabolas are the same.

e) Both parabolas have the same minimum, –2.25.

f) Both parabolas have zeros -2 and 1.

g) When the equation of the parabola is written in the form $y = ax^2 + bx + c$, the value of c is the y-intercept. When the equation is written in the form $y = (x - r)(x - s)$, the values of r and s are the zeros.

5. The graph of a quadratic relation in the form $y = ax^2 + bx + c$ is the same as the graph in the factored form of the same relation.

6. For a quadratic relation of the form $y = ax^2 + bx + c$, c represents the y-intercept of the parabola.

7. For a quadratic relation in the form $y = (x - r)(x - s)$, r and s represent the zeros, or x-intercepts of the parabola.

Examples

- For Example 1, remind students that if a is positive, the parabola opens upward and if a is negative, the parabola opens downward.
- Ensure students understand that the word *zeros* is another name for the x-intercepts. The zeros are the x-coordinates of the points where the parabola intersects the x–axis.

Key Concepts

- Ask students to explain how to find the x- and y-intercepts of the quadratic relation $y = x^2 + 4x - 12$ without graphing.
- Ask students if the equations $y = (x + 3)(x + 1)$ and $y = x^2 + 4x + 3$ represent the same quadratic relation. Encourage them to explain their answers.

Discuss the Concepts

- Have students explain why this statement is true:
  If $(x + 3)(x - 4) = 0$, then either $(x + 3) = 0$ or $(x - 4) = 0$.
- Invite volunteers to come to the board to sketch an example of each parabola, then discuss the characteristics as a class:
  - a parabola that opens upward, with vertex below the x-axis
  - a parabola that has no x-intercepts and opens downward
  - a parabola with an x-intercept and the y-intercept at the same point

Discuss the Concepts Suggested Answers (page 333)

D1. To find the zeros of the relation $y = x^2 - 5x + 6$ without graphing, I would factor the expression to get $y = (x - 3)(x - 2)$. The zeros of this relation are 3 and 2.

D2. The y-intercept of a relation is the value of y when $x = 0$. I would substitute $x = 0$ into the equation $y = (x + 5)(x - 4)$. The y-intercept is (5)(–4), or 20.

D3. The vertex of a quadratic relation does not tell us whether the parabola opens upward or downward. If the parabola opens upward, it has a minimum value of -5. If it opens downward, it has a maximum value of -5.

Practise the Concepts (A)

- Encourage students to refer back to the Examples before asking for assistance.
- Have grid paper and graphing calculators available for students who wish to use them.
- For question 4, students should recognize that the equation in part b) is the factored form of the equation in part c).

Apply the Concepts (B)

- Questions 6 and 7 can be assigned as homework.
- You may wish to complete question 8 as a whole-class activity.
- Question 9 is a Literacy Connect. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information, visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy.
- Question 10 is a Chapter Problem. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up.

Extend the Concepts (C)

- Assign the Extend the Concepts questions to students who are not being challenged by questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.
- For question 12, students should find the greatest common factor before factoring the trinomial.