8.4 Solve Problems Involving Quadratic Relations

Strand
Quadratic Relations of the Form $y = ax^2 + bx + c$

Student Text Pages
344–351

Suggested Timing
80–120 min

Tools
- graphing calculators
- grid paper

Related Resources
BLM 8.4.1 Practice: Solve Problems Involving Quadratic Relations
BLM 8.4.2 Achievement Check Rubric
BLM 8.CO.1 Literacy Link: Mind Map
BLM G1 Grid Paper

Specific Expectations

Solving Problems by Interpreting Graphs of Quadratic Relations

In this section, students will

QR3.01 solve problems involving a quadratic relation by interpreting a given graph or a graph generated with technology from its equation.

Link to Get Ready

Students should have completed all Get Ready questions before proceeding with Section 8.4.

Warm-Up

1. Use a graphing calculator. For each quadratic relation:
- **i)** Identify the maximum or minimum value.
- **ii)** Identify the x- and y-intercepts.
- **iii)** Find the value of y when $x = 4$.

a) $y = x^2 - 4x - 12$
b) $y = -x^2 + 6x$
c) $y = 2x^2 - 12x - 14$
d) $y = -3x^2 + 27$

Warm-Up Answers

1. a) i) minimum: -16 **ii)** x-intercepts: -2 and 6; y-intercept: -12 **iii)** $y = 0$
b) i) maximum: 9 **ii)** x-intercepts: 0 and 6; y-intercept: 0 **iii)** $y = 8$
c) i) minimum: -32 **ii)** x-intercepts: -1 and 7; y-intercept: -14 **iii)** $y = -30$
d) i) maximum: 27 **ii)** x-intercepts: -3 and 3; y-intercept: 27 **iii)** $y = 21$

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class. (10–15 min)

Section Opener

- This section provides an opportunity for students to use graphing calculators to solve problems.
- As a class, brainstorm the different situations in which quadratic relations can be used to model real-life situations (e.g., determining the trajectory of a rocket, throwing a ball, kicking a ball, determining the profit or revenues of a business).

Common Errors

- Some students may have difficulty relating the *x*- and *y*-intercepts and the maximum or minimum to the context of the problem.

R_x Encourage students to graph each quadratic relation. Have them label the *x*-axis both as *x* and with the name of the independent variable. Similarly, have them label the *y*-axis both as *y* and with the name of the dependent variable. Students can then find the *x*- and *y*-intercepts and the maximum or minimum value in terms of *x* and *y*, then use the alternate labels on the axes to help them "translate" the results into the context of the problem.

Ongoing Assessment

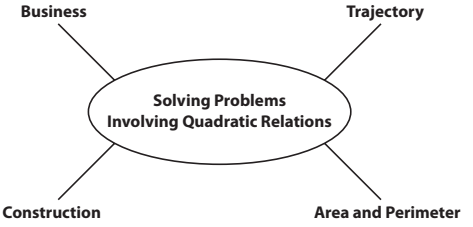
- You may wish to use the Achievement Check, question 6, as a formative or summative assessment. Use **BLM 8.4.2 Achievement Check Rubric** to assess students' responses.

Accommodations

ESL—Encourage students to work with a partner to assist with understanding the language of the problem.

Gifted and Enrichment—Encourage students to solve each problem as many different ways as they can. Challenge students to create and solve their own problem involving quadratic relations.

- Have students make a Mind Map. You may wish to use **BLM 8.CO.1 Literacy Link: Mind Map** for this activity. Within each area, have students write an example of a problem. This may require some coaching; however, the examples students provide do not have to be complicated.



- Use **BLM 8.4.1 Practice: Solve Problems Involving Quadratic Relations** for extra practice or remediation.

Examples

- For Example 1, review how the maximum and minimum are related to the vertex.
- It is important that the window settings be adjusted according to the context of the problem and the shape of the parabola. In real-life situations, negative values may not be reasonable, so the minimum values for *x* and *y* would be set at 0.
- For Example 2, you may wish to have students visit the web site http://www.estesrockets.com/ before working through the Example. Have students read the MathConnect, and ensure they understand the meaning of the term *trajectory*. Have interested students research rockets in the library or on the Internet and report their findings back to the class.
- Encourage students to make a sketch of the situation. This will help them to see that the platform height is the *y*-intercept.
- Indicate to students that the rocket goes up and comes back down. Therefore, it reaches identical heights on the way up and on the way down. There are two times when the rocket is at a particular height after its launch.

Key Concepts

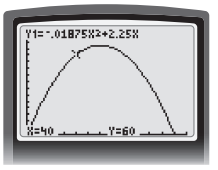
- You may wish to have students use **BLM 8.CO.1 Literacy Link: Mind Map** to brainstorm different problems.

Discuss the Concepts

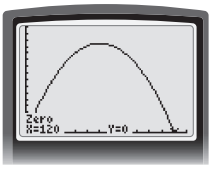
- Have students read the questions and record their solutions before starting a class discussion.

Discuss the Concepts Suggested Answers (page 347)

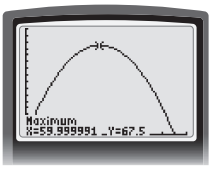
D1. a) Use the equation or a graph to find the height of the ball when it has travelled a horizontal distance of 40 yd. If the height of the ball is greater than the height of the tree, then the ball will clear the tree.



b) Use the equation or a graph to find the horizontal distance the ball has travelled when its height is zero, that is, when the ball lands on the green. If the zero equals 120 yd, the ball lands near the centre of the green.



c) Graph the relation and find the vertex. The *y*-coordinate of the vertex is the maximum height of the ball, and the *x*-coordinate indicates the horizontal distance. From the graph, the maximum height of the ball is 67.5 ft. The ball reaches this height at a horizontal distance of 60 yd.



Practise the Concepts (A)

- Encourage students to refer back to the Examples before asking for assistance.
- Questions 1 and 2 are very similar to the Examples. As students work through these questions individually, circulate to check how they are doing.

Apply the Concepts (B)

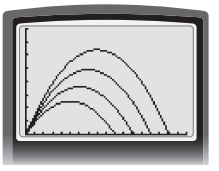
- You may wish to combine questions from Practise the Concepts and Apply the Concepts.
- This is a good opportunity to assign different questions to small groups of students. Have each group share its solution with the class.
- Question 6 is an Achievement Check. It can be used as a form of diagnostic or formative assessment, or assigned as a small summative assessment piece. This provides an opportunity for formative or self-assessment, using **BLM 8.4.2 Achievement Check Rubric**. You may wish to have students use **BLM G1 Grid Paper** for this activity.
- Question 11d) is a Literacy Connect. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy .

Achievement Check Answers (page 349)

6. a) The fireworks explode at the highest point; this is the maximum. Graph each relation and find the maximum.

$h = -5t^2 + 50t$	maximum:125 m; time: 5 s
$h = -5t^2 + 60t$	maximum: 180 m; time: 6 s
$h = -5t^2 + 70t$	maximum: 245 m; time: 7 s
$h = -5t^2 + 80t$	maximum: 320 m; time: 8 s

b)

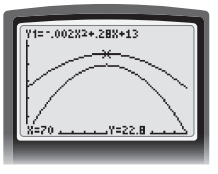


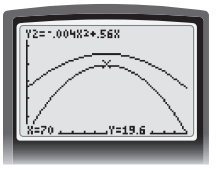
All of the graphs are parabolas that open downward. They all have one common zero. As the coefficient on t increases, the vertex shifts up and to the right.

Extend the Concepts (C)

- Assign the Extend the Concepts questions to students who are not being challenged by questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.
- For question 12, assist students with the window settings:

- Ensure students find where the maximum height of each arch occurs.





- Try to help students interpret 15 m from the centre. Ask, "How many metres is that from one end of the arch?"