9.1 Volume of Prisms and Pyramids

Strand
Measurement and Trigonometry

Student Text Pages
364–371

Suggested Timing
80–160 min

Tools
- calculators
- cardstock paper (optional)
- goggles (optional)
- modelling clay (optional)
- nets of prisms and pyramids with congruent bases and equal heights (optional)
- rulers
- sand
- scissors
- sheets of clear acetate
- tape
- toothpicks or straws (optional)

Related Resources
BLM 9.1.1 Practice: Volume of Prisms and Pyramids
BLM 9.1.2 Investigate: Compare the Volume of a Prism and a Pyramid
BLM 9.1.3 Achievement Check Rubric
BLM A3 Communication General Scoring Rubric
BLM G1 Grid Paper
BLM G3 Centimetre Grid Paper

Specific Expectations

Solving Problems Involving Surface Area and Volume, Using the Imperial and Metric Systems of Measurement

In this section, students will

MT3.01 use the imperial system when solving measurement problems (e.g., problems involving dimensions of lumber, areas of carpets, and volumes of soil or concrete)

MT3.02 perform everyday conversions between the imperial system and the metric system (e.g., millilitres to cups, centimetres to inches) and within these systems (e.g., cubic metres to cubic centimetres, square feet to square yards), as necessary to solve problems involving measurement

MT3.04 solve problems involving the surface areas of prisms, pyramids, and cylinders, and the volumes of prisms, pyramids, cylinders, cones, and spheres, including problems involving the combinations of these figures, using the metric system or the imperial system, as appropriate

Link to Get Ready

Students will learn how to calculate the volume of prisms and pyramids. Have students complete questions 1, 2, and 4 of the Get Ready before proceeding with Section 9.1.

Warm-Up

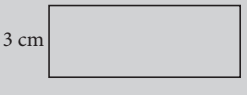
1. On a piece of grid paper, construct the following shapes. Assume that each square is 1 cm.
a) a rectangle 3 cm in width and 8 cm in length
b) a right triangle with legs of 3 cm and 8 cm
c) a circle with a radius of 3 cm

2. Find the area of the shapes in question 1.

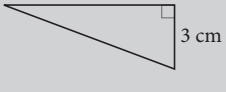
3. If you think of volume as area with a thickness, you could use the formula $V =$ area $\times$ thickness. Use this formula to find the volume of the shapes in question 1 if they were
a) 1 cm thick
b) 2 cm thick

Warm-Up Answers

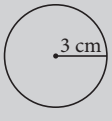
1. a)



b)



c)



2. a) 24 cm^2 **b)** 12 cm^2 **c)** 28.26 cm^2

3. a) rectangle: 24 cm^3; triangle: 12 cm^3; circle: 28.26 cm^3
b) rectangle 48 cm^3; triangle: 24 cm^3; circle: 56.52 cm^3

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Provide students with **BLM G1 Grid Paper** or **BLM G3 Centimetre Grid Paper**, and have them complete the questions independently. Then, discuss the solutions as a class. (15–20 min)

Common Errors

- Some students may forget to use cubic units for volume.

R_x Stress that volume is always measured in cubic units. Demonstrate by pointing out the three dimensions on a solid to indicate that cubic units are used to record the volume.

- Some students may multiply 2×3 when asked to calculate 2^3.

R_x Have students write out the expanded form before completing the multiplication.

Ongoing Assessment

- As students work on the Investigate, circulate and observe how well students work with a partner. Consider recording each student's learning skills: group work, work habits, organization, and initiative.
- Conduct a class discussion for Apply the Concepts question 7. Assess students' communication skills using **BLM A3 Communication General Scoring Rubric**.
- Use Chapter Problem question 8 to assess students' ability to apply their knowledge of volume.

Accommodations

ESL—Pair ESL students with those who have stronger language skills.

Gifted and Enrichment—Challenge students with the Extend the Concepts questions.

Motor—For the Investigate, have students work with a partner.

Section Opener

- Read the Section Opener to the class, and have students brainstorm other examples where one needs to know the volume of an object (e.g., volume of water needed to fill a pool; volume of helium tank needed to fill balloons; volume of punch bowl to hold punch; volume of a fridge to fit inside a kitchen).

Investigate

- Have students work with a partner for the Investigate.
- Provide student pairs with **BLM 9.1.2 Investigate: Compare the Volume of a Prism and a Pyramid**, clear acetate, rulers, scissors, and tape. Before students start, show a 3-D solid with corners and point out a vertex (corner) and a face (surface). Remind students not to tape the edges along the base since they will be filling the solids with sand.
- Wrap up the Investigate by having students discuss their findings. Students should find that it takes three pyramids full of sand to fill a prism with the same base and height.
- As a follow up, have students describe the relationship between the volumes of a prism and a pyramid using mathematical terms (i.e., 3, $\frac{1}{3}$).
- Use **BLM 9.1.1 Practice: Volume of Prisms and Pyramids** for extra practice or remediation.

Investigate Answers (pages 364)

1. The prism has two congruent polygonal bases. The pyramid has one polygonal base, the other faces are triangles.

3. The bases and heights of the prism and the pyramid are the same.

4. The pyramid was filled three times.

6. Wording may vary. Look for the idea that a pyramid has $\frac{1}{3}$ .the volume of a prism.

Examples

- Work through the Examples as a class to ensure that students understand how to calculate the volume of a square-based pyramid, a rectangular pyramid, and a equilateral triangular prism.
- Review the formulas for volume of a prism and a pyramid. Remind students how they know that the volume of a pyramid is $\frac{1}{3}$ the volume of a prism. (It took three times more sand to fill a prism than a pyramid of the same base and height.)
- Emphasize that volume is space measured in cubic units. Stress the importance of using correct units.

Key Concepts

- Ensure that students understand the Key Concepts before assigning the Practise the Concepts questions.

Discuss the Concepts

- Have students complete and then discuss questions D1 and D2 in a class discussion.
- Stress the importance of using correct units for length, area, and volume. For example, you might point out that people often report distance between two points in units of time instead of units of distance. However, the answer is imprecise because time depends on whether someone walks, rides a bike, or gets a drive.
- If you observe students having difficulty with the questions, provide some additional examples of questions for practice. Use **BLM 9.1.1 Practice: Volume of Prisms and Pyramids** for extra practice or remediation.

Discuss the Concepts Suggested Answers (page 367)

D1. Gundeep reported units of area (cm^2) instead of units of volume (cm^3).

D2. Yes. He needs to use the Pythagorean theorem to find the missing dimension of the triangle, which is the height of the triangular face.

Practise the Concepts (A)

- Encourage students to refer back to the Examples before asking for assistance.

Apply the Concepts (B)

- Question 7 is a Literacy Connect. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information, visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy.
- To support question 7, consider providing examples of recently purchased items that include the packaging for students to see how items are packaged efficiently.
- Question 8 is a Chapter Problem. In advance, ask students to bring in a pair of goggles. Provide cardstock paper, scissors, and tape for them to make the packaging. Students can try to fit a pair of goggles inside each package to help them answer the questions. Remind students to keep the solution to this question handy to help with the Chapter Problem Wrap-Up.
- Question 11 is an Achievement Check. It can be used as a form of diagnostic or formative assessment, or assigned as a small summative assessment piece. This provides an opportunity for formative or self-assessment, using **BLM 9.1.3 Achievement Check Rubric**.

Achievement Check Answers (page 371)

11. a) A: 56 cm^3 B:227.5 cm^3 C:600 cm^3

b) A: \$3.50/56 cm^3 = \$0.0625/cm^3

B: \$4.75/227.5 cm^3 = \$0.0209/cm^3

C: \$6.50/600 cm^3 = \$0.0108/cm^3

Wedge C is the best buy. It has the least cost per unit of volume.

Extend the Concepts (C)

- Assign the Extend the Concepts questions to students who are not being challenged by questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.
- Consider adapting question 12 into a hands-on activity. Instead of foam, which requires a knife to cut, students could use toothpicks or straws and modelling clay to form the framework for each pyramid using inches instead of feet. Challenge students who are keen to find out if there is a way of forming more than three pyramids within the 4 by 4 by 12-in. prism.