9.4 Volume of Cones and Spheres

Strand
Measurement and Trigonometry

Student Text Pages
391–397

Suggested Timing
80–160 min

Tools
- calculators
- nets of a cylinder and cone (optional)
- rulers
- sand
- scissors
- sheets of clear acetate
- sheets of paper (optional)
- tape

Related Resources
BLM 9.4.1 Practice: Volumes of Cones and Spheres
BLM 9.4.2 Investigate: Compare the Volume of a Cylinder and a Cone
BLM 9.2.2 Formula Sheet
BLM 9.CO.1 Literacy Link: Discussion Chart

Specific Expectations

Solving Problems Involving Surface Area and Volume, Using the Imperial and Metric Systems of Measurement

In this section, students will

MT3.01 use the imperial system when solving measurement problems (e.g., problems involving dimensions of lumber, areas of carpets, and volumes of soil or concrete)

MT3.02 perform everyday conversions between the imperial system and the metric system (e.g., millilitres to cups, centimetres to inches) and within these systems (e.g., cubic metres to cubic centimetres, square feet to square yards), as necessary to solve problems involving measurement

MT3.04 solve problems involving the surface areas of prisms, pyramids, and cylinders, and the volumes of prisms, pyramids, cylinders, cones, and spheres, including problems involving the combinations of these figures, using the metric system or the imperial system, as appropriate

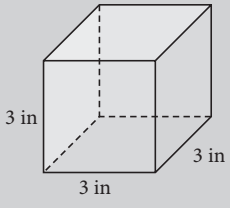
Link to Get Ready

Students will solve problems related to surface area of cones and spheres, including converting measurements. Have students complete questions 1, 2, and 4 of the Get Ready before proceeding with Section 9.4.

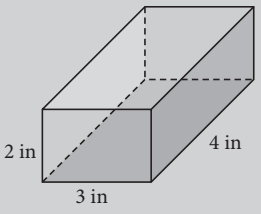
Warm-Up

1. Find the surface area of each shape.

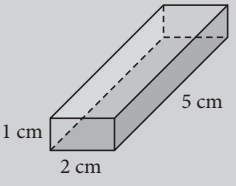
a)



b)



c)



2. Find the volume of each shape in question 1.

3. Find the surface area to volume ratio for each shape in question 1.

4. Order the ratios from question 3 from lowest to highest.

5. Draw a conclusion based on questions 3 and 4.

Warm-Up Answers

1. a) 54 in.^{2} **b)** 52 in.^{2} **c)** 34 cm^2

2. a) 27 in.^{3} **b)** 24 in.^{3} **c)** 10 cm^3

3. a) 2 **b)** 2.17 **c)** 3.4

4. a); b); c)

5. The closer that a shape is to a cube, the lower the ratio of surface area to volume. Therefore, the best shape to use for enclosing material in the shape of a rectangular prism is a cube.

Common Errors

- Some students may confuse or forget the formulas for volume of cones and spheres.

R_x Provide students with **BLM 9.2.2 Formula Sheet** to help them.

- Some students may forget to use cubic units for volume.

R_x Stress that volume is always measured in cubic units.

Ongoing Assessment

- As students work on the Investigate, circulate and observe how well students work with a partner. Consider recording each student's learning skills: group work, work habits, organization, and initiative.
- Use Chapter Problem question 8 to assess students' ability to apply their knowledge of volume of a sphere.

Accommodations

ESL—Pair ESL students with those who have stronger language skills.

Gifted and Enrichment—Challenge students to calculate the height of the cone in D1 using the variables provided.

Refer students to the MathConnect on page 397. Challenge students to research the coffee production worldwide and present their findings in the form of a circle graph.

Memory—Some students may have difficulty processing the steps in multi-step problems and need some coaching. Pair such students with someone who has stronger skills. Remind students to use **BLM 9.2.2 Formula Sheet** for reference.

Motor—For the Investigate, pair students with dexterity problems with those without such difficulties.

Perceptual—Students with perceptual difficulties may have difficulty visualizing cones and spheres. Use a solid cone and sphere to help discuss the Examples.

Visual—For question 7, consider bringing in tennis balls or other spherical objects packaged in a cylinder to help students answer the question.

For question 8, consider modelling a sample container and pair of gloves to help students solve the problem.

Teaching Suggestions

Warm-Up

- Write the Warm-Up questions on the board or on an overhead. Have students complete the questions independently. Then, discuss the solutions as a class. (15–20 min)
- For question 3, explain that the surface area to volume ratio determines the effectiveness of a shape in enclosing an object or material within the shape. The lower the ratio, the more effective a shape is for holding a large volume of material for a small amount of surface.

Section Opener

- Read the Section Opener, and have students brainstorm items that are packaged in cones (e.g., ice-cream cones, party hats, fireworks) and spheres (e.g., snow globes, gumballs). Ask students why they think spheres are used less often than cones for packaging. Prompt them to realize that spheres do not stack well and always move around.

Investigate

- Provide student pairs with **BLM 9.4.2 Investigate: Compare the Volume of a Cylinder and a Cone**, clear acetate, rulers, scissors, and tape. Note that the cylinder and cone have smaller dimensions than the one shown in the student text. Remind students not to tape the edges along the base since they will be filling the solids with sand.
- Before students start, remind them about the Investigate in Section 9.1, and ask them to recall the relationship between the volume of a prism and a pyramid. Ask for any predictions about the relationship between the volume of a cone and a sphere.
- Conclude the Investigate by having students discuss their findings. Students should find that it takes three cones full of sand to fill a sphere with the same base and height.
- As a follow up, have students describe the relationship between the volume of a cone and a sphere using mathematical terms (i.e., 3, $\frac{1}{3}$).
- Use **BLM 9.4.1 Practice: Volume of Cones and Spheres** for extra practice or remediation.

Investigate Answers (page 391)

1. The bases are circular. The faces are the curved side surfaces that taper to a point.

3. The bases and heights of the cylinder and the cone are the same.

4. The cylinder was filled three times.

6. Answers may vary. Look for the idea that a cone has $\frac{1}{3}$ the volume of a cylinder.

Examples

- As a class, work through each of the Examples to ensure that students understand volume of cones and spheres.
- Review the formulas for volume of a cone and a sphere. Remind students how they know that the volume of a cone is $\frac{1}{3}$ the volume of a sphere. (It took three times more sand to fill a sphere than a cone of the same base and height.)
- In Example 3, read the MathConnect about the formula for volume of a sphere using diameter.
- Emphasize the importance of using cubic units.

Key Concepts

- Ensure that students understand the Key Concepts before assigning the Practise the Concepts questions. Have students add the formulas for volume of a cone and a sphere to their Discussion Chart on **BLM 9.CO.1 Literacy Link: Discussion Chart.**

Discuss the Concepts

- Have students work with a partner to complete the questions. In the follow-up discussion, have students recall what they learned about height and slant height in D1 on page 376. Consider having students construct the cone and create and label the sides of the right angle triangle to solve D1.
- For D2, refer students to the MathConnect on page 393 to help answer the question.

Discuss the Concepts Suggested Answers (page 394)

D1. You could use the Pythagorean theorem to solve for height. (The slant height represents the hypotenuse; the radius and height represent the two legs in the triangle.)

D2. The two formulas differ only in one way—one includes diameter, the other includes radius. You would select the formula according to the information you are given.

Practise the Concepts (A)

- Encourage students to refer back to the Examples before asking for assistance.

Apply the Concepts (B)

- Question 7 is a Literacy Connect. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics. This supports general Think Literacy strategies. For more information, visit http://www.edu.gov.on.ca/eng/studentsuccess/thinkliteracy.
- Question 8 is a Chapter Problem. Remind students to keep the solution handy to help with the Chapter Problem Wrap-Up.

Extend the Concepts (C)

- Assign the Extend the Concepts questions to students who are not being challenged by questions in Apply the Concepts.
- Extend the Concepts questions can be used as a diagnostic assessment for those students considering a university-level course in grade 11.
- Questions 10 to 12 present scenarios that work well in hands-on activities. If time permits, consider having students do one or more of these questions hands on.
- Have students read the MathConnect before proceeding with question 10. You may wish to have interested students research on the Internet more about coffee-related social or culturally related practices. Have them report their findings to the class in the form of a brief presentation.